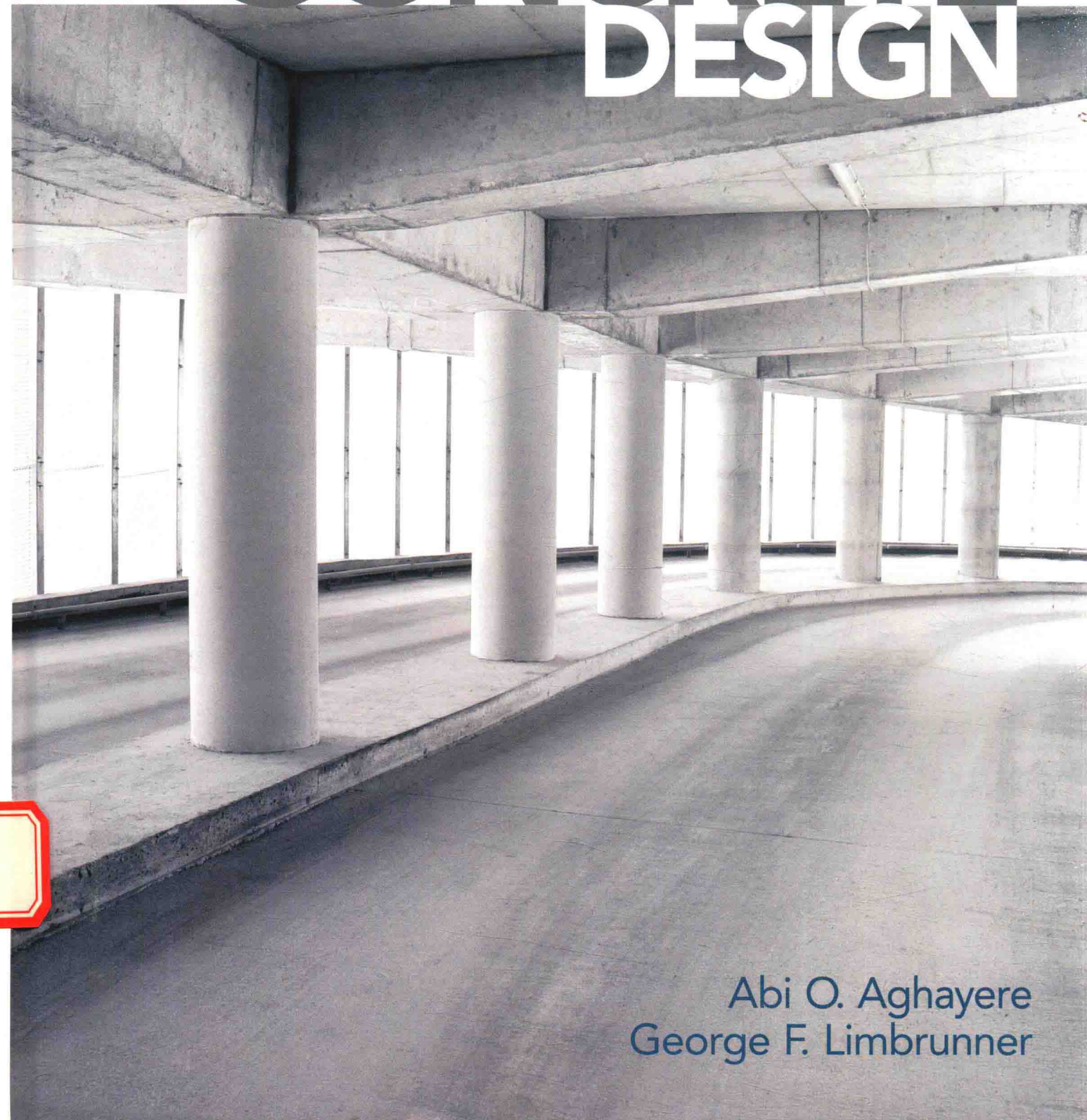


Eighth Edition

REINFORCED CONCRETE DESIGN



Abi O. Aghayere
George F. Limbrunner

Reinforced Concrete Design

EIGHTH EDITION

Abi O. Aghayere, P.Eng.

Drexel University

George F. Limbrunner, PE

Hudson Valley Community College (Emeritus)

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PREFACE

The primary objective of *Reinforced Concrete Design*, eighth edition, remains the same as that of the previous editions: to provide a basic understanding of the strength and behavior of reinforced concrete members and simple reinforced concrete structural systems.

With relevant reinforced concrete research and literature continuing to become available at a rapid rate, it is the intent of this book to translate this vast amount of information and data into an integrated source that reflects the latest information available. It is not intended to be a comprehensive theoretical treatise of the subject, because it is believed that such a document could easily obscure the fundamentals emphasized in engineering technology and applied engineering programs. In addition, it is believed that adequate comprehensive books on reinforced concrete design do exist for those who seek the theoretical background, the research studies, and more rigorous applications.

This eighth edition has been prepared with the primary objective of updating its contents to conform to the latest *Building Code Requirements for Structural Concrete* (ACI 318-11) of the American Concrete Institute. Because the ACI Code serves as the design standard in the United States, it is strongly recommended that the code be used as a companion publication to this book.

In addition to the necessary changes to conform to the new code, some sections have been edited and a new student design project problem has been added and several drawings updated. Answers to selected problems are furnished at the back of the text.

Throughout the eight editions, the text content has remained primarily a fundamental, non-calculus, and practice-oriented approach to the design and analysis of reinforced concrete structural members using numerous examples and a step-by-step solution format. In addition, there are chapters that provide a conceptual approach on such topics as prestressed concrete and detailing of reinforced concrete structures. The metric system (SI) is introduced in Appendix C with several example problems.

Form design is an important consideration in most structural design problems involving concrete members, and Chapter 12 illustrates procedures for the design of job-built forms for slabs, beams, and columns. Appropriate tables are included that will expedite the design process. In Chapter 14, we introduce practical considerations and rules of thumb for the design of reinforced concrete beams, girders, columns and one way slabs, and methods for strengthening existing reinforced concrete structures.

WHAT'S NEW IN THE EIGHTH EDITION:

- The entire text has been revised to conform to the latest ACI Code: ACI 318-11
- The quadratic equation solution approach for the design of rectangular beams is included in Section 2-14
- A new Chapter 14 that discusses practical considerations and rules of thumb for the design of reinforced concrete structures. Guidance is provided for the initial, preliminary sizing and layout of reinforced concrete structures
- The calculation of approximate moment and shears in concrete girders, which cannot be calculated using the ACI coefficients in Chapter 6, is introduced in Chapter 14
- Repair methods for existing reinforced concrete structures is introduced in Chapter 14
- A student reinforced concrete building design project problem has been added in Chapter 14

This book has been thoroughly tested over the years in engineering technology and applied engineering programs and should serve as a valuable design guide and resource for technologists, technicians, engineering and architectural students, and design engineers. In addition, it will aid engineers and architects preparing for state licensing examinations for professional registration.

As in the past, appreciation is extended to students, past and present, and colleagues who, with their constructive comments, criticisms, and enthusiasm, have provided input and encouragement for this edition.

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Abi O. Aghayere

George F. Limbrunner

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1-1 CONCRETE

Concrete consists primarily of a mixture of cement and fine and coarse aggregates (sand, gravel, crushed rock, and/or other materials) to which water has been added as a necessary ingredient for the chemical reaction of curing. The bulk of the mixture consists of the fine and coarse aggregates. The resulting concrete strength and durability are a function of the proportions of the mix as well as other factors, such as the concrete placing, finishing, and curing history.

The compressive strength of concrete is relatively high. Yet it is a relatively brittle material, the tensile strength of which is small compared with its compressive strength. Hence steel reinforcing rods (which have high tensile and compressive strength) are used in combination with the concrete; the steel will resist the tension and the concrete the compression. *Reinforced concrete* is the result of this combination of steel and concrete. In many instances, steel and concrete are positioned in members so that they both resist compression.

1-2 THE ACI BUILDING CODE

The design and construction of reinforced concrete buildings is controlled by the *Building Code Requirements for Structural Concrete* (ACI 318-11) of the American Concrete Institute (ACI) [1]. The use of the term *code* in this text refers to the ACI Code unless otherwise stipulated. The code is revised, updated, and reissued on a 3-year cycle. The code itself has no legal status. It has been incorporated into the building codes of almost all states and municipalities throughout the United States, however. When so incorporated, it has official sanction, becomes a legal document, and is part of the law controlling reinforced concrete design and construction in a particular area.

1-3 CEMENT AND WATER

Structural concrete uses, almost exclusively, hydraulic cement. With this cement, water is necessary for the chemical reaction of *hydration*. In the process of hydration, the cement sets and bonds the fresh concrete into one mass. *Portland cement*, which originated in England, is undoubtedly the most common form of cement. Portland cement consists chiefly of calcium and aluminum silicates. The raw materials are limestones, which provide calcium oxide (CaO), and clays or shales, which furnish silicon dioxide (SiO₂) and aluminum oxide (Al₂O₃). Following processing, cement is marketed in bulk or in 94-lb (1-ft³) bags.

In fresh concrete, the ratio of the amount of water to the amount of cement, by weight, is termed the *water/cement ratio*. This ratio can also be expressed in terms of gallons of water per bag of cement. For complete hydration of the cement in a mix, a water/cement ratio of 0.35 to 0.40 (4 to 4½ gal/bag) is required. To increase the *workability* of the concrete (the ease with which it can be mixed, handled, and placed), higher water/cement ratios are normally used.

1-4 AGGREGATES

In ordinary structural concretes, the aggregates occupy approximately 70% to 75% of the volume of the hardened mass. Gradation of aggregate size to produce close packing is desirable because, in general, the more densely the aggregate can be packed, the better are the strength and durability.

Aggregates are classified as fine or coarse. *Fine aggregate* is generally sand and may be categorized as consisting of particles that will pass a No. 4 sieve (four openings per linear inch). *Coarse aggregate* consists of particles that would be retained on a No. 4 sieve. The maximum size of coarse aggregate in reinforced concrete is governed by various ACI

Code requirements. These requirements are established primarily to ensure that the concrete can be placed with ease into the forms without any danger of jam-up between adjacent bars or between bars and the sides of the forms.

1-5 CONCRETE IN COMPRESSION

The theory and techniques relative to the design and proportioning of concrete mixes, as well as the placing, finishing, and curing of concrete, are outside the scope of this book and are adequately discussed in many other publications [2-5]. Field testing, quality control, and inspection are also adequately covered elsewhere. This is not to imply that these are of less importance in overall concrete construction technology but only to reiterate that the objective of this book is to deal with the design and analysis of reinforced concrete members.

We are concerned primarily with how a reinforced concrete member behaves when subjected to load. It is generally accepted that the behavior of a reinforced concrete member under load depends on the stress-strain relationship of the materials, as well as the type of stress to which it is subjected. With concrete used principally in compression, the compressive stress-strain curve is of primary interest.

The compressive strength of concrete is denoted f'_c and is assigned the units *pounds per square inch* (psi). For calculations, f'_c is frequently used with the units *kips per square inch* (ksi).

A test that has been standardized by the American Society for Testing and Materials (ASTM C39) [6] is used to determine the compressive strength (f'_c) of concrete. The test involves compression loading to failure of a specimen cylinder of concrete. The compressive strength so determined is the highest compressive stress to which the specimen is subjected. Note in Figure 1-1 that f'_c is not the stress that exists in the specimen at failure but that which occurs at a strain of about 0.002. Currently, 28-day concrete strengths (f'_c) range

from 2500 to 9000 psi, with 3000 to 4000 psi being common for reinforced concrete structures and 5000 to 6000 psi being common for prestressed concrete members. Concretes of much higher strengths have been achieved under laboratory conditions. The curves shown in Figure 1-1 represent the result of compression tests on 28-day standard cylinders for varying design mixes.

A review of the stress-strain curves for different-strength concretes reveals that the maximum compressive strength is generally achieved at a unit strain of approximately 0.002 in./in. Stress then decreases, accompanied by additional strain. Higher-strength concretes are more brittle and will fracture at a lower maximum strain than will the lower-strength concretes. The initial slope of the curve varies, unlike that of steel, and only approximates a straight line. For steel, where stresses are below the yield point and the material behaves elastically, the stress-strain plot will be a straight line. The slope of the straight line is the modulus of elasticity. For concrete, however, we observe that the straight-line portion of the plot is very short, if it exists at all. Therefore, there exists no constant value of modulus of elasticity for a given concrete because the stress-strain ratio is not constant. It may also be observed that the slope of the initial portion of the curve (if it approximates a straight line) varies with concretes of different strengths. Even if we assume a straight-line portion, the modulus of elasticity is different for concretes of different strengths. At low and moderate stresses (up to about $0.5f'_c$), concrete is commonly assumed to behave elastically.

The ACI Code, Section 8.5.1, provides the accepted empirical expression for *modulus of elasticity*:

$$E_c = w_c^{1.5} 33 \sqrt{f'_c}$$

where

E_c = modulus of elasticity of concrete in compression (psi)

w_c = unit weight of concrete (lb/ft³)

f'_c = compressive strength of concrete (psi)

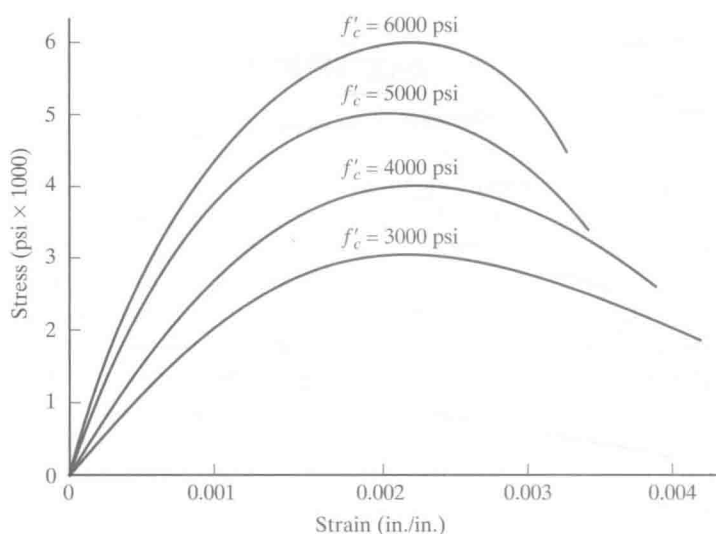


FIGURE 1-1 Typical stress-strain curves for concrete.

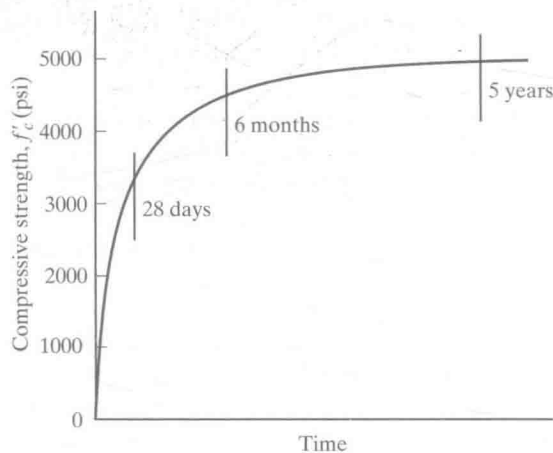


FIGURE 1-2 Strength-time relationship for concrete.

This expression is valid for concretes having w_c between 90 and 160 lb/ft³. For normal-weight concrete, the unit weight w_c will vary with the mix proportions and with the character and size of the aggregates. If the unit weight is taken as 144 lb/ft³, the resulting expression for modulus of elasticity is

$$E_c = 57,000 \sqrt{f'_c} \quad (\text{see Table A-6 for values of } E_c)$$

It should also be noted that the stress-strain curve for the same-strength concrete may be of different shapes if the condition of loading varies appreciably. With different rates of strain (loading), we will have different-shape curves. Generally, the maximum strength of a given concrete is smaller at slower rates of strain.

Concrete strength varies with time, and the specified concrete strength is usually that strength that occurs 28 days after the placing of concrete. A typical strength-time curve for normal stone concrete is shown in Figure 1-2. Generally, concrete attains approximately 70% of its 28-day strength in 7 days and approximately 85% to 90% in 14 days.

Concrete, under load, exhibits a phenomenon termed *creep*. This is the property by which concrete continues to deform (or strain) over long periods of time while under constant load. Creep occurs at a decreasing rate over a period of time and may cease after several years. Generally, high-strength concretes exhibit less creep than do lower-strength concretes. The magnitude of the creep deformations is proportional to the magnitude of the applied load as well as to the length of time of load application.

1-6 CONCRETE IN TENSION

The tensile and compressive strengths of concrete are not proportional, and an increase in compressive strength is accompanied by an appreciably smaller percentage increase in tensile strength. According to the ACI Code Commentary, the tensile strength of normal-weight concrete in flexure is about 10% to 15% of the compressive strength.

The true tensile strength of concrete is difficult to determine. The *split-cylinder test* (ASTM C496) [6] has been

used to determine the tensile strength of lightweight aggregate concrete and is generally accepted as a good measure of the true tensile strength. The split-cylinder test uses a standard 6-in.-diameter, 12-in.-long cylinder placed on its side in a testing machine. A compressive line load is applied uniformly along the length of the cylinder, with support furnished along the full length of the bottom of the cylinder. The compressive load produces a transverse tensile stress, and the cylinder will split in half along a diameter when its tensile strength is reached.

The tensile stress at which splitting occurs is referred to as the *splitting tensile strength*, f_{ct} , and may be calculated by the following expression derived from the theory of elasticity:

$$f_{ct} = \frac{2P}{\pi LD}$$

where

f_{ct} = splitting tensile strength of lightweight aggregate concrete (psi)

P = applied load at splitting (lb)

L = length of cylinder (in.)

D = diameter of cylinder (in.)

Another common approach has been to use the *modulus of rupture*, f_r (which is the maximum tensile bending stress in a plain concrete test beam at failure), as a measure of tensile strength (ASTM C78) [6]. The moment that produces a tensile stress just equal to the modulus of rupture is termed the *cracking moment*, M_{cr} , and may be calculated using methods discussed in Section 1-8. The ACI Code recommends that the modulus of rupture f_r be taken as $7.5\lambda\sqrt{f'_c}$, where f'_c is in psi. Greek lowercase lambda (λ) is a modification factor reflecting the lower tensile strength of lightweight concrete relative to normal-weight concrete. The values for λ are as follows:

Normal-weight concrete—1.0

Sand-lightweight concrete—0.85

All-lightweight concrete—0.75

Interpolation between these values is permitted. See ACI Code Section 8.6.1. for details. If the average splitting tensile strength f_{ct} is specified, then $\lambda = f_{ct}/(6.7\sqrt{f'_c}) \leq 1.0$.

1-7 REINFORCING STEEL

Concrete cannot withstand very much tensile stress without cracking; therefore, tensile reinforcement must be embedded in the concrete to overcome this deficiency. In the United States, this reinforcement is in the form of steel reinforcing bars or welded wire reinforcing composed of steel wire. In addition, reinforcing in the form of structural steel shapes, steel pipe, steel tubing, and high-strength steel tendons is permitted by the ACI Code. Many other approaches have been taken in the search for an economical reinforcement for concrete. Principal among these are the fiber-reinforced

concretes, where the reinforcement is obtained through the use of short fibers of steel or other materials, such as fiberglass. For the purpose of this book, our discussion will primarily include steel reinforcing bars and welded wire reinforcing. High-strength steel tendons are used mainly in prestressed concrete construction (see Chapter 11).

The specifications for steel reinforcement published by the ASTM are generally accepted for the steel used in reinforced concrete construction in the United States and are identified in the ACI Code, Section 3.5.

The steel bars used for reinforcing are, almost exclusively, round deformed bars with some form of patterned ribbed projections rolled onto their surfaces. The patterns vary depending on the producer, but all patterns should conform to ASTM specifications. Steel reinforcing bars are readily available in straight lengths of 60 ft. Smaller sizes are also available in coil stock for use in automatic bending machines. The bars vary in designation from No. 3 through No. 11, with two additional bars, No. 14 and No. 18.

For bars No. 3 through No. 8, the designation represents the bar diameter in eighths of an inch. The No. 9, No. 10, and No. 11 bars have diameters that provide areas equal to 1-in.-square bars, 1½-in.-square bars, and 1¼-in.-square bars, respectively. The No. 14 and No. 18 bars correspond to 1½-in.-square bars and 2-in.-square bars, respectively, and are commonly available only by special order. Round, plain reinforcing bars are permitted for spirals (lateral reinforcing) in concrete compression members.

ASTM specifications require that identification marks be rolled onto the bar to provide the following information: a letter or symbol indicating the producer's mill, a number indicating the size of the bar, a symbol or letter indicating the type of steel from which the bar was rolled, and for grade 60 bars, either the number 60 or a single continuous longitudinal line (called a *grade line*) through at least five deformation spaces. The *grade* indicates the minimum specified yield stress in ksi. For instance, a grade 60 steel bar has a minimum specified yield stress of 60 ksi. No symbol indicating grade

is rolled onto grade 40 or 50 steel bars. Grade 75 bars can have either two grade lines through at least five deformation spaces or the grade mark 75. Reference [7] is an excellent resource covering the various aspects of bar identification.

Reinforcing bars are usually made from newly manufactured steel (billet steel). Steel types and ASTM specification numbers for bars are tabulated in Table A-1. Note that ASTM A615, which is billet steel, is available in grades 40, 60, 75, and 80. Grade 80 steel is allowed for non-seismic applications per ASTM 615 and ASTM 706 [8]. (The full range of bar sizes is not available in grades 40, 75 and 80, however.) Grade 75 steel is approximately 20% stronger than Grade 60 steel requiring a corresponding reduction in the required area of reinforcement, though the installed cost of Grade 75 steel reinforcement is slightly higher than the cost for Grade 60 steel. ASTM A706, low-alloy steel, which was developed to satisfy the requirement for reinforcing bars with controlled tensile properties and controlled chemical composition for weldability, is available in only one grade. Tables A-2 and A-3 contain useful information on cross-sectional areas of bars.

The most useful physical properties of reinforcing steel for reinforced concrete design calculations are yield stress (f_y) and modulus of elasticity. A typical stress-strain diagram for reinforcing steel is shown in Figure 1-3a. The idealized stress-strain diagram of Figure 1-3b is discussed in Chapter 2.

The yield stress (or yield point) of steel is determined through procedures governed by ASTM standards. For practical purposes, the yield stress may be thought of as that stress at which the steel exhibits increasing strain with no increase in stress. The yield stress of the steel will usually be one of the known (or given) quantities in a reinforced concrete design or analysis problem. See Table A-1 for the range of f_y .

The modulus of elasticity of carbon reinforcing steel (the slope of the stress-strain curve in the elastic region) varies over a very small range and has been adopted as 29,000,000 psi (ACI Code, Section 8.5.2).

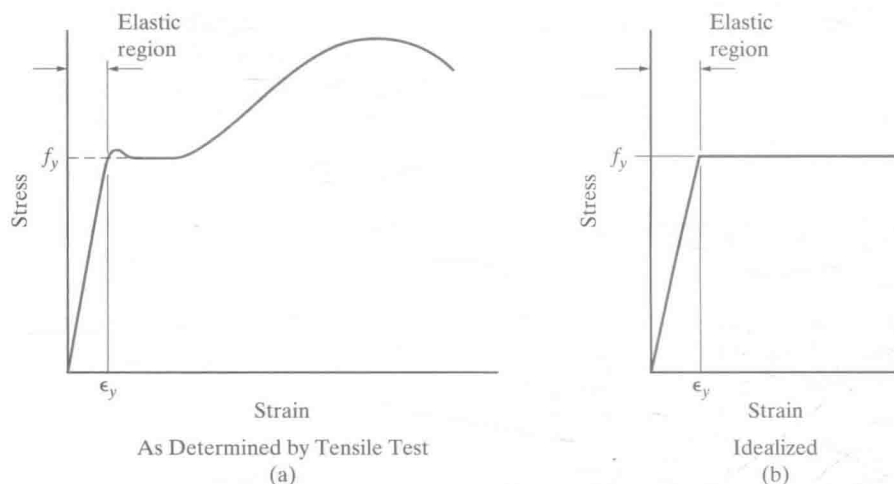


FIGURE 1-3 Stress-strain diagram for reinforcing steel.

Unhindered corrosion of reinforcing steel will lead to cracking and spalling of the concrete in which it is embedded. Quality concrete, under normal conditions, provides good protection against corrosion for steel embedded in the concrete with adequate cover (minimum requirements are discussed in Chapter 2). This protection is attributed to, among other factors, the high alkalinity of the concrete. Where reinforced concrete structures (or parts of structures) are subjected to corrosive conditions, however, some type of corrosion protection system should be used to prevent deterioration. Examples of such structures are bridge decks, parking garage decks, wastewater treatment plants, and industrial and chemical processing facilities.

One method used to minimize the corrosion of the reinforcing steel is to coat the bars with a suitable protective coating. The protective coating can be a nonmetallic material such as epoxy or a metallic material such as zinc (galvanizing). The ACI Code requires epoxy-coated reinforcing bars to comply with ASTM A775 or ASTM A934 and galvanized bars to comply with ASTM A767. The bars to be epoxy coated or zinc coated (galvanized) must meet the code requirements for uncoated bars as tabulated in Table A-1.

Welded wire reinforcing (WWR) (commonly called *mesh*) is another type of reinforcement. It consists of cold-drawn wire in orthogonal patterns, square or rectangular, resistance welded at all intersections. It may be supplied in either rolls or sheets, depending on wire size. WWR with wire diameters larger than about $\frac{1}{4}$ in. is usually available only in sheets.

Both plain and deformed WWR products are available. Plain WWR must conform to ASTM A185 and be made of wire conforming to ASTM A82. Deformed WWR must

conform to ASTM A497 and be made of wire conforming to ASTM A496. Both materials have a yield strength of 70,000 psi. For both materials, the code has assigned a yield strength value of 60,000 psi but makes provision for the use of higher-yield strengths provided the stress corresponds to a strain of 0.35%. The deformed wire is usually more expensive, but it can be expected to have an improved bond with the concrete.

A rational method of designating wire sizes to replace the formerly used gauge system has been adopted by the wire industry. Plain wires are described by the letter W followed by a number equal to 100 times the cross-sectional area of the wire in square inches. Deformed wire sizes are similarly described, but the letter D is used. Thus a W9 wire has an area of 0.090 in.² and a D8 wire has an area of 0.080 in.². A W8 wire has the same cross-sectional area as the D8 but is plain rather than deformed. Sizes between full numbers are given by decimals, such as W9.5.

Generally, the material is indicated by the symbol WWR, followed by spacings first of longitudinal wires, then of transverse wires, and last by the sizes of longitudinal and transverse wires. Thus WWR6 \times 12-W16 \times W8 indicates a plain WWR with 6-in. longitudinal spacing, 12-in. transverse spacing, and a cross-sectional area equal to 0.16 in.² for the longitudinal wires and 0.08 in.² for the transverse wires.

Additional information about WWR, as well as tables relating size number with wire diameter, area, and weight, may be obtained through the Wire Reinforcement Institute [9] or the Concrete Reinforcing Steel Institute [9 and 10]. ACI 318-11 contains a useful chart that gives area (in.²/ft) for various WWR spacings (see Appendix E).

Most concrete is reinforced in some way to resist tensile forces [Figure 1-4]. Some structural elements, particularly



FIGURE 1-4 Concrete construction in progress. Note formwork, reinforcing bars, and pumping of concrete. (George Limbrunner)

footings, are sometimes made of *plain concrete*, however. Plain concrete is defined as structural concrete with no reinforcement or with less reinforcement than the minimum amount specified for reinforced concrete. Plain concrete is discussed further in Chapter 10.

1-8 BEAMS: MECHANICS OF BENDING REVIEW

The concept of bending stresses in homogeneous elastic beams is generally discussed at great length in all strength of materials textbooks and courses. Beams composed of material such as steel or timber are categorized as homogeneous, with each exhibiting elastic behavior up to some limiting point. Within the limits of elastic behavior, the internal bending stress distribution developed at any cross section is linear (straight line), varying from zero at the neutral axis to a maximum at the outer fibers.

The accepted expression for the maximum bending stress in a beam is termed the *flexure formula*,

$$f_b = \frac{Mc}{I}$$

where

f_b = calculated bending stress at the outer fiber of the cross section

M = the applied moment

c = distance from the neutral axis to the outside tension or compression fiber of the beam

I = moment of inertia of the cross section about the neutral axis

The flexure formula represents the relationship between bending stress, bending moment, and the geometric properties of the beam cross section. By rearranging the flexure formula, the maximum moment that may be applied to the beam cross section, called the *resisting moment*, M_R , may be found:

$$M_R = \frac{F_b I}{c}$$

where F_b = the allowable bending stress.

This procedure is straightforward for a beam of known cross section for which the moment of inertia can easily be found. For a reinforced concrete beam, however, the use of the flexure formula presents some complications, because the beam is not homogeneous and concrete does not behave elastically over its full range of strength. As a result, a somewhat different approach that uses the beam's internal bending stress distribution is recommended. This approach is termed the *internal couple method*.

Recall from strength of materials that a couple is a pure moment composed of two equal, opposite, and parallel forces separated by a distance called the *moment arm*, which is commonly denoted Z . In the internal couple method, the couple represents an internal resisting moment and is

composed of a compressive force C above the neutral axis (assuming a single-span, simply supported beam that develops compressive stress above the neutral axis) and a parallel internal tensile force T below the neutral axis.

As with all couples, and because the forces acting on any cross section of the beam must be in equilibrium, C must equal T . The internal couple must be equal and opposite to the bending moment at the same location, which is computed from the external loads. It represents a couple developed by the bending action of the beam.

The internal couple method of determining beam strength is more general and may be applied to homogeneous or nonhomogeneous beams having linear (straight-line) or nonlinear stress distributions. For reinforced concrete beams, it has the advantage of using the basic resistance pattern found in the beam.

The following three analysis examples dealing with plain (unreinforced) concrete beams provide an introduction to the internal couple method. Note that the unreinforced beams are considered homogeneous and elastic. This is valid if the moment is small and tensile bending stresses in the concrete are low (less than the tensile bending strength of the concrete) with no cracking of the concrete developing. For this condition, the entire beam cross section carries bending stresses. Therefore, the analysis for bending stresses in the uncracked beam can be based on the properties of the gross cross-sectional area using the elastic-based flexure formula. The use of the flexure formula is valid as long as the maximum tensile stress in the concrete does not exceed the modulus of rupture f_r . If a moment is applied that causes the maximum tensile stress just to reach the modulus of rupture, the cross section will be on the verge of cracking. This moment is called the *cracking moment*, M_{cr} .

These examples use both the internal couple approach and the flexure formula approach so that the results may be compared.

Example 1-1

A normal-weight plain concrete beam is 6 in. \times 12 in. in cross section, as shown in Figure 1-5. The beam is simply supported on a span of 4 ft and is subjected to a midspan concentrated load of 4500 lb. Assume $f'_c = 3000$ psi.

- Calculate the maximum concrete tensile stress using the internal couple method.
- Repeat part (a) using the flexure formula approach.
- Compare the maximum concrete tensile stress with the value for modulus of rupture f_r using the ACI-recommended value based on f'_c .

Solution:

Calculate the weight of the beam (weight per unit length):

$$\begin{aligned} \text{weight of beam} &= \text{volume per unit length} \times \text{unit weight} \\ &= \frac{6 \text{ in.}(12 \text{ in.})}{144 \text{ in.}^2/\text{ft}^2} (150 \text{ lb/ft}^3) \\ &= 75 \text{ lb/ft} \end{aligned}$$

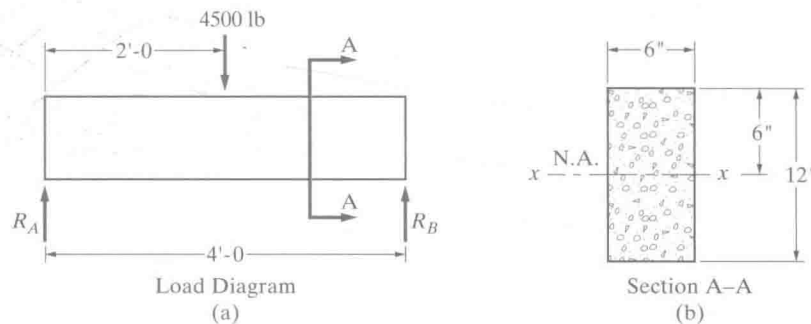


FIGURE 1-5 Sketch for Example 1-1.

Calculate the maximum applied moment:

$$\begin{aligned}
 M_{\max} &= \frac{PL}{4} + \frac{wL^2}{8} \\
 &= \frac{4500 \text{ lb}(4 \text{ ft})}{4} + \frac{75 \text{ lb/ft}(4 \text{ ft})^2}{8} \\
 &= 4650 \text{ ft-lb}
 \end{aligned}$$

a. Internal couple method

1. Because the beam is homogeneous, elastic, and symmetrical with respect to both the X-X and Y-Y axes, the neutral axis (N.A.) is at midheight. Stresses and strains vary linearly from zero at the neutral axis (which is also the centroidal axis) to a maximum at the outer fiber. As the member is subjected to positive moment, the area above the N.A. is stressed in compression and the area below the N.A. is stressed in tension. These stresses result from the bending behavior of the member and are shown in Figure 1-6.
2. C represents the resultant compressive force above the N.A. T represents the resultant tensile force below the N.A. C and T each act at the centroid of their respective triangles of stress distribution. Therefore $Z = 8$ in. C and T must be equal (since

$\Sigma H_F = 0$). The two forces act together to form the internal couple (or internal resisting moment) of magnitude CZ or TZ .

3. The internal resisting moment must equal the bending moment due to external loads at any section. Therefore

$$M = CZ = TZ$$

$$4650 \text{ ft-lb} (12 \text{ in./ft}) = C (8 \text{ in.})$$

from which

$$C = 6975 \text{ lb} = T$$

4. $C = \text{average stress} \times \text{area of beam on which stress acts}$

$$C = \frac{1}{2} f_{\text{top}} (6 \text{ in.})(6 \text{ in.}) = 6975 \text{ lb}$$

Solving for f_{top} yields

$$f_{\text{top}} = 388 \text{ psi} = f_{\text{bott}}$$

b. Flexure formula approach

$$I = \frac{bh^3}{12} = \frac{6(12^3)}{12} = 864 \text{ in.}^4$$

$$f_{\text{top}} = f_{\text{bott}} = \frac{Mc}{I} = \frac{4650(12)(6)}{864} = 388 \text{ psi}$$

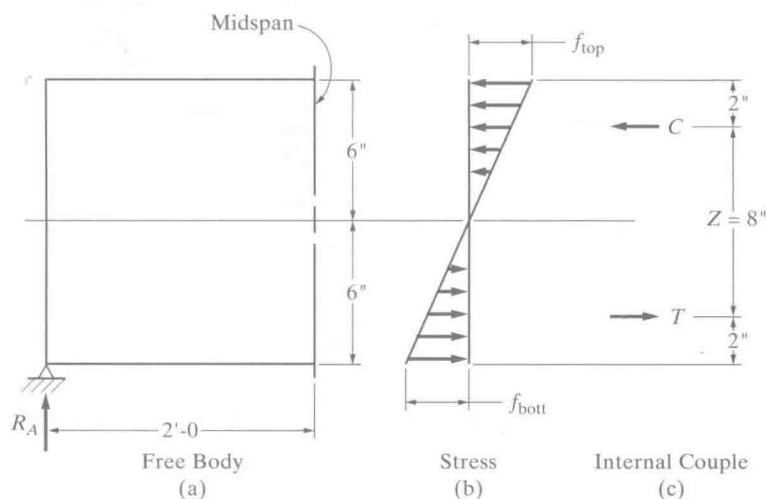


FIGURE 1-6 Sketch for Example 1-1.