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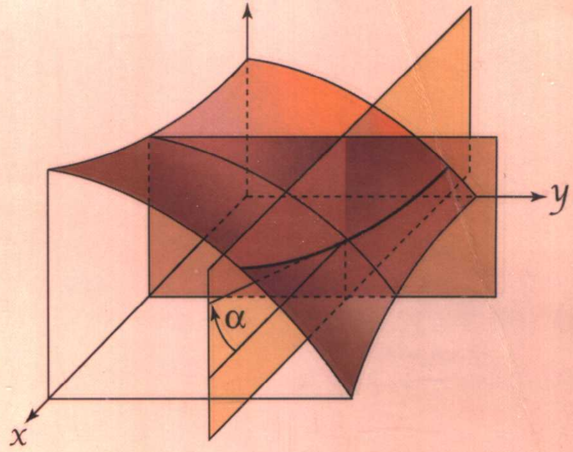
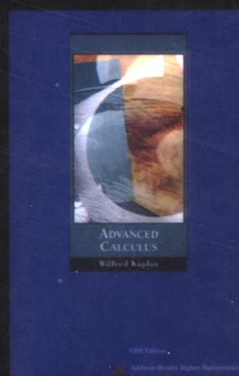


高等微积分学

（第五版）（英文版）

Advanced Calculus

Fifth Edition



[美] Wilfred Kaplan 著

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内容简介

本书全面地介绍了矢量和矩阵、矢量分析以及偏微分方程。本书不仅介绍了理论知识，还涉及到数值方法。全书共分为10章。前两章介绍了线性代数和偏微分。第3章介绍了散度、旋度和一些基本的恒等式，并简要介绍了直角坐标，最后的几节中还介绍了*n*维空间中的张量。其余的章节则分别介绍了积分、无穷级数、解析函数、线性系统以及偏微分方程等。书中的定义都有明确标示，所有的重要结果都作为定理以公式的形式给出。书中提供了大量的习题，并给出了答案。此外，本书还提供了大量的参考文献，并在每章的末尾给出了推荐阅读的书目。

本书的读者应具有大学低年级的微积分学基础。本书适合作为高等微积分学和实解析等课程的研究生或高年级本科生双语教学的教材和课后参考书，也可供有关的研究人员参考。

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Preface to the Fifth Edition

SOME HISTORY

As I recall, it was in 1948 that Mark Morkovin, a colleague in engineering, approached me to suggest that I write a text for engineering students needing to proceed beyond elementary calculus to handle the new applications of mathematics. World War II had indeed created many new demands for mathematical skills in a variety of fields.

Mark was persuasive and I prepared a book of 265 pages, which appeared in lithoprinted form, and it was used as the text for a new course for third-year students. The typesetting was done using a "varityper," a new typewriter that had keys for mathematical symbols.

In the summer of 1949 I left Ann Arbor for a sabbatical year abroad, and we rented our home to a friend and colleague Eric Reissner, who had a visiting appointment at the University of Michigan. Eric was an adviser to a new publisher, Addison-Wesley, and learned about my lithoprinted book when he was asked to teach a course using it. He wrote to me, asking that I consider having it published by Addison-Wesley.

Thus began the course of this book. For the first edition the typesetting was carried out with lead type and I was invited to watch the process. It was impressive to see how the type representing the square root of a function was created by physically cutting away at an appropriate type showing the square root sign and squeezing type for the function into it. How the skilled person carrying this out would have marveled at the computer methods for printing such symbols!

ABOUT THIS EDITION

This edition differs from the previous one in that the chapter on ordinary differential equations included in the third edition but omitted in the fourth edition has been restored as Chapter 9. Thus the present book includes all the material present in the previous editions, with the exception of the introductory review chapter of the first edition.

A number of minor changes have been made throughout, especially some updating of the references.

The purpose of including all the topics is to make the book more useful for reference. Thus it can serve both as text for one or more courses and as a source of information after the courses have been completed.

ABOUT THE BOOK

The background assumed is that usually obtained in the freshman-sophomore calculus sequence. Linear algebra is not assumed to be known but is developed in the first chapter. Subjects discussed include all the topics usually found in texts on advanced calculus. However, there is more than the usual emphasis on applications and on physical motivation. Vectors are introduced at the outset and serve at many points to indicate geometrical and physical significance of mathematical relations.

Numerical methods are touched upon at various points, both because of their practical value and because of the insights they give into the theory. A sound level of rigor is maintained throughout. Definitions are clearly labeled as such and all important results are formulated as theorems. A few of the finer points of real variable theory are treated at the ends of Chapters 2, 4, and 6. A large number of problems (with answers) are distributed throughout the text. These include simple exercises as well as complex ones planned to stimulate critical reading. Some points of the theory are relegated to the problems, with hints given where appropriate. Generous references to the literature are given, and each chapter concludes with a list of books for supplementary reading. Starred sections are less essential in a first course.

TOPICAL SUMMARY

Chapter 1 opens with a review of vectors in space, determinants, and linear equations, and then develops matrix algebra, including Gaussian elimination, and n-dimensional geometry, with stress on linear mappings. The second chapter takes up partial derivatives and develops them with the aid of vectors (gradient, for example) and matrices; partial derivatives are applied to geometry and to maximum-minimum problems. The third chapter introduces divergence and curl and the basic identities; orthogonal coordinates are treated concisely; final sections provide an introduction to tensors in n-dimensional space.

The fourth chapter, on integration, reviews definite and indefinite integrals, using numerical methods to show how the latter can be constructed; multiple integrals are treated carefully, with emphasis on the rule for change of variables; Leibnitz's Rule for differentiating under the integral sign is proved. Improper integrals are also covered; the discussion of these is completed at the end of Chapter 6, where they are

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related to infinite series. Chapter 5 is devoted to line and surface integrals. Although the notions are first presented without vectors, it very soon becomes clear how natural the vector approach is for this subject. Line integrals are used to provide an exceptionally complete treatment of transformation of variables in a double integral. Many physical applications, including potential theory, are given.

Chapter 6 studies infinite series without assumption of previous knowledge. The notions of upper and lower limits are introduced and used sparingly as a simplifying device; with their aid, the theory is given in almost complete form. The usual tests are given: in particular, the root test. With its aid, the treatment of power series is greatly simplified. Uniform convergence is presented with great care and applied to power series. Final sections point out the parallel with improper integrals; in particular, power series are shown to correspond to the Laplace transform.

Chapter 7 is a complete treatment of Fourier series at an elementary level. The first sections give a simple introduction with many examples; the approach is gradually deepened and a convergence theorem is proved. Orthogonal functions are then studied, with the aid of inner product, norm, and vector procedures. A general theorem on complete systems enables one to deduce completeness of the trigonometric system and Legendre polynomials as a corollary. Closing sections cover Bessel functions, Fourier integrals, and generalized functions.

Chapter 8 develops the theory of analytic functions with emphasis on power series, Laurent series and residues, and their applications. It also provides a full treatment of conformal mapping, with many examples and physical applications and extensive discussion of the Dirichlet problem.

Chapter 9 assumes some background in ordinary differential equations. Linear systems are treated with the aid of matrices and applied to vibration problems. Power series methods are treated concisely. A unified procedure is presented to establish existence and uniqueness for general systems and linear systems.

The final chapter, on partial differential equations, lays great stress on the relationship between the problem of forced vibrations of a spring (or a system of springs) and the partial differential equation

$$\rho u_{tt} + h u_t - k^2 \nabla^2 u = F(x, y, z, t).$$

By pursuing this idea vigorously the discussion uncovers the physical meaning of the partial differential equation and makes the mathematical tools used become natural. Numerical methods are also motivated on a physical basis.

Throughout, a number of references are made to the text *Calculus and Linear Algebra* by Wilfred Kaplan and Donald J. Lewis (2 vols., New York, John Wiley & Sons, 1970–1971), cited simply as CLA.

Suggestions on the Use of This Book as the Text for a Course

The chapters are independent of each other in the sense that each can be started with a knowledge of only the simplest notions of the previous ones. The later portions of the chapter may depend on some of the later portions of earlier ones. It is thus possible to construct a course using just the earlier portions of several chapters. The following is an illustration of a plan for a one-semester course, meeting four hours a week: 1.1 to 1.9, 1.14, 1.16, 2.1 to 2.10, 2.12 to 2.18, 3.1 to 3.6, 4.1 to 4.9, 5.1 to 5.13, 6.1 to 6.7, 6.11 to 6.19. If it is desired that one topic be stressed, then the corresponding chapters can be taken up in full detail. For example, Chapters 1, 3, and 5 together provide a very substantial training in vector analysis; Chapters 7 and 10 together contain sufficient material for a one-semester course in partial differential equations; Chapter 8 provides sufficient text for a one-semester course in complex variables.

I express my appreciation to the many colleagues who gave advice and encouragement in the preparation of this book. Professors R. C. F. Bartels, F. E. Hohn, and J. Lehner deserve special thanks and recognition for their thorough criticisms of the first manuscript; a number of improvements were made on the basis of their suggestions. Others whose counsel has been of value are Professors R.V. Churchill, C. L. Dolph, G. E. Hay, M. Morkovin, G. Piranian, G. Y. Rainich, L. L. Rauch, M. O. Reade, E. Rothe, H. Samelson, R. Buchi, A. J. Lohwater, W. Johnson, and Dr. G. Béguin.

For the preparation of the third edition, valuable advice was provided by Professors James R. Arnold, Jr., Douglas Cameron, Ronald Guenther, Joseph Horowitz, and David O. Lomen. Similar help was given by Professors William M. Boothby, Harold Parks, B. K. Sachveva, and M. Z. Nashed for the fourth edition and by Professors D. Burkett, S. Deckelman, L. Geisler, H. Greenwald, R. Lax, B. Shabell and M. Smith for the present edition.

To Addison-Wesley publishers I take this occasion to express my appreciation for their unfailing support over many decades. Warren Blaisdell first represented them, and his energy and zeal did much to get the project under way. Over the years many others carried on the high standards he had set. I mention David Geggis, Stephen Quigley, and Laurie Rosatone as ones whose fine cooperation was typical of that provided by the company.

To my wife I express my deeply felt appreciation for her aid and counsel in every phase of the arduous task and especially for maintaining her supportive role for this edition, even when conditions have been less than ideal.

Wilfred Kaplan
Ann Arbor, Michigan

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1

Vectors and Matrices

1.1 INTRODUCTION

Our main goal in this book is to develop higher-level aspects of the calculus. The calculus deals with functions of one or more variables. The simplest such functions are the linear ones: for example, $y = 2x + 5$ and $z = 4x + 7y + 1$. Normally, one is forced to deal with functions that are not linear. A central idea of the differential calculus is the approximation of a nonlinear function by a linear one. Geometrically, one is approximating a curve or surface or similar object by a tangent line or plane or similar linear object built of straight lines. Through this approximation, questions of the calculus are reduced to ones of the algebra associated with lines and planes—linear algebra.

This first chapter develops linear algebra with these goals in mind. The next four sections of the chapter review vectors in space, determinants, and simultaneous linear equations. The following sections then develop the theory of matrices and some related geometry. A final section shows how the concept of vector can be generalized to the objects of an arbitrary "vector space."

1.2 VECTORS IN SPACE

We assume that mutually perpendicular x, y, and z axes are chosen as in Fig. 1.1, and we assume a common unit of distance along these axes. Then every point P in space has coordinates (x, y, z) with respect to these axes, as in Fig. 1.1. The origin O has coordinates $(0, 0, 0)$.

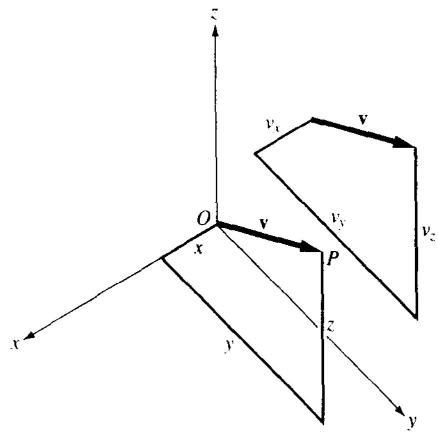

Figure 1.1 Coordinates in space.

A vector **v** in space has a magnitude (length) and direction but no fixed location. We can thus represent **v** by any one of many directed line segments in space, all having the same length and direction (Fig. 1.1). In particular, we can represent **v** by the directed line segment from O to a point P, provided that the direction from O to P is that of **v** and that the distance from O to P equals the length of **v**, as suggested in Fig. 1.1. We write simply

$$\mathbf{v} = \overrightarrow{OP}. \tag{1.1}$$

The figure also shows the components v_x, v_y, v_z of **v** along the axes. When (1.1) holds, we have

$$v_x = x, \qquad v_y = y, \qquad v_z = z. \tag{1.2}$$

We assume the reader's familiarity with addition of vectors and multiplication of vectors by numbers (scalars). With the aid of these operations a general vector **v** can be represented as follows:

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}. \tag{1.3}$$

Here **i**, **j**, **k** are *unit vectors* (vectors of length 1) having the directions of the coordinate axes, as in Fig. 1.2. By the Pythagorean theorem, **v** then has magnitude, denoted by $|\mathbf{v}|$, given by the equation

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}. \tag{1.4}$$

In particular, for $\mathbf{v} = \overrightarrow{OP}$ the distance of P: (x, y, z) from O is

$$|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}. \tag{1.5}$$

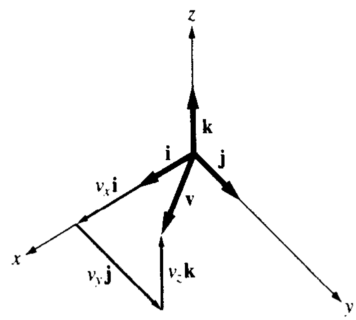

Figure 1.2 Vector **v** in terms of **i**, **j**, **k**.

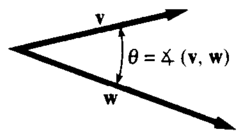

Figure 1.3 Definition of dot product.

More generally, for $\mathbf{v} = \overrightarrow{P_1P_2}$, where P_1 is (x_1, y_1, z_1) and P_2 is (x_2, y_2, z_2), one has $\mathbf{v} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = (x_2 - x_1)\mathbf{i} + \cdots$ and the distance between P_1 and P_2 is

$$d = |\mathbf{v}| = |\overrightarrow{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}. \tag{1.6}$$

The vector **v** can have 0 length, in which case $\mathbf{v} = \overrightarrow{OP}$ only when P coincides with O. We then write

$$\mathbf{v} = \mathbf{0} \tag{1.7}$$

and call **v** the zero vector.

The vector **v** is completely specified by its components v_x, v_y, v_z. It is often convenient to write

$$\mathbf{v} = (v_x, v_y, v_z) \tag{1.8}$$

instead of Eq. (1.3). Thus we think of a vector in space as an *ordered triple of numbers*. Later we shall consider such triples as matrices (row vectors or column vectors).

The *dot product* (or *inner product*) of two vectors **v**, **w** in space is the number

$$\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}|\,|\mathbf{w}| \cos\theta, \tag{1.9}$$

where $\theta = \sphericalangle(\mathbf{v}, \mathbf{w})$, chosen between 0 and π inclusive (see Fig. 1.3). When **v** or **w** is **0**, the angle θ is indeterminate, and $\mathbf{v} \cdot \mathbf{w}$ is taken to be 0. We also have $\mathbf{v} \cdot \mathbf{w} = 0$ when **v**, **w** are *orthogonal* (perpendicular) vectors, $\mathbf{v} \perp \mathbf{w}$. We agree to say that the **0** vector is orthogonal to all vectors (and parallel to all vectors). With this convention

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