ALM 2

Advanced Lectures in Mathematics

Asymptotic Theory in Probability and Statistics with Applications

Editors: Tze Leung Lai • Lianfen Qian • Qi-Man Shao

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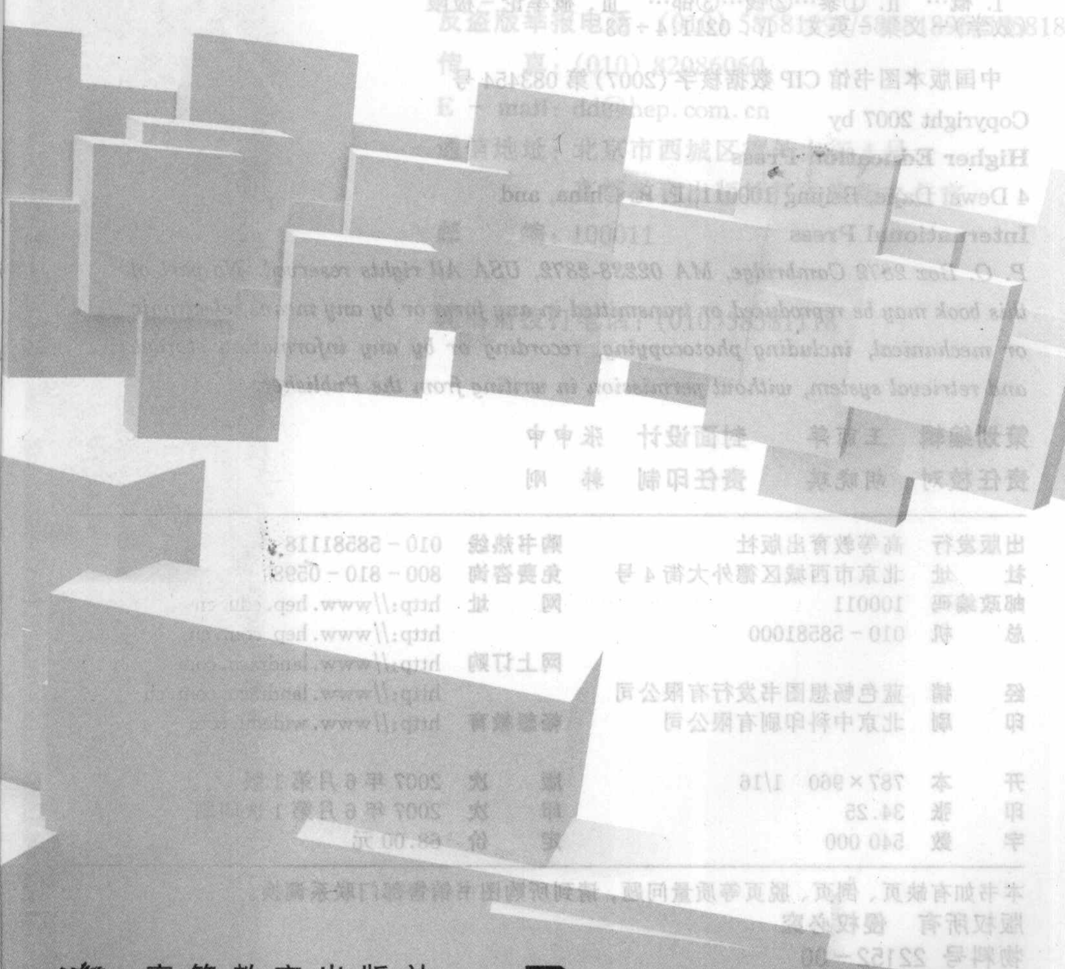

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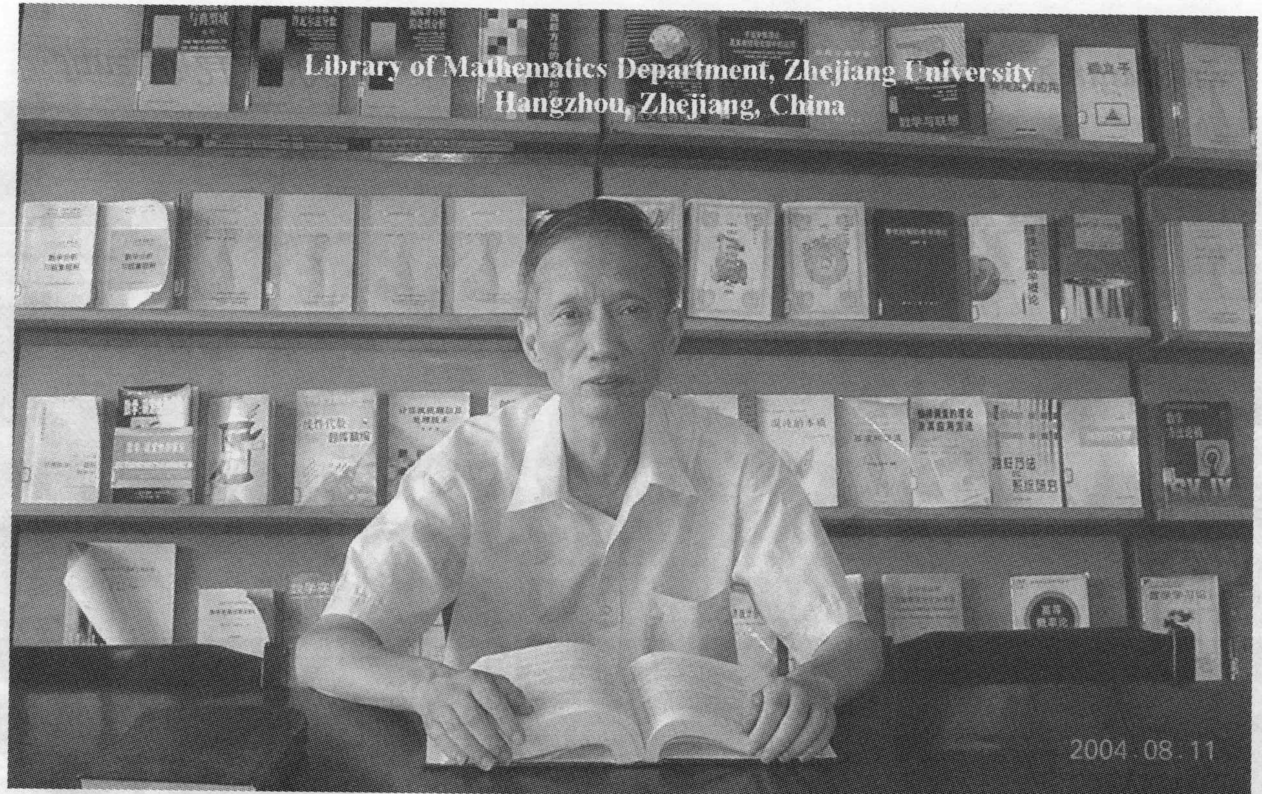
ADVANCED LECTURES IN MATHEMATICS



Professor Zhengyan Lin

June 19-21, 2006, Center of Mathematical Sciences, Zhejiang university

Preface

To celebrate the 65th birthday of Professor Zhengyan Lin, an International Conference on Asymptotic Theory in Probability and Statistics was held at the Center of Mathematical Sciences and Department of Mathematics of Zhejiang University, Hangzhou, China, in the summer of 2006. One of the aims of the conference was to provide a platform for the exchange of new ideas and recent developments in asymptotic theory and applications. Many speakers of the conference were invited to contribute to this volume, which consists of expository papers based on their invited talks or related research areas. All papers were carefully peer refereed. We would like to dedicate this book to Professor Zhengyan Lin and wish him continuing success in many years to come!

Professor Lin is a leading probabilist in China. He has made significant contributions to the development of asymptotic theory, especially, limit theorems for mixing dependent random variables and self-normalized sums, and sample path properties of Gaussian processes. Professor Lin has published over 140 papers and 7 books. He and Professor Chuanrong Lu have supervised over 160 graduate students at Hangzhou University (now merged with Zhejiang University) since 1982.

An objective of the present volume of 18 papers by the invited speakers and contributors to the conference is to introduce graduate students to some active research areas in probability and statistics. Most papers are survey papers so that the present volume can provide readers with a valuable resource in probability, statistics and their applications. Obviously, we cannot cover all of the important topics of current research.

The volume consists of three parts: (I) Limit Theorems, (II) Statistics and Applications, and (III) Mathematical Finance and Insurance.

Part I has 8 papers, focusing on limit theory through various angles. It starts with the probability theory of self-normalization (Lai and Shao), followed by random partitions (Su), adaptive designs (Zhang), Gaussian processes (Wang), Gaussian random fields (Xiao), large deviations theory for two-parameter Gaussian processes (Chen and Csörgő), intersection local times (Chen), and ends with limit theorems for U-statistics (JING) to serve as a link between probability and statistics.

Part II contains 7 papers, exploring various research areas in statistics and its applications. It starts with the classical inverse problem for the t-Statistics (Yang, Fang and Kotz), followed by statistical issues in rounded data (Bai, Zheng, Zhang and Hu). Then it introduces a variety of useful statistical models such as piecewise regression models (Qian), partially linear models (Wang, Liang and Jin) and nonlinear time series models (H. Yu). Finally it reviews recent work on classification and illustrates a probabilistic classifier with environmental and remote sensing applications (J. Yu and Ranneby), mixed linear model approaches for complex trait analysis (Yang and Zhu).

Part III consists of 3 papers. It begins with introducing inference and computation for stochastic volatility models related to option pricing (Ji), and then gives an overview of Choquet integrals and their applications to risk theory (Wang and Yan). The volume ends with a paper by Yang on actuarial science and its recent developments.

We thank all authors for their superb contributions and the referees for their thorough and timely work. Thanks also go to the Center of Mathematical Sciences at Zhejiang University for providing conference facilities and financial support, to the executive editors, S. T. Yau, K. F. Liu and L. Z. Ji, for their interests in publishing the volume for the Advanced Lecture in Mathematics, and to the Chinese National Science Foundation for financial support. Last but not the least, we thank the local organizers for their tremendous contributions to the conference.

T. L. Lai, L. F. Qian and Q. M. Shao
March 2007

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PartI: Limit Theorems

Part I: Limit Theorems

Self-normalized Limit Theorems in Probability and Statistics

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Abstract

The normalizing constants in classical limit theorems are usually sequences of real numbers. Moment conditions or other related assumptions are necessary and sufficient for many classical limit theorems. However, the situation becomes very different when the normalizing constants are sequences of random variables. A self-normalized large deviation shows that no any moment condition is needed for such large deviation type results. A self-normalized law of the iterated logarithm remains valid for all distributions in the domain of attraction of a normal or stable law. This reveals that self-normalization preserves essential properties much better than deterministic normalization does. In this chapter we review some important developments on self-normalized limit theorems in the last decade, especially on self-normalized large deviations, self-normalized saddle point approximations, self-normalized moderate deviations, self-normalized Cramér type large deviations for independent random variables, self-normalized law of the iterated logarithm and increments, self-normalized large and moderate deviations in $\mathbb{R}^d$, large and moderate deviations of Hotelling's T^2 statistic, large and moderate deviations for self-normalized empirical processes; limiting distributions of self-normalized sums, weak invariance principle for self-normalized partial sum processes, Darling-Erdős type theorem, asymptotic distributions of non-central self-normalized sums, the pseudo-maximization approach for self-normalized stochastic processes.

1 Introduction

Let X, $X_1, X_2, \cdots$ be independent and identically distributed random variables. Put

$$S_n = \sum_{i=1}^{n} X_i, \quad V_n^2 = \sum_{i=1}^{n} X_i^2. \tag{1.1}$$

The standardized sum usually means $b_n(S_n - a_n)$, where a_n and b_n are non-random sequences, while self-normalized sum refers to S_n/V_n. It is well-known that moment conditions or other related conditions are necessary and sufficient for many classical limit theorems. For example, the strong law of large numbers holds if and only if the mean of X is finite; the central limit theorem holds if and only if $EX^2I(|X| \leqslant x)$ is slowing varying as $x \to \infty$; and a necessary and sufficient condition for the large deviation is that the moment generating function of X is finite in a neighborhood of zero. On the other hand, limit theorems for self-normalized sums S_n/V_n put a totally new countenance upon the classical limit theorems. In contrast to the well-known Hartman-Wintner law of the iterated logarithm and its converse by Strassen (1966), Griffin and Kuelbs (1989) obtained a self-normalized law of the iterated logarithm for all distributions in the domain of attraction of a normal or stable law. Shao (1997) showed that no moment conditions are needed for a self-normalized large deviation result $P(S_n/V_n \geqslant x\sqrt{n})$ and that the tail probability of S_n/V_n is Gaussian like when X_1 is in the domain of attraction of the normal law and sub-Gaussian like when X is in the domain of attraction of a stable law, while Giné, Götze and Mason (1997) proved that the tails of S_n/V_n are uniformly sub-Gaussian when the sequence is stochastically bounded. Shao (1999) established a Cramér type result for self-normalized sums only under a finite third moment condition. These results strongly show that self-normalized partial sums preserve desirable properties much better than non-randomized partial sums. Self-normalization is also commonly used in statistics. Many statistical inferences require the use of classical limit theorems. However, these classical results often involve some unknown parameters, one needs to first estimate the unknown parameters and then substitute the estimators into the classical limit theorems. This commonly used practice is exactly the self-normalization. A typical case is the Student t-statistic.

The close relationship between the Student t-statistic T_n and the self-normalized sum S_n/V_n can be seen below:

$$T_n := \frac{\bar{X}}{s/\sqrt{n}} = \frac{S_n}{V_n}\Big(\frac{n-1}{n-(S_n/V_n)^2}\Big)^{1/2} \tag{1.2}$$

and

$$\{T_n \geqslant t\} = \Big\{\frac{S_n}{V_n} \geqslant t\Big(\frac{n}{n+t^2-1}\Big)^{1/2}\Big\}, \tag{1.3}$$

where $\bar{X}$ is the sample mean and s is the sample standard deviation.

In this chapter we review some important developments in the self-normalized limit theorems in the last 20 years. We will focus on the following topics:

1. Self-normalized large deviations;
2. Self-normalized saddlepoint approximations;
3. Self-normalized moderate deviations;
4. Cramér type large deviations for independent random variables;
5. Self-normalized law of the iterated logarithm and increments;
6. Self-normalized large and moderate deviations in $\mathbb{R}^d$;
7. Large and moderate deviations of Hotelling's T^2 statistic;
8. Large and moderate deviations for self-normalized empirical processes;
9. Limiting distributions of self-normalized sums;
10. Weak invariance principle for self-normalized partial sum processes;
11. Darling-Erdős type theorem;
12. Asymptotic distributions of non-central self-normalized sums;
13. Self-normalized processes: a pseudo-maximization approach;
14. Applications to statistics.

The review is based on three survey papers by Shao (1998), Shao (2004) and de la Pena, Klass and Lai (2007).

2 Self-normalized large deviations

Let $X, X_1, X_2, \cdots$ be a sequence of independent and identically distributed (i.i.d.) random variables. The classical Cramér-Chernoff large deviation [8] states that if