

Digital Signal Processing

System Analysis and Design



**Paulo S. R. Diniz, Eduardo A. B. da Silva,
and Sergio L. Netto**

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Federal University of Rio de Janeiro



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Digital Signal Processing

System Analysis and Design

Digital signal processing lies at the heart of the communications revolution and is an essential element of key technologies such as mobile phones and the Internet. This book covers all the major topics in digital signal processing (DSP) design and analysis, supported by MATLAB® examples and other modeling techniques. The authors explain clearly and concisely why and how to use digital signal processing systems; how to approximate a desired transfer function characteristic using polynomials and ratios of polynomials; why an appropriate mapping of a transfer function onto a suitable structure is important for practical applications; and how to analyze, represent, and explore the trade-off between time and frequency representation of signals. An ideal textbook for students, it will also be a useful reference for engineers working on the development of signal processing systems.

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To our families, our parents, and our students

Preface

This book originated from a training course for engineers at the research and development center of TELEBRAS, the former Brazilian telecommunications holding. That course was taught by the first author back in 1987, and its main goal was to present efficient digital filter design methods suitable for solving some of their engineering problems. Later on, this original text was used by the first author as the basic reference for the digital filters and digital signal processing courses of the Electrical Engineering Program at COPPE/Federal University of Rio de Janeiro.

For many years, former students asked why the original text was not transformed into a book, as it presented a very distinct view that they considered worth publishing. Among the numerous reasons not to attempt such task, we could mention that there were already a good number of well-written texts on the subject; also, after many years of teaching and researching on this topic, it seemed more interesting to follow other paths than the painful one of writing a book; finally, the original text was written in Portuguese and a mere translation of it into English would be a very tedious task.

In recent years, the second and third authors, who had attended the signal processing courses using the original material, were continuously giving new ideas on how to proceed. That was when we decided to go through the task of completing and updating the original text, turning it into a modern textbook. The book then took on its present form, updating the original text, and including a large amount of new material written for other courses taught by the three authors during the past few years.

This book is mainly written for use as a textbook on a digital signal processing course for undergraduate students who have had previous exposure to basic discrete-time linear systems, or to serve as textbook on a graduate-level course where the most advanced topics of some chapters are covered. This reflects the structure we have at the Federal University of Rio de Janeiro, as well as at a number of other universities we have contact with. The book includes, at the end of most chapters, a brief section aimed at giving a start to the reader on how to use MATLAB[®] as a tool for the analysis and design of digital signal processing systems. After many discussions, we decided that having explanations about MATLAB inserted in the main text would in some cases distract the reader, making him or her lose focus on the subject.

The distinctive feature of this book is to present a wide range of topics in digital signal processing design and analysis in a concise and complete form, while allowing the reader to fully develop practical systems. Although this book is primarily intended

as an undergraduate and graduate textbook, its origins on training courses for industry warrant its potential usefulness to engineers working in the development of signal processing systems. In fact, our objective is to equip the readers with the tools that enable them to understand why and how to use digital signal processing systems; to show them how to approximate a desired transfer function characteristic using polynomials and ratios of polynomials; to teach them why an appropriate mapping of a transfer function into a suitable structure is important for practical applications; to show how to analyze, represent, and explore the tradeoff between the time and frequency representations of signals. For all that, each chapter includes a number of examples and end-of-chapter problems to be solved, aimed at assimilating the concepts as well as complementing the text.

Chapters 1 and 2 review the basic concepts of discrete-time signal processing and z transforms. Although many readers may be familiar with these subjects, they could benefit from reading these chapters, getting used to the notation and the authors' way of presenting the subject. In Chapter 1, we review the concepts of discrete-time systems, including the representation of discrete-time signals and systems, as well as their time- and frequency-domain responses. Most importantly, we present the sampling theorem, which sets the conditions for the discrete-time systems to solve practical problems related to our real continuous-time world. Chapter 2 is concerned with the z and Fourier transforms which are useful mathematical tools for representation of discrete-time signals and systems. The basic properties of the z and Fourier transforms are discussed, including a stability test in the z transform domain.

Chapter 3 discusses discrete transforms, with special emphasis given to the discrete Fourier transform (DFT), which is an invaluable tool in the frequency analysis of discrete-time signals. The DFT allows a discrete representation of discrete-time signals in the frequency domain. Since the sequence representation is natural for digital computers, the DFT is a very powerful tool, because it enables us to manipulate frequency-domain information in the same way as we can manipulate the original sequences. The importance of the DFT is further increased by the fact that computationally efficient algorithms, the so-called fast Fourier transforms (FFTs), are available to compute the DFT. This chapter also presents real coefficient transforms, such as cosine and sine transforms, which are widely used in modern audio and video coding, as well as in a number of other applications. A section includes a discussion on the several forms of representing the signals, in order to aid the reader with the available choices.

Chapter 4 addresses the basic structures for mapping a transfer function into a digital filter. It is also devoted to some basic analysis methods and properties of digital filter structures.

Chapter 5 introduces several approximation methods for filters with finite-duration impulse response (FIR), starting with the simpler frequency sampling method and the widely used windows method. This method also provides insight to the windowing

strategy used in several signal processing applications. Other approximation methods included are the maximally flat filters and those based on the weighted-least-squares (WLS) method. This chapter also presents the Chebyshev approximation based on a multivariable optimization algorithm called the Remez exchange method. This approach leads to linear-phase transfer functions with minimum order given a prescribed set of frequency response specifications. This chapter also discusses the WLS–Chebyshev method which leads to transfer functions where the maximum and the total energy of the approximation error are prescribed. This approximation method is not widely discussed in the open literature but appears to be very useful for a number of applications.

Chapter 6 discusses the approximation procedures for filters with infinite-duration impulse response (IIR). We start with the classical continuous-time transfer-function approximations, namely the Butterworth, Chebyshev, and elliptic approximations, that can generate discrete-time transfer functions by using appropriate transformations. Two transformation methods are then presented which are the impulse-invariance and the bilinear transformation methods. The chapter also includes a section on frequency transformations in the discrete-time domain. The simultaneous magnitude and phase approximation of IIR digital filters using optimization techniques is also included, providing a tool to design transfer functions satisfying more general specifications. The chapter closes by addressing the issue of time-domain approximations.

Chapter 7 includes the models that account for quantization effects in digital filters. We discuss several approaches to analyze and deal with the effects of representing signals and filter coefficients with finite wordlength. In particular, we study the effects of quantization noise in products, signal scaling that limits the internal signal dynamic range, coefficient quantization in the designed transfer function, and the nonlinear oscillations which may occur in recursive realizations. These analyses are used to indicate the filter realizations that lead to practical finite-precision implementations of digital filters.

Chapter 8 deals with basic principles of discrete-time systems with multiple sampling rates. In this chapter, we emphasize the basic properties of multirate systems, thoroughly addressing the decimation and interpolation operations, giving examples of their use for efficient digital filter design.

Chapter 9 presents several design techniques for multirate filter banks, including several forms of 2-band filter banks, cosine-modulated filter banks, and lapped transforms. It also introduces the concept of multiresolution representation of signals through wavelet transforms, and discusses the design of wavelet transforms using filter banks. In addition, some design techniques to generate orthogonal, as well as biorthogonal bases for signal representation, are presented.

In Chapter 10, we present some techniques to reduce the computational complexity of FIR filters with demanding specifications. In particular, we introduce the prefilter and interpolation methods which are mainly useful in designing narrowband lowpass

and highpass filters. In addition, we present the frequency response masking approach, for designing filters with narrow transition bands satisfying more general specifications, and the quadrature method, for narrow bandpass and bandstop filters.

Chapter 11 presents a number of efficient realizations for IIR filters. For these filters, a number of realizations considered efficient from the finite-precision effects point of view are presented and their salient features are discussed in detail. These realizations will equip the reader with a number of choices for the design of good IIR filters. Several families of structures are considered in this chapter, namely: parallel and cascade designs using direct-form second-order sections; parallel and cascade designs using section-optimal and limit-cycle-free state-space sections; lattice filters; and several forms of wave digital filters.

In Chapter 12, the most widely used implementation techniques for digital signal processing are briefly introduced. This subject is too large to fit in a chapter of a book; in addition, it is changing so fast that it is not possible for a textbook on implementation to remain up to date for long. To cope with that, we have chosen to analyze the most widely used implementation techniques which have been being employed for digital signal processing in the last decade and to present the current trends, without going into the details of any particular implementation strategy. Nevertheless, the chapter should be enough to assist any system designer in choosing the most appropriate form of implementing a particular digital signal processing algorithm.

This book contains enough material for an undergraduate course on digital signal processing and a first-year graduate course. There are many alternative ways to compose these courses; however, we recommend that an undergraduate course should include most parts of Chapters 1, 2, 3, and 4. It could also include the non-iterative approximation methods of Chapters 5 and 6, namely, the frequency sampling and window methods described in Chapter 5, the analog-based approximation methods, and also the continuous-time to discrete-time transformation methods for IIR filtering of Chapter 6. At the instructor's discretion the course could also include selected parts of Chapters 7, 8, 10, and 11.

As a graduate course textbook, Chapters 1 to 4 could be seen as review material, and the other chapters should be covered in depth.

This book would never be written if people with a wide vision of how an academic environment should be were not around. In fact, we were fortunate to have Professors L. P. Calôba and E. H. Watanabe as colleagues and advisors. The staff of COPPE, in particular Mr M. A. Guimarães and Ms F. J. Ribeiro, supported us in all possible ways to make this book a reality. Also, the first author's early students, J. C. Cabezas, R. G. Lins, and J. A. B. Pereira (in memoriam) wrote, with him, a computer package that generated several of the examples in this book. The engineers of CPqD helped us to correct the early version of this text. In particular, we would like to thank the engineer J. Sampaio for his complete trust in this work. We benefited from working in an environment with a large signal processing group where our colleagues

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Paulo S. R. Diniz
Eduardo A. B. da Silva
Sergio L. Netto

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