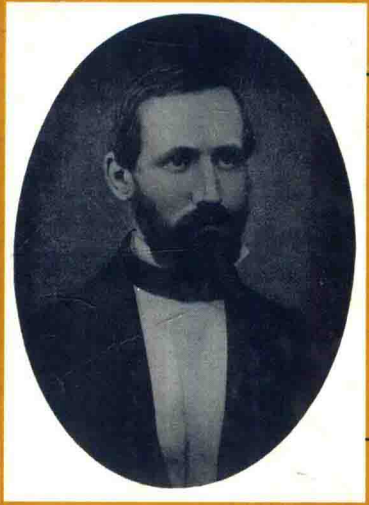
Bernhard Riemann
1826–1866



Turning Points in the Conception of Mathematics

Detlef Laugwitz

Translated by
Abe Shenitzer

Birkhäuser

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With the Editorial Assistance of the Author,
Hardy Grant, and Sarah Shenitzer

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Bernhard Riemann
1826–1866

BERNHARD RIEMANN
(1826–1866)

Preface

It is precisely in the case of Riemann, who has no equals in the intellectual penetration of mathematical problems, that it pays to trace the underlying integrated conception.

(*Gerade bei Riemann, der an gedanklicher Durchdringung mathematischer Probleme nicht Seinesgleichen hat, lohnt es sich, der zugrunde liegenden einheitlichen Konzeption nachzuspüren.*)

—H. Weyl (1925)

The degeneration of mathematics began with the ideas of Riemann, Dedekind, and Cantor, which progressively repressed the reliable genius of Euler, Lagrange, and Gauss.

(*Die Entartung der Mathematik begann mit den Ideen von Riemann, Dedekind, und Cantor, durch die der solide Geist von Euler, Lagrange, und Gauss mehr und mehr zurückgedrängt wurde.*)

—C.L. Siegel (1959)

One of the most profound and imaginative mathematicians of all time, he had a strong inclination to philosophy, indeed, was a great philosopher.

—H. Freudenthal (1975)

No one person is capable of a full analysis of Riemann's work, its history, its development and its influence on current mathematics.

—R. Narasimhan (1990)

The idea for a book on Riemann, to appear in the series *Vita Mathematica*, came from E. Fellmann exactly ten years ago on the occasion of the Leibniz Colloquium in Noordwijkerhout. One evening I discussed with Hans Freudenthal problems of the historiography of mathematics, and he told me with great enthusiasm about the intense joy he experienced when writing biographies, especially the ones of Cauchy and Riemann, which he had contributed to the *Dictionary of Scientific Biography*. The atmosphere of this encounter, marked in equal measure by themes of philosophy and of mathematics and its history, made me think that it was not entirely hopeless to try to write an essay that would be an approximation to Riemann's consistent conceptions, an essay that

would bring together the available materials and prepare the ground for further work. This was the best I was able to strive for. Encouraging assistance came from the accounts of Weyl and Freudenthal as well as from more recent research, especially that of U. Bottazzini, J. Gray, and E. Scholz, and from R. Narasimhan's preface to N.

My objective, as well as my limitations, implied certain restrictions. I left out details of Riemann's mathematics that seemed to me unnecessary for the understanding of his global conception. This means that a mathematician may not find in my account certain parts of Riemann's work that strike him as significant for his own research. The need for selectivity applied as well to biography and to contemporary history. Here the choices were made easier by the fact that Dedekind's biography of Riemann, published in 1876, remains the most important source to this day. It is readily available in N., and so are additional materials bearing primarily on Riemann's schooling. Recent publications have changed next to nothing in the overall picture. Letters by members of the Dedekind family published by W. Scharlau were very helpful, and so too were individual references in the papers of E. Neuenschwander.

Riemann's qualifying papers — his doctoral dissertation of 1851 on complex analysis, his habilitation paper of 1854 on real analysis, and his habilitation lecture devoted to geometry and physics and rich in philosophical allusions — suggested a natural ordering of the contents of the book. I thought it necessary to supplement the brief introductory account of Riemann in his time with a sketch of analysis in the decades prior to his creative period. It is remarkable that the views of mathematicians in the early part of the nineteenth century were so different from our present conceptions that without some familiarity with these views it is hardly possible to understand the turning points in Riemann's conception that I tried to present in the last chapter.

This book will have achieved its purpose if it encourages others to produce further commentaries on Riemann and his work, as well as on the intellectual history of mathematics.

I could not have written the book without manifold assistance. I recall my Göttingen teachers Theodor Kaluza (senior) and Gerhard Lyra, who taught me as a young student to appreciate Riemann's world of thought in the spirit of Hermann Weyl and Richard Courant, and I remember the lectures of Carl Ludwig Siegel, in which he expressed his admiration for Riemann's analytical techniques. During the various stages of writing the book I profited from discussions with, and advice from, my colleagues U. Bottazzini, H. Harborth, W. Luh, E. Neuenschwander, and, in Darmstadt, P. Dintelmann, Dr. L. Schönefuss, and Dr. Th. Walter. The latter also rendered expert assistance during the final stages of editing. In dealing with archival materials I was helped by Dr.

E. Neuenschwander, by Dr. Rohlfing in the manuscript department, and by Dr. U. Hunger in the university archive in Göttingen. My daughter Annette Laugwitz, M.A., in Hamburg, prepared materials on Riemann's home country which contribute to an understanding of his scientific development. I am grateful to the editor, Dr. E. Fellmann, for his constant encouragement and professional advice. He and the publisher were able to surmount unexpected difficulties. Mrs. R. Jaschik patiently retyped the frequently modified manuscript.

I wish to take this opportunity to express my thanks to all persons mentioned in the preface. I will thank those not mentioned here in other ways.

Baltrum/Nordsee, August 1994 DETLEF LAUGWITZ

* * *

I wish to thank Birkhäuser for their willingess to publish an English translation of my book. My warm thanks to Abe Shenitzer and his "team" for a careful and fluent translation. As a result of our fruitful collaboration, a few passages in the translation are clearer than the corresponding passages in the original.

Darmstadt, July 1997 DETLEF LAUGWITZ

Note to the Reader

Some of the most important sources have been collected in the work published by R. Narasimhan in 1990. It is referred to in the sequel as N. followed by a page number (see the bibliography). When reading the present book the reader should, if possible, have ready access to Narasimhan's work. This work includes a reprint of the second edition of Riemann's collected works, published in 1892, with unaltered pagination. The second edition is referred to in the sequel as W. followed by a page number. The first edition of Riemann's collected works, published in 1876, is difficult to come by and is not referred to.

The bibliography includes only works that are frequently referred to. Infrequently mentioned source materials and items that go beyond our coverage of particular topics are quoted in the main text. N. 869–910 contains extensive bibliographies by W. Purkert and E. Neuenschwander. I am grateful to E. Neuenschwander for giving me the opportunity to consult another bibliography he is working on.

As a rule, when quoting texts I have retained the spelling of the original.

As usual, the *Journal für die reine und angewandte Mathematik* is referred to as *Crelle* or *Crelle's Journal.*

DETLEF LAUGWITZ

List of Illustrations

Sources of the illustrations

1–9: Original photographs by Annette Laugwitz

10 and 15–17: Hans-Heinrich Himme, *Stich-haltige Beiträge zur Geschichte der Georgia Augusta in Göttingen*, Göttingen: Vandenhoeck & Ruprecht 1987

12, 14, 29: Prof. Dr. H. Harborth

13: Universitätsbibliotheck Göttingen

20: Euler-Archiv Basel

23, 35: Niedersächsische Staats- und Universitätsbibliotheck Göttingen, department of manuscripts

25: Universitäts-Archiv, Göttingen

All others: Archiv Birkhäuser Verlag

Translator's Remarks

On this book

I don't always agree with the author, but I find him stimulating and enlightening. The book is an intellectual panorama of mathematics from Leibniz to Bourbaki.

On usage

(a) I hope that readers are not baffled by the following sentence:

Already in his dissertation, Riemann introduced a topological invariant.

Try the same thing without using "already" in this way. I think that one needs quite a few words to do the job. (By the way, this is *not* a Yiddishism. "So go already" is.)

(b) I say "what follows is stories" rather than "what follows are stories."

(c) Eric Partridge, author of *Usage and Abusage* (last revised in 1957) prefers "besides other things" and "in addition to other things" to "among other things." I follow Partridge.

German spelling

Some of the German texts go back to the 18th century. The reader who keeps this in mind won't jump to the conclusion that *bey* should be *bei, direct* should be *direkt*, and so on.

Thanks

In addition to the people whose help is acknowledged on the title page, I wish to thank my friend Abe Achtman for calling my attention to a number of linguistic rough spots.

ABE SHENITZER

Contents

0. Introduction

0.1 Bernhard Riemann in his time

0.1.1 His life and the development of his personality

The external circumstances of Riemann's short life can be quickly set down. He was born on 17 September 1826 into the family of the pastor of Breselenz near Dannenberg in the kingdom of Hanover. He attended the gymnasiums in Hanover (1840–1842) and in Lüneburg (1842–1846) and studied at Göttingen (1846–1847, 1849–1851) and at Berlin (1847–1849). He obtained his doctorate in 1851, and habilitated, i.e., took the qualifying examination for lecturing at a university, in 1854, both at Göttingen under Gauss. He became an associate professor (at Göttingen) in 1857 and a full professor in 1859. In June of 1862 he married Elise Koch, a friend of his sisters. Always in poor health, he spent a large part of the years that followed in Italy. His only daughter, Ida, was born there. In 1866, in the first days of the war with Prussia, he again decided to go south. He died on 20 July 1866 in Selasca on Lago Maggiore.

We will try to supplement this summary of the main events of his life so as to form an image of his personality.

To understand his development one must keep in mind that until the age of 14 Riemann lived in the circle of his family in the isolated Hanover Wendland, a thinly settled and undeveloped area on the Elbe river. His father, Friedrich Bernhard Riemann, came from Boizenburg, on the Mecklenburg shore of the Elbe. A few years after Bernhard's birth he took over the parish in Quickborn, not far from Breselenz. Riemann's mother was a daughter of a Hofrat in Hanover. In Section 1.4 of the Introduction we will comment on photographs of his native place.

All his life Riemann found it difficult to associate with people. Both as a student and as a docent he was always attracted by the security and solitude of Quickborn. He lost this anchorage when his father died in 1855.

He seems not to have sought contacts with people. In fact, he seems to have stubbornly discouraged attempts at greater closeness on the part of those — such as Eisenstein in Berlin — who were close to him in spirit. Dedekind, who occasionally managed to get him out of the loneliness of his room, has nothing to report about the concrete contents of scientific contacts. Riemann's scientific thought reflects his introvert makeup. He becomes absorbed in self-