Fifth Edition

Transform Circuit Analysis for Engineering and Technology

William D. Stanley

TRANSFORM CIRCUIT ANALYSIS FOR ENGINEERING AND TECHNOLOGY

FIFTH EDITION

William D. Stanley

Old Dominion University

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PREFACE

As in the case of the first four editions, this fifth edition was designed for an advanced undergraduate circuit analysis course in an applied engineering curriculum or in an upper-division engineering technology curriculum. The book could also serve as a self-study reference for engineers and technologists.

The reader should be familiar with the fundamentals of differential and integral calculus and with basic dc circuit analysis techniques. It is anticipated that most readers will also be familiar with steady-state ac circuit theory. However, the latter condition is not a necessity, since a major portion of the book may be mastered without a background in ac circuits.

The first four chapters are devoted to time-domain considerations. Chapter 1 is an introduction to the general philosophy of the book. The fundamentals of waveform analysis are presented in Chapter 2. The reader should find that his or her knowledge of differentiation and integration will be strengthened after mastering this chapter, particularly in regard to graphical techniques. The voltage–current relationships for each of the basic circuit elements are explained in Chapter 3 and developed fully in Chapter 4.

The next four chapters are devoted to transform-domain considerations. Following a detailed development of the Laplace transform and inverse transform in Chapter 5, the use of transform techniques in obtaining complete circuit responses is presented in Chapter 6. In Chapter 7, the emphasis shifts to the system concepts of circuit theory. Among the topics considered are transfer functions, impedance functions, convolution, and stability. In Chapter 8, sinusoidal steady-state techniques are developed and compared with Laplace transform techniques. The frequency response concept is developed, and the use of pole–zero methods for obtaining frequency response plots is explored.

Chapter 9 deals with Fourier analysis and the concept of a spectrum. Both the Fourier series and the Fourier transform are covered.

Chapter 10 provides an introduction to discrete-time systems. Topics covered include sampled signals, the sampling theorem, difference equations, and the z-transform. This chapter represents a modern and timely supplement to the continuous-time material predominant through the remainder of the book.

Among the topics presented in the appendices are complex algebra and several proofs of transform theorems. These topics may be considered at the discretion of the instructor whenever possible.

Depending on the nature of the course and the background of the students, there is reasonable latitude available on the depth and rigor with which the material in this book can be presented. On the one hand, the derivation and formulation of the principles involved can be emphasized. On the other hand, the use of principles as tools for solving and interpreting practical problems can be emphasized, with only casual consideration of the mathematical fine points. Any suitable compromise between these limits should be possible.

Multisim and MATLAB

The circuit analysis program featured in the previous edition of the text was Electronics Workbench. This program has now evolved into a new product with the name Multisim. The result has been a continued improvement in the product. It served as the basis for computer-aided circuit analysis in this fifth edition. The Electronics Workbench examples of the fourth edition have been modified to work with Multisim and all instructions have been modified accordingly. This coverage is optional and is not a prerequisite for other topics within the text.

Multisim continues to be one of the most widely employed Windows-based circuit analysis programs, particularly in educational institutions. Its success is based largely on the fact that it is very user-friendly and is quickly learned by computer-literate students.

The text contains several dozen Multisim examples, located near the end of each chapter. Most of these Multisim examples are based on previous chapter examples that had been analyzed earlier by standard circuit analysis methods. Thus, the reader can compare results from the two drastically different approaches. Readers with no background in Multisim should carefully study Appendix D before considering the Multisim examples in the text.

The text also contains a number of MATLAB examples. As in the case of Multisim, this coverage is optional and is not a prerequisite for other topics within the text. MATLAB examples are also located near the end of the appropriate chapters.

MATLAB is one of the most popular software packages for performing mathematical operations. It has a wide variety of capabilities, including operations in algebra, calculus, differential equations, and matrix manipulations. Some of the most useful operations that support circuit analysis will be illustrated.

Appendix E, providing a brief introduction to MATLAB, has been added for the benefit of readers who are unfamiliar with the program. Most of the programming details, however, are provided in the MATLAB examples throughout the text.

Acknowledgments

I would like to thank the following reviewers for their invaluable feedback: Stephen T. Adams, University of Southern Mississippi, and Ronald H. Rockland, New Jersey Institute of Technology.

CONTENTS

CHAPTER 1

Introductory Considerations

OVERVIEW

Chapter 1 is by far the shortest chapter in the book. Unlike all other chapters, it has no example problems or final problems for the reader. The purpose of this brief chapter is to introduce a few definitions and conventions that are fundamental in establishing the approach for all subsequent chapters. In addition, a list of the concepts with which the reader should be familiar for further study will be given.

OBJECTIVES

After completing this chapter, the reader should be able to

- State the three types of circuit parameters and show their schematic symbols.
- State the two forms of ideal electrical energy sources and show their schematic symbols.
- Show the two forms for representing an actual source with internal resistance.
- Discuss the terms *excitation* and *response.*
- Discuss the difference between circuit analysis and circuit synthesis.
- Define the four types of dependent sources and show their schematic forms.

1–1 CIRCUIT ELEMENTS

A linear electric circuit consists of some combination of three types of *passive circuit parameters* and two types of *energy sources.* The three types of circuit parameters are (a) *resistance,* (b) *inductance,* and (c) *capacitance.* The schematic representations of the three circuit parameter components are shown in Figure 1–1. An extensive treatment of these parameters is presented in Chapter 3.

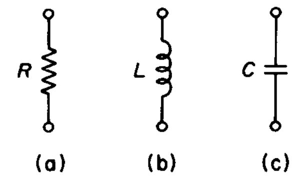

Figure 1–1
Circuit parameters: (a) resistance, (b) inductance, (c) capacitance.

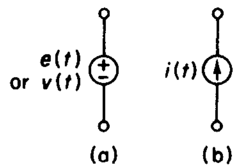

Figure 1–2
Ideal energy sources: (a) voltage, (b) current.

The two ideal types of energy sources are (a) the ideal *voltage source* and (b) the ideal *current source*. The schematic representations of the ideal sources are shown in Figure 1–2. The hypothetical ideal voltage source is assumed to maintain the same voltage across its terminals regardless of the load. The hypothetical ideal current source is assumed to deliver the same current regardless of the load.

No actual electrical energy source quite fits the ideal models, although many are sufficiently close that they may be approximated by ideal sources under many conditions. An actual energy source will neither maintain a constant voltage across its terminals nor deliver a constant current from its terminals for all values of load. The actual behavior of such sources can usually be described by a model composed of a hypothetical ideal source and one or more passive circuit parameters. The effect of the passive elements is to control the voltage or current available to the remainder of the circuit in essentially the same manner as the terminals of the actual sources.

In a wide variety of cases, a single resistance in conjunction with an ideal energy source is sufficient to characterize the external behavior of the actual source. The two possible forms for this case are shown in Figure 1–3. These forms are: (a) an ideal voltage source in series with a resistance and (b) an ideal current source in parallel with a resistance. In general, a given source may be represented by either form.

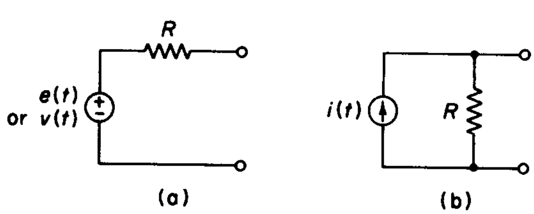

Figure 1–3
Two models for real energy sources.

1–2 EXCITATION AND RESPONSE

Assume that a given circuit is excited by one or more voltage and/or current sources. Throughout the circuit, currents will begin to flow through passive elements, and voltages will appear across them. In general, all voltages and currents will vary with *time* in some fashion. Letting t represent time, we can assert that, in general, all voltages and currents in the circuit are functions of the independent variable t. In most cases in this text, we will use the symbol e to represent a voltage source and the symbol v to represent a voltage drop across a passive element. The symbol i will be used to represent a current, without regard to whether it is a source or the current through a passive element. For special reasons there are a few exceptions to this notation, so it should not be interpreted as ironclad.

To emphasize that voltages and currents are functions of the variable t, we will often write these quantities in the forms $e(t)$, $v(t)$, and $i(t)$. The quantity in the parentheses is called the *argument* of the voltage or current, which, of course, is the variable t for the quantities just stated. In an algebraic expression involving several operations, one must be careful not to confuse this notation with multiplication, which is also expressed in some cases by parentheses.

The voltage and/or current sources that excite a circuit are called the *excitations,* and the resulting voltages and currents associated with passive components are called the *responses.* In many applications, the excitations are regarded as the *inputs,* and the responses are regarded as the *outputs.*

1–3 ANALYSIS AND SYNTHESIS

The study of circuit theory can be divided into two separate areas: (a) *circuit analysis* and (b) *circuit synthesis.* The main topics in this text will be devoted to circuit analysis. The basic theme of circuit analysis is as follows: If the voltage and current source excitations and the network are known, one can determine the voltage and current responses within the network. Frequently, the analysis theme is slightly modified in that perhaps a source or a component must be determined, but a basic underlying property of analysis problems is that the circuit configuration is known.

The basic theme of circuit synthesis is as follows: If we are given the response and the excitation, or a relationship between the response and the excitation, one can determine the circuit. In essence, the principles of circuit synthesis permit one to determine a circuit that will provide a prescribed processing on a given signal. Practical circuit design differs from the science of network synthesis in the sense that actual circuit design frequently uses the concepts of both analysis and synthesis (along with engineering economics) to accomplish its goal.

1–4 DEPENDENT SOURCES

The voltage and current source models introduced in Section 1–1 represent the common independent source models. *Independent* sources are those whose values are independent of the levels of any voltages or currents in the circuit. The values of all voltage and current sources encountered thus far have either been independent constant values (corresponding to dc sources) or independent time-varying quantities.

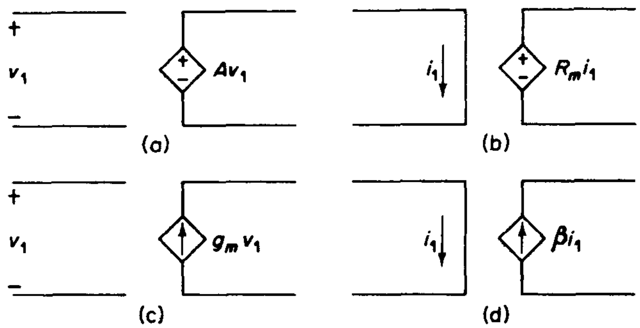

Figure 1–4
Four possible models of ideal dependent (or controlled) sources.

Dependent (or *controlled*) source models are a special class in which the voltage or current is dependent on some other voltage or current within the circuit. Such sources arise from complex physical interactions in many electronic and electrical devices. For example, the base current of a bipolar junction transistor causes a much larger collector current to flow, and this process is modeled by a dependent source.

A dependent source, like an independent source, may be either a voltage source or a current source. However, it may also be controlled by either a voltage or a current. This results in four different combinations for dependent source models, illustrated in Figure 1–4. The symbol used for dependent source models is in accordance with an increasing portion of the literature, although earlier texts employ the same symbol (circle) as for independent source models. Each of the four forms will now be discussed briefly.

Voltage-Controlled Voltage Source

The model for the *voltage-controlled voltage source* (VCVS) is shown in Figure1–4(a). An independent control voltage v_1 is assumed to exist across certain control terminals. This voltage controls a dependent voltage whose value is Av_1. The quantity A is dimensionless and corresponds to a voltage gain when the model represents a voltage amplifier.

Current-Controlled Voltage Source

The model for the *current-controlled voltage source* (ICVS) is shown in Figure 1–4(b). An independent control current i_1 is assumed to be flowing in certain control terminals. This current controls a dependent voltage whose value is $R_m i_1$. The quantity R_m, relating the dependent voltage to the controlling current, has the dimension of ohms and is called the *transresistance.*

Voltage-Controlled Current Source

The model for the *voltage-controlled current source* (VCIS) is shown in Figure 1–4(c). An independent control voltage v_1 is assumed to exist across certain control terminals. This voltage controls a dependent current whose value is $g_m v_1$. The quantity g_m, relating the dependent current to the controlling voltage, has the dimension of siemens and is called the *transconductance*.

Current-Controlled Current Source

The model for the *current-controlled current source* (ICIS) is shown in Figure 1–4(d). An independent control current i_1 is assumed to be flowing in certain control terminals. This current controls a dependent current whose value is βi_1. The quantity β is dimensionless and corresponds to a current gain when the model represents a current amplifier.

1–5 OUTLINE OF CIRCUIT ANALYSIS

The study of basic linear circuit analysis may be arbitrarily classified in the following four categories:

1. Steady-state dc resistive circuit analysis
2. Steady-state sinusoidal ac circuit analysis
3. General linear circuit analysis from the classical differential equation approach
4. General linear circuit analysis from the Laplace transform approach

The steady-state responses of a linear circuit excited by dc sources are all constants, independent of time, and the main parameter of interest is usually resistance, since the primary effects of inductances and capacitances disappear in this case. The study of electric circuits usually begins with a treatment of dc resistive circuits.

If a linear circuit is excited only by sinusoidal excitations, all steady-state responses in the circuit will also be sinusoidal. An elegant body of theory has been built around this concept, and a number of texts are devoted solely to this topic. In a sense, category 1 may be regarded as a special case of category 2, in which the frequency of dc is considered to be zero.

Categories 3 and 4 actually form the most general approaches for linear circuit analysis, since they apply to a circuit with arbitrary excitations, and the responses obtained by using these methods are complete solutions containing both *transient* and *steady-state* responses. (The concepts of transient and steady state will be explained in Chapter 4.) Actually, categories 3 and 4 both accomplish the same general task. However, the Laplace transform approach is popular for dealing with electric circuits, primarily because (a) one can learn to apply transform methods to solve circuit problems using only algebraic manipulations in conjunction with tables; (b) the procedure governing transform methods may be readily developed as an extension of the methods of steady-state analysis; and (c) the use of transform methods permits the engineering analyst to simplify, analyze, and interpret problems of greater complexity than with the differential equation approach.

From the title of this book it is evident that our primary goal is the development of transform analysis techniques. However, it would be a serious mistake to study this method without first developing a suitable background to help the reader understand the overall perspective.

First, it is assumed that the reader has a reasonable background in dc resistive circuit analysis. It is desirable that the reader has a working knowledge of at least the following basic concepts for the dc case:

1. Ohm's law
2. Kirchhoff's voltage law (including mesh current equations)
3. Kirchhoff's current law (including node voltage equations)
4. Thevenin's theorem
5. Norton's theorem
6. Determination of the equivalent resistance of a passive network

In addition, the author believes that most readers of this book will also have a background in steady-state ac circuit theory using complex algebra; such a background will enhance the understanding of the transform method. However, the latter, though desirable, is not an absolute necessity since Appendix A is devoted to complex algebra, and a basic treatment of steady-state ac concepts is developed along with the transform method within the text.

As far as the classical differential equation approach is concerned, enough is included to provide the reader with a basic understanding of this approach. In particular, the reader will be taught to express and interpret the differential equation relationships for circuit components and networks, as such concepts are employed in the development of the transform approach.

CHAPTER 2

Waveform Analysis

OVERVIEW

Before developing a detailed treatment of transient circuit analysis, it is necessary to understand the mathematical forms of the waveforms that are used in describing the phenomena. Virtually all voltage and current waveforms that occur in network analysis can be described in terms of a few basic mathematical functions. These functions are investigated in detail in this chapter and will be used to describe voltage and current waveforms throughout the remainder of the book.

OBJECTIVES

After completing this chapter, the reader should be able to

- State the units and abbreviations for the most common circuit quantities.
- Sketch a step function and express its mathematical form.
- Sketch the form of a switched function and express its mathematical form.
- Sketch a ramp function and express its mathematical form.
- State the derivative and integral relationships between step, ramp, and parabolic functions.
- Sketch the exponential function and express its mathematical form.
- Define the damping constant and the time constant of an exponential function, and state the relationship between them.
- Sketch the sinusoidal function and express its mathematical form.
- Define the angular frequency, repetition frequency, phase, angle, and amplitude of a sine wave.
- State the relationship between the frequency and period of a sine wave.
- Draw a diagram showing the relative phase shifts between the different forms of sine and cosine functions.
- Express sine or cosine functions with any phase angle in terms of the different forms of sine and cosine functions.

- Add any number of sinusoids of the same frequency, but with arbitrary phase angles, to obtain a single sinusoid at the same frequency.
- Sketch the damped sinusoid function and express its mathematical form.
- Sketch the form of a shifted function and express its mathematical form.
- Express various complex waveforms in terms of simple waveforms such as steps, ramps, and sinusoids.
- Define the concept of an impulse function and express its mathematical form.
- State the mathematical relationships between step and impulse functions.
- Define the condition for a function to be periodic and sketch a representative periodic function.
- Determine the average or dc value of a periodic waveform and discuss its significance.
- Determine the rms or effective value of a periodic waveform and discuss its significance.
- Determine the average power dissipated in a resistor by a periodic voltage or current.

2–1 NOTATION AND UNITS

Most of the mathematical functions presented in this chapter are considered to be functions of the independent variable, time. Whenever the framework is that of a general waveform without regard to either voltage or current, general symbols such as $f(t)$, $g(t)$, and $h(t)$ will be employed. As stated in Chapter 1, the argument (t) identifies such waveforms as functions of time. Whenever a waveform is to be used to represent a voltage or current, specific symbols such as $v(t)$, $e(t)$, or $i(t)$ will be employed. As a general rule, lowercase letters will be used to specify time-dependent quantities. Occasionally, the argument (t) will be omitted, but the presence of the lowercase letter will still identify the quantity as a possible time-dependent function.

The use of uppercase letters for voltages and currents will be reserved for fixed quantities such as peak and average values, and for steady-state phasors and transforms. More will be said about phasors and transforms later.

The primary quantities of interest in this text, along with their symbols, basic units, and abbreviations for units, are summarized in Table 2–1. Prefixes and their abbreviations are summarized in Table 2–2.

For simplicity in notation and expression, we will frequently omit units from the expressions for complicated waveform functions and circuit diagrams. The absence of specific units in a given problem will be understood to mean that *all quantities involved are expressed in their basic units given in* Table 2–1. However, when a quantity requires a prefix from Table 2–2, it will be necessary to specify the required unit. A general waveform such as $f(t)$ will usually not require units to be specified. When in doubt about handling prefixed units, the reader should always convert such units to their most basic form.

2–2 STEP FUNCTION

In the solution of general transient problems, it is often necessary to specify a particular value of the independent variable time at which the excitation is applied to the circuit. Since this is usually arbitrary, the time $t = 0$ is often chosen for convenience. Suppose, then, that