Relativistic Quantum Fields

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Preface

The propagator approach to a relativistic quantum theory pioneered in 1949 by Feynman has provided a practical, as well as intuitively appealing, formulation of quantum electrodynamics and a fertile approach to a broad class of problems in the theory of elementary particles. The entire renormalization program, basic to the present confidence of theorists in the predictions of quantum electrodynamics, is in fact dependent on a Feynman graph analysis, as is also considerable progress in the proofs of analytic properties required to write dispersion relations. Indeed, one may go so far as to adopt the extreme view that the set of all Feynman graphs *is* the theory.

We do not advocate this view in this book nor in its companion volume, "Relativistic Quantum Mechanics," nor indeed do we advocate any single view to the exclusion of others. The unsatisfactory status of present-day elementary particle theory does not allow one such a luxury. In particular, we do not wish to minimize the importance of the progress achieved in formal quantum field theory nor the considerable understanding of low-energy meson-nucleon processes given by dispersion theory. However, we give first emphasis to the development of the Feynman rules, proceeding directly from a particle wave equation for the Dirac electron, integrated with hole-theory boundary conditions.

Three main convictions guiding us in this approach were the primary motivation for undertaking these books:

1. The Feynman graphs and rules of calculation summarize quantum field theory in a form in close contact with the experimental numbers one wants to understand. Although the statement of the theory in terms of graphs may imply perturbation theory, use of graphical methods in the many-body problem shows that this formalism is flexible enough to deal with phenomena of nonperturbative character (for example, superconductivity and the hard-sphere Bose gas).

2. Some modification of the Feynman rules of calculation may well outlive the elaborate mathematical structure of local canonical quantum field theory, based as it is on such idealizations as fields defined at points in space-time. Therefore, let us develop these rules first, independently of the field theory formalism which in time may come to be viewed more as a superstructure than as a foundation.

3. Such a development, more direct and less formal—if less compelling—than a deductive field theoretic approach, should bring quantitative calculation, analysis, and understanding of Feynman graphs into the bag of tricks of a much larger community of physicists than the specialized narrow one of second quantized theorists. In particular, we have in mind our experimental colleagues and students interested in particle physics. We believe this would be a healthy development.

Our original idea of one book has grown in time to two volumes. In the first book, "Relativistic Quantum Mechanics," we develop a propagator theory of Dirac particles, photons, and Klein-Gordon mesons and perform a series of calculations designed to illustrate various useful techniques and concepts in electromagnetic, weak, and strong interactions. These include defining and implementing the renormalization program and evaluating effects of radiative corrections, such as the Lamb shift, in low-order calculations. The necessary background for this book is provided by a course in nonrelativistic quantum mechanics at the general level of Schiff's text "Quantum Mechanics."

In the second book, "Relativistic Quantum Fields," we develop canonical field theory, and after constructing closed expressions for propagators and for scattering amplitudes with the *LSZ* reduction technique, return to the Feynman graph expansion. The perturbation expansion of the scattering amplitude constructed by canonical field theory is shown to be identical with the Feynman rules in the first book. With further graph analysis we study analyticity properties of Feynman amplitudes to arbitrary orders in the coupling parameter and illustrate dispersion relation methods. Finally, we prove the finiteness of renormalized quantum electrodynamics to each order of the interaction.

Without dwelling further on what we do, we may list the major topics we omit from discussion in these books. The development of action principles and a formulation of quantum field theory from a variational approach, spearheaded largely by Schwinger, are on the whole ignored. We refer to action variations only in search of symmetries. There is no detailed discussion of the powerful developments in axiomatic field theory on the one hand and the purely *S*-matrix approach, divorced from field theory, on the other. Aside from a discussion of the Lamb shift and the hydrogen atom spectrum in the first book, the bound-state problem is ignored. Dynamical applications of the dispersion relations are explored only minimally. A formulation of a quantum field theory for massive vector mesons is not given—nor is a formulation of any quantum field theory with derivative couplings. Finally, we have not prepared a bibliography of all the significant original papers underlying many of the developments recorded in these books. Among the following recent excellent books or monographs is to be found the remedy for one or more of these deficiencies:

Schweber, S.: "An Introduction to Relativistic Quantum Field Theory," New York, Harper & Row, Publishers, Inc., 1961.

Jauch, J. M., and F. Rohrlich: "The Theory of Photons and Electrons," Cambridge, Mass., Addison-Wesley Publishing Company, Inc., 1955.

Bogoliubov, N. N., and D. V. Shirkov: "Introduction to the Theory of Quantized Fields," New York, Interscience Publishers, Inc., 1959.

Akhiezer, A., and V. B. Bereztetski: "Quantum Electrodynamics," 2d ed., New York, John Wiley & Sons, Inc., 1963.

Umezawa, H.: "Quantum Field Theory," Amsterdam, North Holland Publishing Company, 1956.

Hamilton, J.: "Theory of Elementary Particles," London, Oxford University Press, 1959.

Mandl, F.: "Introduction to Quantum Field Theory," New York, Interscience Publishers, Inc., 1960.

Roman, P.: "Theory of Elementary Particles," Amsterdam, North Holland Publishing Company, 1960.

Wentzel, G.: "Quantum Theory of Field," New York, Interscience Publishers, Inc., 1949.

Schwinger, J.: "Quantum Electrodynamics," New York, Dover Publications, Inc., 1958.

Feynman, R. P.: "Quantum Electrodynamics," New York, W. A. Benjamin, Inc., 1962.

Klein, L. (ed.): "Dispersion Relations and the Abstract Approach to Field Theory," New York, Gordon and Breach, Science Publishers, Inc., 1961

Screaton, G. R. (ed.): "Dispersion Relations; Scottish Universities Summer School," New York, Interscience Publishers, Inc., 1961.

Chew, G. F.: "S-Matrix Theory of Strong Interactions," New York, W. A. Benjamin, Inc., 1962.

In conclusion, we owe thanks to the many students and colleagues who have been invaluable critics and sounding boards as our books evolved from lectures into chapters, to Prof. Leonard I. Schiff for important initial encouragement and support to undertake the writing of these books, and to Ellen Mann and Rosemarie Stampfel for marvelously cooperative secretarial help.

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General Formalism

ntuitive and correspondence arguments were used in "Relativistic Quantum Mechanics" in developing the propagator approach and giving practical rules for calculating, in perturbation theory, interactions of relativistic particles. We now turn to a systematic derivation of these rules from the formalism of quantized fields. Our motivation is first to "patch up the holes" in our arguments in the propagator approach and then to develop a formalism which might be applied to problems for which perturbation theory is not adequate, such as processes involving strongly coupled mesons and nucleons.

Our approach is best illustrated by the electromagnetic field. The potentials $A^\mu(x)$ satisfy the Maxwell wave equations and may be considered as describing a dynamical system with an infinite number of degrees of freedom. By this we mean that $A^\mu(x)$ at each point of space may be considered an independent generalized coordinate. To make the transition from classical to quantum theory, we must, according to the general principles proclaimed in Chap. 1,[1] elevate coordinates and their conjugate momenta to operators in the Hilbert space of possible physical states and impose quantum conditions upon them. This is the canonical quantization procedure. It is a straightforward extension to field functions, which obey differential wave equations derivable from a lagrangian, of the quantization procedure of nonrelativistic mechanics. When it is done, there emerges a particle interpretation of the electromagnetic field—in the sense of Bohr's principle of complementarity.

If photons emerge in such a natural way from the quantization of the Maxwell field, one is led to ask whether other particles whose existence is observed in nature are also related to force fields by the same quantization procedure. On this basis Yukawa predicted the existence of the π meson from knowledge of the existence of nuclear forces. Conversely, it is natural from this point of view to associate with each kind of observed particle in nature a field $\varphi(x)$ which satisfies an assumed wave equation. A particle interpretation of the field φ is then obtained when we carry through the canonical quantization program.

In such a program we must first define the momenta $\pi(x)$ conjugate to the field coordinates $\varphi(x)$. We do this in terms of a lagrangian, from which the wave equation for each field $\varphi(x)$ as well as the conjugate momenta are derivable. Applying the canonical quantization procedure with the commutator condition of Chap. 1, we obtain field quanta, such as photons, which obey Bose statistics. In order to

[1] References to Chaps. 1 to 10 or parts thereof are references to the companion volume, "Relativistic Quantum Mechanics."

describe Fermi particles which obey an exclusion principle with a similar quantum field formalism, it turns out to be necessary only to replace the quantum commutator conditions by anticommutator relations.

In this way a unified formalism which provides a basis for the description of both kinds of particles can be constructed. An additional attractive feature of the lagrangian approach which will be seen shortly is that it leads directly to the conservation laws.

11.1 Implications of a Description in Terms of Local Fields

Before continuing and exploring the consequences of applying the quantization procedure to classical fields which satisfy wave equations, it is perhaps worthwhile to discuss the implications of such a program. The first is that we are led to a theory with differential wave propagation. The field functions are continuous functions of continuous parameters $\mathbf{x}$ and t, and the changes in the fields at a point $\mathbf{x}$ are determined by properties of the fields infinitesimally close to the point $\mathbf{x}$.

For most wave fields (for example, sound waves and the vibrations of strings and membranes) such a description is an idealization which is valid for distances larger than the characteristic length which measures the granularity of the medium. For smaller distances these theories are modified in a profound way.

The electromagnetic field is a notable exception. Indeed, until the special theory of relativity obviated the necessity of a mechanistic interpretation, physicists made great efforts to discover evidence for such a mechanical description of the radiation field. After the requirement of an "ether" which propagates light waves had been abandoned, there was considerably less difficulty in accepting this same idea when the observed wave properties of the electron suggested the introduction of a new field $\psi(x)$. Indeed there is no evidence of an ether which underlies the electron wave $\psi(\mathbf{x},t)$. However, it is a gross and profound extrapolation of present experimental knowledge to assume that a wave description successful at "large" distances (that is, atomic lengths $\approx 10^{-8}$ cm) may be extended to distances an indefinite number of orders of magnitude smaller (for example, to less than nuclear lengths $\approx 10^{-13}$ cm).

In the relativistic theory, we have seen that the assumption that the field description is correct in arbitrarily small space-time intervals has led—in perturbation theory—to divergent expressions for the electron self-energy and the "bare charge." Renormalization theory has sidestepped these divergence difficulties, which may be indicative of the failure of the perturbation expansion. However, it is widely felt that the divergences are symptomatic of a chronic disorder in the small-distance behavior of the theory.

We might then ask why local field theories, that is, theories of fields which can be described by differential laws of wave propagation, have been so extensively used and accepted. There are several reasons, including the important one that with their aid a significant region of agreement with observations has been found, examples of which have already appeared in the discussions of the companion volume. But the foremost reason is brutally simple: there exists no convincing form of a theory which avoids differential field equations.

A theory of the interaction of relativistic particles is necessarily of great mathematical complexity. Because of the existence of creation and annihilation processes it is at once a theory of the many-body problem. At the present time one knows how to develop only approximate solutions to this problem, and therefore the predictions of any such theory are incomplete and at best somewhat ambiguous.

Faced with this situation, the most reasonable course to steer in constructing theories is to retain the general principles which have worked before in a more restricted domain. In this case, this includes the prescription for quantization which strongly involves the existence of a hamiltonian H. However, since H generates infinitesimal time displacements according to the Schrödinger equation, we are led to a description with differential development in time. Lorentz invariance then requires a differential development in space as well. A hamiltonian may well not exist for a nonlocal "granular" theory; if it does not, the link connecting us with the quantization methods of nonrelativistic theories is broken.

If we simply retain the notion of a Lorentz-invariant microscopic description in terms of continuous coordinates $\mathbf{x}$ and t, we expect that the influence of interactions should not propagate through space-time with velocity faster than c. This notion of "microscopic causality" strongly forces us into the field concept. Even if there is a granularity at small distances, if we are to retain microcausality the influence of one "granule" upon the next must be retarded; the most natural way to describe this is with additional fields. The problem thus becomes more complicated, without corresponding gain in understanding.

There is no concrete experimental evidence of a granularity at small distances.[1] There is likewise nothing but positive evidence that

[1] In quantum electrodynamics there exists an agreement between theory and experiment to very great precision in both low- and high-energy processes. See, for example, R. P. Feynman, *Rept. Solvay Congr., Brussels*, Interscience Publishers, Inc., New York, 1961.

special relativity is correct in the high-energy domain, and furthermore, there is, if anything, positive evidence[1] that the notion of microscopic causality is a correct hypothesis. Since there exists no alternative theory which is any more convincing, we shall hereafter restrict ourselves to the formalism of local, causal fields. It is undoubtedly true that a modified theory must have local field theory as an appropriate large-distance approximation or correspondence. However, we again emphasize that the formalism we develop may well describe only the large-distance limit (that is, distances $> 10^{-13}$ cm) of a physical world of considerably different submicroscopic properties.

11.2 The Canonical Formalism and Quantization Procedure for Particles

To preface our development, we recall the familiar path to the quantization of a classical dynamical system in particle mechanics. For purposes of illustration consider the one-dimensional motion of a particle in a conservative force field. We let q be the (generalized) coordinate of the particle, $\dot{q} = dq/dt$ the velocity, and $L(q,\dot{q})$ the lagrangian. According to Hamilton's principle, the dynamics of the particle is determined by the condition

$$\delta J = \delta \int_{t_1}^{t_2} L(q,\dot{q})\, dt = 0 \tag{11.1}$$

Equation (11.1) states that the actual physical path $q(t)$ which the particle follows in traversing the interval from (q_1,t_1) to (q_2,t_2) is that along which the action J is stationary. Thus small variations from this path, $q(t) \to q(t) + \delta q(t)$, as shown in Fig. 11.1, leave the action unchanged to first order in the variation.

Hamilton's principle leads directly to the Euler-Lagrange equations of motion[2]

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \tag{11.2}$$

In order to carry out the formal quantization of this equation, we rewrite it in hamiltonian form. We do so by defining the momentum p conjugate to q,

$$p = \frac{\partial L}{\partial \dot{q}} \tag{11.3}$$

[1] We mean by this the experimental verification of the dispersion relations for forward pion-nucleon scattering, to be discussed in Chap. 18.

[2] Cf. H. Goldstein, "Classical Mechanics," Addison-Wesley Publishing Company, Inc., Reading, Mass., 1950. The form of (11.2) applies for no higher than the first derivative of the coordinates appearing in L.

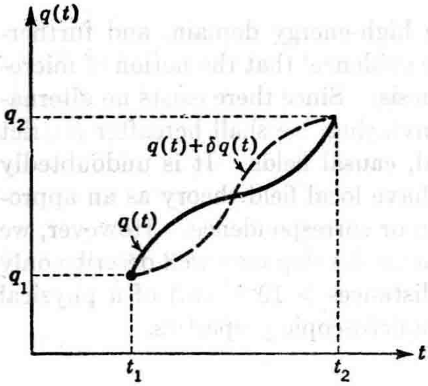

Fig. 11.1 Variation in path with fixed end points for action principle.

and introducing the hamiltonian by the Legendre transformation

$$H(p,q) = p\dot{q} - L(q,\dot{q}) \tag{11.4}$$

In terms of H, the equation of motion (11.2) becomes

$$\{H,q\}_{\rm PB} = \frac{\partial H}{\partial p} = \dot{q} \qquad \text{and} \qquad \{H,p\}_{\rm PB} = -\frac{\partial H}{\partial q} = \dot{p} \tag{11.5}$$

where $\{\ \}_{\rm PB}$ means a Poisson bracket.

To quantize (11.5), we let q become a hermitian operator in a Hilbert space and replace p by $-i\,\partial/\partial q$ so that the conjugate momentum and coordinate satisfy a commutator relation

$$[p,q] = -i \tag{11.6}$$

corresponding to the classical Poisson bracket $\{p,q\}_{\rm PB} = 1$. With this definition, p is also hermitian. The dynamics of the particle is contained in the Schrödinger equation

$$H(p,q)\Psi(t) = i\frac{\partial\Psi(t)}{\partial t} \tag{11.7}$$

where Ψ is the wave function, or state vector, in the Hilbert space. If we specify the initial state Ψ at an arbitrary time, say $t = 0$, the Schrödinger equation determines the state and hence physical expectation values at all future times.

This formulation of the time development of the motion of the particle, with the time dependence carried in Ψ while the operators p and q are not time-dependent, is known as the Schrödinger picture. Alternatively, we may express the time development of the motion in a different language in which the operators $p(t)$ and $q(t)$ carry the time dependence instead of the state vectors Ψ. This is known as the