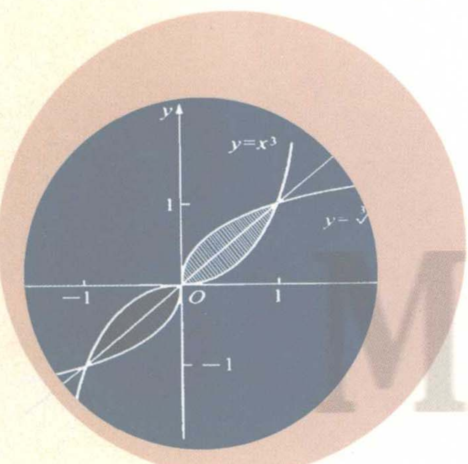
全国高等农林院校“十一五”规划教材

高等数学 英文版

ADVANCED MATHEMATICS

梁保松　叶耀军　主编



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Abstract

This text, the fruit of teaching revolution item of higher education in Henan Province, is completed in English edition according to our country's curriculum system and education nowadays. The main contents conclude: Limits and Continuity of Functions, Derivatives and Differentials, Mean Value Theorem of Differentials and Applications of Derivatives, Indefinite Integrals, Definite Integrals, Differential Calculus of Functions of Several Variables, Double Integrals, Infinite Series, Differential Equations and Difference Equations; also it includes some useful applications in biology, management of the economy and social science. This text is abundant in contents; it pays more attention to stuff education and students' innovation ability as well as mathematics application and develops the modeling ideas in utilizing mathematics.

As to this text, the structure is precise, the logic is quite clear, and the state is in detail, all of which are easy to understand. Besides, it can be used as a teaching material for higher education in fields like agriculture, forestry, biology, economics and finance; it also can be a good reference to all kinds of technicians.

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Preface

This text is the fruit of teaching revolution item of higher education in Henan Province.

At present age, the internationalization of higher education is the main trend of education development . Certainly, one of our primary goal in China's higher teaching revolution is to foster and train more and more highly special talents with international eyes, awareness and association abilities; while, the bilingual teaching is just a very good and effective way in realizing such a big goal.

In light of a document named "Suggestions on strengthening the higher undergraduate education and enhancing the quality in teaching", the ministry of education in 2001 proposed precisely that it is quite necessary in every common and specialized courses to provide conditions for undergraduate education in using English as a tool ; further, it strove in three years to make the curriculum in foreign language covering 5%- 10% among all courses being given. In addition, the evaluating scheme for general higher undergraduate education issued by the ministry of education in 2002 bring the bilingual teaching an important check index into the assessment index system, requiring the hour for the bilingual teaching taking up more than 50% of a special course. Then , from the year 2002, the bilingual teaching becomes one of the hottest topics in higher teaching revolution. Therefore, many universities propose policies encouraging teachers to study foreign teaching materials; accordingly, a new trying and exploration develop quickly in light of the practical course.

In 2003, we've already made some exploration and try on bilingual teaching, which created an important factor, the environment for applying English, but not merely learn it, also, such practice is accepted by many students, from which we feel deep that teachers in English well are fundamental to the bilingual teaching; besides, to the foreign materials, only those contents satisfying our country's situation can be a crucial factor and would be more helpful.

Nowadays, the foreign texts introduced generally have common problems as follows:

1. The price is too high;
2. Many cases provided break away from China's social and life environment;
3. The contents and curriculum systems are quite different from that of our country.

Therefore, the problems referred above always impede the development of our country's bilingual teaching. In effect; only after making native transformation can the foreign teaching materials be applied much more effectively. Thus, in terms of the fostering characteristics for universities in our country combining with nowadays' curriculum contents and systems, we complete this "Higher Mathematics" in English edition after making some digestion, assimilation and recreation to the foreign text.

Accepting sole responsibility for the final manuscript, I would like to thank the following editors for their plunging: Yaojun Ye, Dang Luo, Dianli Cao, Zhen Chen, Xianmin Hou, Lina Ji, Jianping Wang, Rui Wang, Keqin Su, Yawie Wang.

In publishing, I warmly thank the vice-headmaster Professor Baoan Cui, Henan Agricultural University, Mingzeng Yu, institution of higher education and Xiaoying Hu, for their highly support and help.

This edition of the Higher Mathematics in English edition is a big try and exploration, some mistakes may not be avoided despite of the most carefully arrangement and revision, so it will be our great pleasure to get criticism and comments from all experts, coteries and readers.

Baosong Liang

Feb. 2008

Contents

Chapter 1 Limits and Continuity of Functions

Functions which build a correspondence play crucial roles in higher mathematics, for which our main tool is the limit. Since that the studying on mathematics requires the very basic ideas of functions, their properties and graphs. Therefore, our book begins with the definition of functions, the limit and the continuity.

1.1 Concept for Functions

From elementary mathematics, we have already known something on the functions, here, we make further summaries and give some supplements.

1.1.1 Definition

Definition 1 In **R**, **a function** f from a nonempty subset D to E is a rule that assigns a unique element y in E to each element x in D, that is, for any $x \in D$, there is a unique value $y \in E$ corresponding to x in terms of a certain rule f, and we say y a **function** of x, denoted by $y=f(x)$. The set D is the **domain** of the function f, x is called **independent variable**, and y is **dependent variable.**

If x takes on $x_0 \in D$, then the value correspondingly is called a **functional value** of $y=f(x)$ at x_0, denoted $f(x_0)$. Thus, if only x chooses numbers throughout the domain D, then, all functional values correspondingly form a set:

$$W = \{y \mid y = f(x), \ x \in D\},$$

which is called to be the **range.**

Through the definition, we know a function is merely determined by its domain D and the rule f, so any letter is possible in the expression, and then we get:

The two equal functions have the same domain and corresponding rule.

For example, the functions $f(x)=\sqrt[3]{x^4-x^3}$ and $g(x)=x\sqrt[3]{x-1}$ have the same domain $(-\infty, +\infty)$ and corresponding rule, so we say they are equal. While, $f(x)=x$ and $g(x)=\frac{x^2-x}{x-1}$ do not equal, for the domain of $f(x)$ is $(-\infty, +\infty)$ and that of $g(x)$ is $(-\infty, 1) \cup (1, +\infty)$. In addition, $f(x)=1+x^2$ and $g(t)=1+t^2$ are also express the same function.

From definition 1, we know for every $x \in D$, the points $(x, f(x))$ in the plane with coordinates pairs of function $f(x)$ forms the function's **graph**. See the following example.

Example 1 As to any real number x, $[x]$ shows the greatest integer that less than x, and the corresponding graph is continuous line segments provided that x chooses successive integers.

For instance, $[\sqrt{2}]=1$, $[-\pi]=-4$, $[\pi]=3$, $[0]=0$, the function $f(x)=[x]$ is called **integer function.** Easy to see it extends continuously on $(-\infty, +\infty)$, and the range is a set of integers (Figure 1. 1)

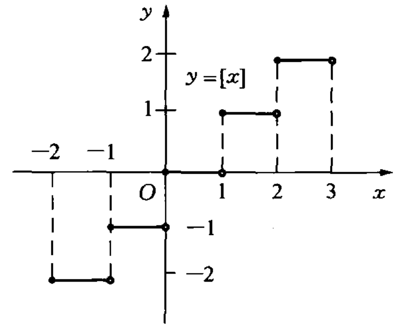

Figure 1. 1

1. 1. 2 Piecewise Functions

Functions involving more than one rule, as in the following examples, are called **piecewise functions.** Graphing such kinds of functions involves making each rule over the appropriate portion of the domain.

Example 2 The **absolute - value function**

The function

$$y=f(x)=|x|=\begin{cases} x, & x\geqslant 0 \\ -x, & x<0 \end{cases},$$

with the domain $D=(-\infty, +\infty)$ and the range $W=[0, +\infty)$ assigning each real number x to a nonnegative number $|x|$ and its figure is symmetric with the y - axis(Figure 1. 2).

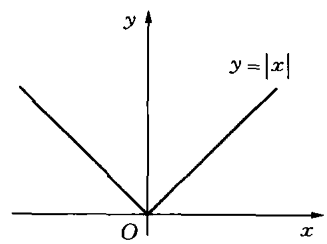

Figure 1. 2

Example 3 The **piecewise function**

The function

$$y=f(x)=\begin{cases} -1+x^2, & x<0 \\ 0, & x=0 \\ 1+x^2, & x>0 \end{cases}$$

whose domain is $D=(-\infty, +\infty)$ and the range is $W=(-1, +\infty)$ (Figure 1. 3) .

What's common for functions in Example 2 and Example 3 is that the corresponding rules are distinct in different ranges, such functions we call **piecewise functions.**

Example 4 The function

$$y=f(x)=\begin{cases} 2\sqrt{x}, & 0\leqslant x\leqslant 1 \\ 1+x, & x>1 \end{cases}$$

is a piecewise function, whose domain is $D=[0, +\infty)$. From which we see the corresponding rule is determined by the range independent variable lies, which is just a standard for evaluating the corresponding value to the function. Such as for $\frac{1}{2}\in$

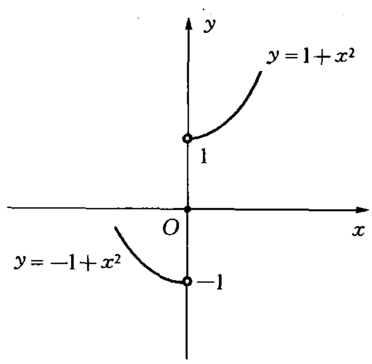

Figure 1. 3

$[0, 1]$, we have $f\left(\frac{1}{2}\right)=2\sqrt{\frac{1}{2}}=\sqrt{2}$ and $f(4)=1+4=5$ because of $4\in(1, +\infty)$.

1.1.3 The Compositions

Definition 2 Suppose that E and D, the domain of $y=f(u)$ and $u=\varphi(x)$, are subsets of $\mathbf{R}$, and the range of the latter is W, if $W\cap E\neq\emptyset$, then, the function $y=f[\varphi(x)]$ is a **composite** of $y=f(u)$ and $u=\varphi(x)$ in the order of first g, then f, denoted $f\circ g$, where u is an **intermediate variable.**

With the composite function, we can either obtain a new function from several or decompose a function into finite numbers.

Example 5 Let $y=f(u)=u^2$, $u=\varphi(x)=1-x^2$, by which we obtain the composite function is

$$y=f[\varphi(x)]=(1-x^2)^2 \quad (-\infty<x<+\infty).$$

Example 6 Let $y=f(u)=\sqrt{u}$, $u=\varphi(x)=1-x^2$, then the composite function is

$$y=f[\varphi(x)]=\sqrt{1-x^2}\ (-1\leqslant x\leqslant 1).$$

Notice that though the domain of $u=\varphi(x)=1-x^2$ is $(-\infty, +\infty)$, to guarantee that the composite function is well defined, we let $u\geqslant 0$, accordingly, the domain of the composite function is $[-1, 1]$.

Example 7 Let $y=f(u)=\arcsin u$, $u=\varphi(x)=3+x^2$, then, we have $u\geqslant 3$ regardless of how to choose the value x, therefore, the range of $u=\varphi(x)$ is $W=[3, +\infty)$, and the domain of $y=f(u)$ is $E=[-1, 1]$, that is $W\cap E=\emptyset$, then, $y=f[\varphi(x)]$ does not be defined actually.

Remark Not any two functions can produce a composite function, and moreover, the domain of the composite may differ from that of the two given functions.

Generally, the composite functions $f[g(x)]$ and $g[f(x)]$ are not equal, see as follows:

Example 8 Let $f(x)=x^2$, $g(x)=3x$, then find $f[g(x)]$ and $g[f(x)]$.

Solution

$$f[g(x)]=f(3x)=(3x)^2=9x^2,\ g[f(x)]=g(x^2)=3x^2.$$

In particular, we may take the composition of three or more functions. For instance, the composite function $f\circ g\circ h$ is founded by first h, then g, and finally f as $(f\circ g\circ h)(x)=f(g(h(x)))$.

Find $f\circ g\circ h$ where $h(x)=\sqrt{x}$, $g(x)=\cot x$ and $f(x)=1+x^2$.

By definition,

$$(f\circ g\circ h)(x)=f(g(h(x)))=f(g(\sqrt{x}))=f(\cot\sqrt{x})=1+(\cot\sqrt{x})^2.$$

So far, applying the composition, we build complicated functions with simpler ones. As to the composite functions, it is quite helpful to be decomposed into simpler functions.

For instance, given $h(x)=\sqrt{1+x^2}$, then find functions f and g satisfying $h=f\circ g$.