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DISCRETE MATHEMATICS

FIFTH EDITION

离散数学

（第5版）

Kenneth A. Ross
Charles R. B. Wright 著

Pearson Education

清华大学出版社

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Preface to the Fifth Edition

In writing this book we have had in mind both computer science students and mathematics majors. We have aimed to make our account simple enough that these students can learn it and complete enough that they won't have to learn it again.

The most visible changes in this edition are the 274 new supplementary exercises and the new chapters on probability and on algebraic structures. The supplementary exercises, which have complete answers in the back of the book, ask more than 700 separate questions. Together with the many end-of-section exercises and the examples throughout the text, these exercises let students practice using the material they are studying.

One of our main goals is the development of mathematical maturity. Our presentation starts with an intuitive approach that becomes more and more rigorous as the students' appreciation for proofs and their skill at building them increase.

Our account is careful but informal. As we go along, we illustrate the way mathematicians attack problems, and we show the power of an abstract approach. We and our colleagues at Oregon have used this material successfully for many years to teach students who have a standard precalculus background, and we have found that by the end of two quarters they are ready for upperclass work in both computer science and mathematics. The math majors have been introduced to the mathematics culture, and the computer science students have been equipped to look at their subject from both mathematical and operational perspectives.

Every effort has been made to avoid duplicating the content of mainstream computer science courses, but we are aware that most of our readers will be coming in contact with some of the same material in their other classes, and we have tried to provide them with a clear, *mathematical* view of it. An example of our approach can be seen first in Chapter 4, where we give a careful account of `while` loops. We base our discussion of mathematical induction on these loops, and also, in Chapter 4 and subsequently, show how to use them to design and verify a number of algorithms. We have deliberately stopped short of looking at implementation details for our algorithms, but we have provided most of them with time complexity analyses. We hope in this way to develop in the reader the habit of automatically considering the running time of any algorithm. In addition, our analyses illustrate the use of some of the basic tools we have been developing for estimating efficiency.

The overall outline of the book is essentially that of the fourth edition, with the addition of two new chapters and a large number of supplementary exercises. The first four chapters contain what we regard as the core material of any serious discrete mathematics course. These topics can readily be covered in a quarter. A semester course can add combinatorics and some probability or can pick up graphs, trees, and recursive algorithms.

We have retained some of the special features of previous editions, such as the development of mathematical induction from a study of `while` loop invariants, but we have also looked for opportunities to improve the presentation, sometimes by changing notation. We have gone through the book section by section looking for ways to provide more motivation, with the result that many sections now begin where they used to end, in the sense that the punch lines now appear first as questions or goals that get resolved by the end of the section.

We have added another "Office Hours" section at the end of Chapter 1, this one emphasizing the importance of learning definitions and notation. These sections, which we introduced in the fourth edition, allow us to step back a bit from our role as text authors to address the kinds of questions that our own students have asked. They give us a chance to suggest how to study the material and focus on what's important. You may want to reinforce our words, or you may want to take issue with them when you talk with your own students. In any case, the Office Hours provide an alternative channel for us to talk with our readers without being formal, and perhaps they will help your students open up with their own questions in class or in the office.

We have always believed that students at this level learn best from examples, so we have added examples to the large number already present and have revised others, all to encourage students to read the book. Our examples are designed to accompany and illustrate the mathematical ideas as we develop them. They let the instructor spend time on selected topics in class and assign reading to fill out the presentation. Operating in this way, we have found that we can normally cover a section a day in class. The instructor's manual, available from Prentice Hall, indicates which sections might take longer and contains a number of suggestions for emphasis and pedagogy, as well as complete answers to all end-of-section exercises.

The end-of-chapter supplementary questions, which are a new feature of this edition, are designed to give students practice at thinking about the material. We see these exercises as representative of the sorts of questions students should be able to answer after studying a chapter. We have deliberately not arranged them in order of difficulty, and we have deliberately also not keyed them to sections—indeed, many of the exercises bring together material from several sections. To see what we mean, look at the supplementary exercises for Chapter 5, on combinatorics, where we have included an especially large number of problems, many of which have a variety of essentially different parts. A few of the supplementary questions, such as the ones in Chapter 12 on algorithms to solve the Chinese Remainder and Polynomial Interpolation problems, also extend the text account in directions that would have interrupted the flow of ideas if included in the text itself. Some of the questions are very easy and some are harder, but none of them are meant to be unusually difficult. In any case, we have provided complete answers to all of them, not just the odd-numbered ones, in the back of the book, where students can use them to check their understanding and to review for exams.

The new chapters on probability and algebraic structures respond to requests from current and past users who were disappointed that we had dropped these topics in going from the third edition to the fourth. Since those were two of our favorite chapters, we were happy to reinstate them and we have taken this opportunity to completely revise each of them. In Chapter 9 we now work in the setting of discrete probability, with only tantalizing, brief allusions to continuous probability, most notably in the transition to normal distributions from binomial distributions. The material on semigroups, rings, and fields in Chapter 12 is not changed much from the account in the third edition, but the discussion of groups is dramatically different. The emphasis is still on how groups act on sets, but in the context of solving some intriguing combinatoric problems we can develop basic abstract ideas of permutation group theory without getting bogged down in the details of cycle notation. As another response to reader feedback, we have moved the section on matrix multiplication from Chapter 3 to Chapter 11, which is the first place we need it.

Naturally, we think this edition is a substantial improvement and worth all of the effort it has taken. We hope you will agree. We welcome any comments and of course especially welcome reports of errors or misprints that we can correct in subsequent printings.

Supplements

The Instructor's Resource Manual, which course instructors may obtain gratis from Prentice Hall, contains complete answers to all exercises in the text. In addition, Prentice Hall publishes inexpensive student workbooks of practice problems on discrete mathematics, with full solutions to all exercises. The Prentice Hall Companion Web site for this text contains information about such materials.

Acknowledgments

This is a better book because of the many useful suggestions from our colleagues and correspondents Patrick Brewer, William Kantor, Richard M. Koch, Eugene Luks, George Matthews, Christopher Phillips, and Brad Shelton. Dick Koch's gentle questions and suggestions, based on his incisive ideas and point of view, were especially helpful. We also benefitted from suggestions provided by the following reviewers and other anonymous reviewers: Dr. Johannes Hattingh, Georgia State University; Bharti Temkin, Texas Tech; Marjorie Hicks, Georgia State University; and Timothy Ford, Florida Atlantic University.

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To the Student Especially

You may find this course the hardest mathematics class you have ever taken, at least in part because you won't be able to look through the book to find examples just like your homework problems. That's the way it is with this subject, and you'll need to use the book in a way that you may not be used to. You'll have to read it. [In computer science there's an old saying, usually abbreviated as RTFM, which stands for "read the friendly manual."] When you do read the book, you'll find that some places seem harder than others. We've done our best to write clearly, but sometimes what we think is clear may not be so obvious to you. In many cases, if you don't find a passage obvious or clear, you are probably making the situation too complicated or reading something unintended into the text. Take a break; then back up and read the material again. Similarly, the examples are meant to be helpful. In fact, in this edition we have made a special effort to put even more examples early in each section to help you see where the discussion is leading. If you are pretty sure you know the ideas involved, but an example seems much too hard, skip over it on first reading and then come back later. If you aren't very sure of the ideas, though, take a more careful reading.

Exercises are an important part of the book. They give you a chance to check your understanding and to practice thinking and writing clearly and mathematically. As the book goes on, more and more exercises ask you for proofs. We use the word "show" most commonly when a calculation is enough of an answer and "prove" to indicate that some reasoning is called for. "Prove" means "give a convincing argument or discussion to show why the assertion is true." What you write should be convincing to an instructor, to a fellow student, and to yourself the next day. Proofs should include words and sentences, not just computations, so that the reader can follow your thought processes. Use the proofs in the book as models, especially at first. The discussion of logical proofs in Chapter 2 will also help. Perfecting the ability to write a "good" proof is like perfecting the ability to write a "good" essay or give a "good" oral presentation. Writing a good proof is a lot like writing a good computer program. Using words either too loosely or extensively (when in doubt, just write) leads to a very bad computer program and a wrong or poor proof. All this takes practice and plenty of patience. Don't be discouraged when one of your proofs fails to convince an expert (say a teacher or a grader). Instead, try to see what failed to be convincing.

Now here's some practical advice, useful in general, but particularly for this course. The point of the homework is to help you learn by giving you practice thinking correctly. To get the most out of it, keep a homework notebook. It'll help you review and will also help you to organize your work. When you get a problem wrong, rework it in your notebook. When you get a problem right, ask yourself what your method was and why it worked. Constantly retracing your own successes will help to embed correct connections in the brain so that when you need them the right responses will pop out.

Read ahead. Look over the material before class to get some idea of what's coming and to locate the hard spots. Then, when the discussion gets to the tough points, you can ask for clarification, confident that you're not wasting class time on things that would be obvious after you read the book. If you're prepared, you can take advantage of your instructor's help and save yourself a lot of struggling. After class, rewrite your class notes while they're still fresh in your mind.

At strategic places in the book, we have inserted very short "Office Hours" sections with the kinds of questions our own students have asked us. These informal sections address questions about how to study the material and what's important to get out of it. We hope they will lead to your own questions that you may want to raise in class or with your instructor.

Study for tests, even if you never did in high school. Prepare review sheets and go over them with classmates. Try to guess what will be on the exams. That's one of the best ways to think about what the most important points have been in the course. Each chapter ends with a list of the main points it covers and with some suggestions for how to use the list for review. One of the best ways to learn material that you plan to use again is to tie each new idea to as many familiar concepts and situations as you can and to visualize settings in which the new fact would be helpful to you. We have included lots of examples in the text to make this process easier. The review lists can be used to go over the material in the same way by yourself or with fellow students.

The supplementary exercises at the ends of the chapters are also a good way to check your command of the chapter material. You don't need to work them all, though it's a good idea to look them all over just to see what kinds of questions one can be expected to answer. Our best advice on doing exercises is that doing a few thoughtfully is better than trying to do a lot in a hurry, and it will probably take you less time overall.

Answers or hints to most odd-numbered exercises in the sections, as well as complete answers to all supplementary exercises, are given in the back of the book. Wise students will look at the answers only after trying seriously to do the problems. When a proof is called for, we usually give a hint or an outline of a proof, which you should first understand and then expand upon. A symbols index appears on the inside front cover of the book. At the back of the book there is an index of topics. After Chapter 13 there is a brief dictionary of some terms that we use in the text without explanation, but which readers may have forgotten or never encountered. Look at these items right now to see where they are and what they contain and then join us for Chapter 1.

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读者意见反馈

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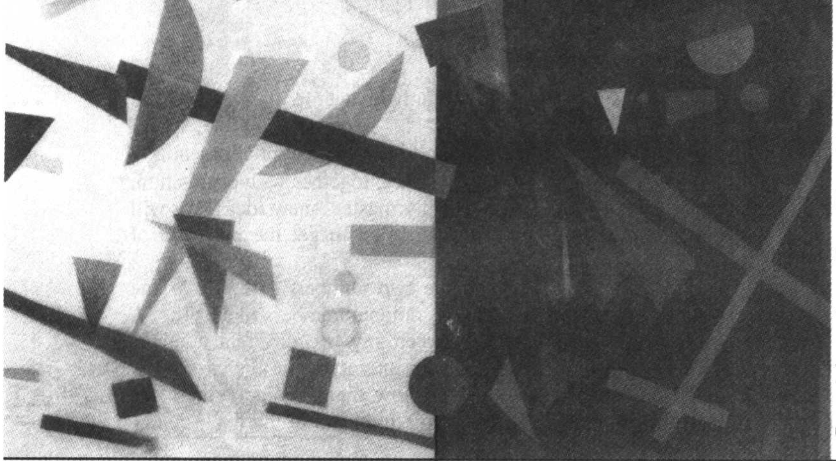
Contents



CHAPTER 1

Sets, Sequences, and Functions

This chapter is introductory and contains a relatively large number of fundamental definitions and notations. Much of the material may be familiar, though perhaps with different notation or at a different level of mathematical precision. Besides introducing concepts and methods, this chapter establishes the style of exposition that we will use for the remainder of the book.

1.1 Some Warm-up Questions

In this section we point out the importance of precision, abstraction and logical thinking. We also introduce some standard terms and notation. The mathematical content is less important than the ways of thinking about things.

How many numbers are there between 1 and 10? Between 1 and 1000? Between 1000 and 1,000,000? Between 6357 and 924,310? These questions are all just special cases of the more general question, "How many numbers are there between m and n?" For example, the question about how many numbers there are between 1000 and 1,000,000 is the case where $m = 1000$ and $n = 1{,}000{,}000$. If we learn how to answer the general question, then we'll be able to apply our method to solve the four specific [and fairly artificial] problems above and also any others like them that we come across in practice. By the end of this section we'll even be able to answer some substantially more challenging questions by using what we've learned about these simpler ones.

The process of going from specific cases to general problems is called **abstraction**. One of our goals in this book is to convince you that *abstraction is your friend*. This assertion may seem hard to believe right now, but you will see lots of examples to prove our point. For now, we claim that abstraction is valuable for at least three reasons.

First, the process of abstraction strips away inessential features of a specific problem to focus attention on the core issues. In this way, abstraction often makes a problem easier to analyze and hence easier to solve. Indeed, one can view abstraction as just one standard step in analyzing any problem.

Second, the solution to the abstracted problem applies not just to the problem we started with but to others like it as well, i.e., to all problems that have the same abstract type. By solving the abstract problem, we solve a whole class of specific problems at no extra cost.

Moreover, using abstraction means that you don't have to do a lot of essentially similar homework problems. A few well-chosen exercises, together with abstraction, can provide all the practice necessary to master—really master—new ideas. We will have more to say later about how you can use abstraction to get the most out of your homework.

So how many numbers *are* there between the two numbers m and n? That depends on what we mean by "number." If we allow rational numbers like $311/157$, then there are already infinitely many of them between any two different numbers m and n. The answer is always the same and not very interesting in this case.

Let's make the question interesting by asking how many *integers* there are between m and n. [Recall that an **integer** is one of the numbers in the list

$$\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots$$

which extends without bound in both directions.] How many integers are there between 1 and 10? If we mean "strictly between," then the numbers are 2, 3, 4, 5, 6, 7, 8, 9, and there are 8 of them, but if we mean "between, but allowing 1 and 10, too," then the answer is $8+2 = 10$. We can't answer the question until we have a clear, precise statement of it. Usually, when people say "think of a number between 1 and 10," they mean to allow both 1 and 10, so for the rest of this section let's agree that "between" includes both ends of the range. Then our general problem is

How many integers i are there with $m \leq i \leq n$?

Lots of people will guess that the answer is $n - m$. To show that this guess is wrong in general, it's enough to exhibit one pair of numbers m and n for which it's wrong. How about $m = 1$ and $n = 10$, i.e.,

How many integers i are there with $1 \leq i \leq 10$?

The right answer is 10, as we observed above, not $10 - 1$, so the guess $n - m$ is wrong.

In fact, our example with $m = 1$ and $n = 10$ shows how to get the correct answer in lots of cases. If $m = 1$ and n is any positive integer, then the integers i with $m = 1 \leq i \leq n$ are just $1, 2, \ldots, n$, and there are exactly n of them. We record this modest piece of information.

Fact 1 If n is a positive integer, then there are n integers i such that $1 \leq i \leq n$.

Now to find out how many integers i there are with $1000 \leq i \leq 1{,}000{,}000$ we could list them all and then count them, but there's an easier way that leads to a general method: count the i's with $1 \leq i \leq 1{,}000{,}000$ and then throw away the ones with $i < 1000$, i.e., with $i \leq 999$. Fact 1 gives us our answer: $1{,}000{,}000 - 999 = 999{,}001$. This method leads us to another general fact.

Fact 2 If m and n are positive integers with $m \leq n$, then the number of integers i such that $m \leq i \leq n$ is $n - (m - 1) = n - m + 1$.

Before you read on, stop and make sure that you see where the $m - 1$ came from here and why this result really does follow from Fact 1.

Fact 2 is a statement in the form "if . . . , then . . . ." The part between "if" and "then" contains its **hypotheses**, that m and n are positive integers and that $m \leq n$, and the part after "then" is its **conclusion**, in this case a statement about a certain set of integers. If . . . then . . . statements of this sort come up in all kinds of situations, not just in mathematics, and the ability to handle them correctly can be crucial. We will work with such statements throughout this book, and we will pay special attention to them in Chapter 2 on logic and arguments.

Given an implication such as Fact 2, it's natural to ask whether the hypotheses are really necessary. Would the conclusion still be true if we dropped a hypothesis? If not, how would it change? Can we say anything at all useful under weaker hypotheses?

If we leave out the second hypothesis of Fact 2 and allow $m > n$, then we get an easy answer, since there are no i's at all with *both* $m \le i$ *and* $i \le n$. If a computer program with inputs m and n prints something for each integer between m and n, and if we want to know how many things it prints, then we may need to consider the case $m > n$. For our discussion now, though, let's keep $m \le n$ as a hypothesis.

What would happen if we allowed either m or n to be negative or 0? For example, how many integers i are there with $-10 \le i \le 90$? If we simply add 11 to everything, then the question becomes how many integers $i + 11$ are there with $-10 + 11 = 1 \le i + 11 \le 90 + 11 = 101$? Then Fact 2 says that there are $101 - 1 + 1$ possible $i + 11$'s, so that's how many i's there are, too. This answer is $(90 + 11) - (-10 + 11) + 1 = 90 - (-10) + 1 = n - m + 1$ still. It didn't really matter that we chose to add 11; any integer bigger than 10 would have let us use Fact 2 and would have canceled out at the end. The method also didn't depend on our choice of $m = -10$. The hypothesis that m and n are positive is unnecessary, and we have the following more general result.

Fact 3 If m and n are integers with $m \le n$, then there are $n - m + 1$ integers i with $m \le i \le n$.

Let's try a harder question now. How many integers between 1 and 100 are even? How many between 10 and 100? Between 11 and 101? Between 17 and 72? Recall that an integer is **even** if it's twice some integer. The even integers, then, are the ones in the list

$$\ldots, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, \ldots$$

and the **odd** integers are the ones that aren't even, i.e., the ones in the list

$$\ldots, -7, -5, -3, -1, 1, 3, 5, 7, 9, \ldots.$$

Our new general question is, "How many even integers i are there that satisfy $m \le i \le n$?" Let's assume first that m and n are positive and worry later about negative possibilities. Since Fact 2 says that there are $n - m + 1$ integers between m and n, and since about half of them are even, we might guess that the answer is $(n - m + 1)/2$. We see that this guess can't be right in general, though, because it's not a whole number unless $n - m + 1$ is even. Still, we can figure that $(n - m + 1)/2$ is probably *about* right.

The strategy that worked before to get the correct answer still seems reasonable: count the even integers from 1 to n and subtract the number of even integers from 1 to $m - 1$. Without actually listing the integers from 1 to n, we can describe the even ones that we'd find. They are $2, 4, \ldots, 2s$ for some integer s, where $2s \le n$ but $n < 2(s + 1)$, i.e., where $s \le n/2 < s + 1$. For example, if $n = 100$, then $s = 50$, and if $n = 101$, then $n/2 = 50.5$ and again $s = 50$. Our answer is s, which we can think of as "the integer part" of $n/2$.

There's a standard way to write such integer parts. In general, if x is any real number, the **floor** of x, written $\lfloor x \rfloor$, is the largest integer less than or equal to x. For example $\lfloor \pi \rfloor = \lfloor 3.14159265\cdots \rfloor = 3$, $\lfloor 50.5 \rfloor = 50$, $\lfloor 17 \rfloor = 17$, and $\lfloor -2.5 \rfloor = -3$. The **ceiling** of x, written $\lceil x \rceil$, is similarly defined to be the smallest integer that is at least as big as x, so $\lceil \pi \rceil = 4$, $\lceil 50.5 \rceil = 51$, $\lceil 72 \rceil = 72$, and $\lceil -2.5 \rceil = -2$. In terms of the number line, if x is not an integer, then $\lfloor x \rfloor$ is the nearest integer to the left of x and $\lceil x \rceil$ is the nearest integer to the right of x.

We've seen that the number of even integers in $1, 2, \ldots, n$ is $\lfloor n/2 \rfloor$, and essentially the same argument shows the following.

Fact 4 Let k and n be positive integers. Then the number of multiples of k between 1 and n is $\lfloor n/k \rfloor$.

Again, stop and make sure that you see how the reasoning we used for the even numbers with $k = 2$ would work just as well for any positive integer k. What does Fact 4 say for $k = 1$?

It follows from Fact 4 that if m is a positive integer with $m \le n$, then the number of multiples of k in the list $m, m+1, \ldots, n$ is $\lfloor n/k \rfloor - \lfloor (m-1)/k \rfloor$. It's natural to wonder whether there isn't a nicer formula that doesn't involve the floor function. For instance, with $k = 2$ the number of even integers between 10 and 100 is $\lfloor 100/2 \rfloor - \lfloor 9/2 \rfloor = \lfloor 50 \rfloor - \lfloor 4.5 \rfloor = 50 - 4 = 46$, and the number between 11 and 101 is $\lfloor 101/2 \rfloor - \lfloor 10/2 \rfloor = \lfloor 50.5 \rfloor - \lfloor 5 \rfloor = 50 - 5 = 45$. In both cases $(n - m + 1)/2 = 91/2 = 45.5$, which is close to the right answer.

In fact, as we will now show, $(n - m + 1)/2$ is *always* close to our answer $\lfloor n/k \rfloor - \lfloor (m-1)/k \rfloor$, which is the actual number of multiples of k between m and n. Notice that we will need to use a little algebra here to manipulate inequalities. As we go along in the book, we'll need more algebraic tools, though probably nothing harder than this. If your algebra skills are a little rusty, now would be a good time to review them. Here's the argument.

In general, $\lfloor x \rfloor \le x < \lfloor x \rfloor + 1$, so $x - 1 < \lfloor x \rfloor \le x$. Thus, with n/k for x, we get

$$\frac{n}{k} - 1 < \left\lfloor \frac{n}{k} \right\rfloor \le \frac{n}{k} \tag{1}$$

and similarly, with $(m-1)/k$ for x, we get

$$\frac{m-1}{k} - 1 < \left\lfloor \frac{m-1}{k} \right\rfloor \le \frac{m-1}{k}. \tag{2}$$

Multiplying (2) through by -1 reverses the inequalities to give

$$-\frac{m-1}{k} \le -\left\lfloor \frac{m-1}{k} \right\rfloor < -\frac{m-1}{k} + 1. \tag{3}$$

Finally, adding inequalities (1) and (3) term by term gives

$$\frac{n}{k} - 1 - \frac{m-1}{k} < \left\lfloor \frac{n}{k} \right\rfloor - \left\lfloor \frac{m-1}{k} \right\rfloor < \frac{n}{k} - \frac{m-1}{k} + 1,$$

which just says that

$$\frac{n-m+1}{k} - 1 < \text{our answer} < \frac{n-m+1}{k} + 1.$$

We've shown that the number of multiples of k between m and n is always an integer that differs from $(n - m + 1)/k$ by less than 1, so $(n - m + 1)/k$ was a pretty good guess at the answer after all.

To finish off our general question, let's allow m or n to be negative. How does the answer change? For example, how many even integers are there between -11 and 72? The simple trick is to add a big enough even integer to everything, for instance to add 20. Then, because i is even with $-11 \le i \le 72$ if and only if $i + 20$ is even with $-11 + 20 \le i + 20 \le 72 + 20$, and because the number of even integers between $-11 + 20 = 9$ and $72 + 20 = 92$ is $\lfloor 92/2 \rfloor - \lfloor 8/2 \rfloor = 46 - 4 = 42$, this is the answer to the question for -11 and 72, too.

The same trick will work for multiples of any positive integer k. Moreover, if we add a multiple of k to everything, say tk for some integer t, then we have

$$\begin{aligned}\left\lfloor \frac{n+tk}{k} \right\rfloor - \left\lfloor \frac{m-1+tk}{k} \right\rfloor &= \left\lfloor \frac{n}{k} + t \right\rfloor - \left\lfloor \frac{m-1}{k} + t \right\rfloor \\ &= \left\lfloor \frac{n}{k} \right\rfloor + t - \left(\left\lfloor \frac{m-1}{k} \right\rfloor + t \right) \qquad \text{[why?]} \\ &= \left\lfloor \frac{n}{k} \right\rfloor - \left\lfloor \frac{m-1}{k} \right\rfloor,\end{aligned}$$