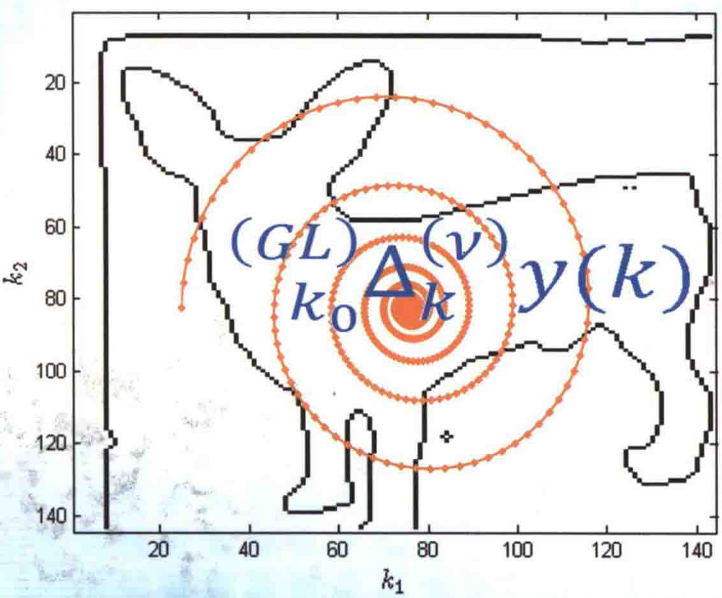
Discrete Fractional Calculus

Applications in Control and Image Processing

Series in Computer Vision - Vol. 4



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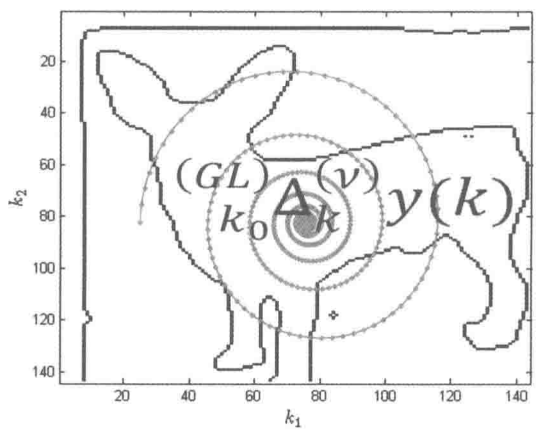

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by Piotr Ostalczyk

Rien n'est parfait.

Antoine de Saint-Exupéry, 'Le Petit Prince'

Preface

The statement of professor Katsuyuki Nishimoto expressed in his book entitled "An Essence of Nishimoto's Fractional Calculus, Calculus of the 21st Century: Integrations and Differentiations of Arbitrary Order" [Nishimoto (1991b)] dated 1991 appeard as the prohecy of the end of XX century. This apt opinion confirms the Science Watch of Thomson Reuters stating in October, 2009 that the fractional calculus nowadays is an 'Emerging Research Front'.

Since the mid-twentieth century, the fractional calculus consequently has been introduced at 'the mathematical and technical scientific fora' of the world by scientists out of whom there should be mentioned Oldham, K. B., Jerome Spannier [Oldham and Spannier (1974)], Katsuiuki Nishimoto [Nishimoto (1984, 1986, 1989)], Stefan Samko, Anatoly A. Kilbas and Oleg Marichev [Samko *at al.* (1986)], Kenneth S. Miller and Bertram Ross [Miller and Ross (1993)]. It is worth noting the great contribution of professors: Alain Oustaloup [Oustaloup (1983, 1991, 1995)], Virginia Kiryakova [Kiryakova (1994)], Igor Podlubny [Podlubny (1999b)] and their teams contributing to the development of the fractional calculus.

According to professor Katsuyuki Nishimoto's forecast the beginning of twenty-first century brought a rapid growth of works devoted to the theoretical and applied fractional calculus. Among many important monographs one should mention the works of professors: Shantanu Das [Das (2009); Das and Pan (2012)], Tenreiro Machado, Ivo Petráš [Petráš (2011)], YangQuan Chen [Monje *et al.* (2010)], Dumitru Baleanu [Baleanu *et al.* (2011)] and Alain Oustaloup [Oustaloup (2014)].

The precise historical review is contained in the articles by Professors: J. Tenreiro Machado, Virginia Kiryakova, Francesco Mainardi [Machado *at al.* (2011)] and J. Tenreiro Machado, Alexandra M. Galhado and Juan J. Truijllo [Machado *at al.* (2013)].

Now, also in Poland, the fractional calculus is the subject of research in many scientific centers, among which one should list the following professors and their teams: Małgorzata Klimek at Częstochowa University of Technology (Częstochowa) [Klimek (2009)], Tadeusz Kaczorek [Kaczorek (2011)] and recently deceased Professor Mikołaj Busłowicz at Bialystok University of Technology, Jerzy Klamka at Silesian University of Technology (Gliwice), Andrzej Dzieliński at Warsaw

University of Technology, Wojciech Mitkowski at The AGH University of Science and Technology (Kraków), Stefan Domek [Domek (2013)] at West Pomeranian University of Technology (Szczecin), Krzysztof Latawiec and Rafał Stanisławski at Opole University of Technology.

At this point the Author would like to apologise for not menitoning the remaining prominent scientists working on the fractional calculus developement and applications in different scientific areas.

The fractional calculus originally concerned continuous-variable functions. Such functions describing the so-called analog signals of real world are continuous functions of the temporal variable t. One of the first fractional-order derivatives application appeared in the anomalous diffusion models. In the early sixties of the last century there was proposed the fractional model of the ultra-capacitor. In mechanics, the viscoelasticity phenomenon [Freed *et al.* (2002); Meral *et al.* (2010)] particularly accurately describes mathematical models based on the fractional calculus.

The inerposition of the digital computers to signal processing which can deal with immense quantities of information expressed by numbers, not signals, forced a conversion of the analog signal to a sequence of samples expressed as a set of digital words. This sampling process is usually performed in a digital-to-analog converter (under the well-known restriction expressed by the Shannon theorem). Therefore, one establishes a relation between the continuous variable function and its discrete-variable counterpart. In a discrete version of the fractional calculus the continuous-variable functions are substituted by discrete-variable ones, the fractional-order derivatives are replaced by fractional-order differences and the fractional-order integerals by fractional-order sums. One should admit, that operating on fractional-order differences and sums is more complicated in comparison with the integer-order case. The complications are related to longer signal processing time and larger computer memory needed. The huge development of computers, converters and the memory size compensates for this inconvenience.

This book presents selected applications of the discrete fractional calculus in the discrete system control theory and discrete image processing. In the discrete system identification, analysis and synthesis one can consider integer or fractional models. Usually fractional models are more simple, with lower number of parameters including the fractional orders. In the closed-loop system analysis and synthesis one can consider fractional order controllers or compensators with the integer order plants or more general case where both the regulator and the controlled plant are described by the fractional-order difference equations. The classical problems in the control theory ranges from transient and frequency response analysis of dynamic elements and systems, fractional-oredr PID controllers, fractional-order systems' stability, fractional-order systems' robustness, the optimal control of the fractional-order systems including the variational problem. The classical digital filters can be generalised to the fractional-order case, where backward differences are generalised to the fractional-order case. Also in the digital image processing one can successfully generalise classical methods to the fractional ones. Here, one should mention early works of the CRONE team at the University of Bordeaux on image edge detection.

The monograph is organised as follows. Basing upon the discrete calculus given in Chapters 1 and 2 fundamental properties of the fractional-order backward-diference and sum are presented. Althought as a fundametal form of the fractional-order backward difference/sum the Grünwald-Letnikov one is used, its five equivalents: Horner, Riemann-Liouville, Caputo, Polynomial-Like Matrix and Laguerre-based are introduced in Chapter 3. The simple graphical interpretation of the Grünwald-Letnikov form is proposed in Chapter 4. Chapter 5 contains the fundamental proprties of the fractional-order backward difference/sum. In the equations only FO backward differences without forward shifts are considered. An application of the discrete fractional calculus starts in Chapter 6 where the non-linear fractional-order dynamical system linearisation procedure is presented. The linearised system descriptions by: the fractional-order difference equation, the state-space equations, discrete fractional transfer function and polynoimial like matrix are given. The linear time-invariant FOE solution $y(k)$ at the discrete time instant k depends only on antecedent solution values $y(k-1), y(k-2), \ldots$. This creates the problem of initial conditions denoted as $y_{k_0-1}, y_{k_0-2}, \ldots$. No forward shift is addmitted. For the four equivalent forms mentioned above homogenous and forced solutions are studied in Chapter 7. Numerous transient responses are presented. This chapter contains also the FOS frequency characterisitcs analysis represented by the discrete Nyquist plots, amplitude and phase discrete characterisitcs and related discrete Bode plots. The FOS reachability and observability supplement the FOS dynamic properties analysis. The FOS stability tests based on the frequency characteristics finish the selected applications of the discrete fractional calculus in the linear discrete system analysis. In Chapter 8 fundamental dynamical elements are analysed. They follow those known in the clasical linear discrete-time: summator, differentiator, lag and oscillation ones. The connections considered in Chapter 9: parallel, serial and with the negative feedback enable building more complicated dynamical strucutres. The closed-loop system structure with the fractional order PID controllers is considered in Chapter 10. Three last chapters presents applications of the discrete fractional calculus in image processing. Considering an image as two variable discrete functions, in Chapters 11, 12, 13 one applies fractional order backward difference and sum to build an image of the fractional potential, detect an image edge and to filter an image.

The book is dedicated to students and engineers working on automatic control, dynamic system identification and image processing. Like in any monograph, there are inevitable errors. Therefore the Author encourages all readers to send any information about them and suggestions improving the topics dealt with in the book.

Here, it's a honour to thank everyone without whom this book would not have appeared. First of all, the Author is very grateful to Professor Chi Hau Chen at the University of Massachssetts Dartmouth (Dartmouth, Massachussetts, USA) for enabling the results presentation the Author have gained for over twenty years of work.

Next, the Author would like to thank Professor Dominik Sankowski, the head of The Institute of Applied Computer Science, Lodz University of Technology for his permanent spiritual encouragement in the Author's research activities and building extremely convenient conditions for scientific activities and especially for this monograph's preparing.

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Piotr Władysław Ostalczyk
Piotrków Tryb., 2015

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