STATISTICAL TABLES FOR THE SOCIAL, BIOLOGICAL AND PHYSICAL SCIENCES

F. C. Powell

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compiled by

F. C [illegible]OWELL

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The Rand Corporation. *A Million Random Digits* (The Free Press, Glencoe, Ill. 1955)

Conover, W. J. *Practical Nonparametric Statistics* (Wiley, New York 1971)

In addition, many of the tables are based on material which has appeared in the following:

Journals

Annals of Mathematical Statistics

Biometrics

Biometrika

Journal of the American Statistical Association

Books

Comrie, L. J. *Chambers's Shorter Six-Figure Mathematical Tables* (Chambers, Edinburgh 1966)

Harter, H. L. & Owen, D. B. *Selected Tables in Mathematical Statistics*, vol. 1, sponsored by the Institute of Mathematical Statistics (Markham, Chicago 1970)

Harvard University Computation Laboratory *Annals*, vol. 35: *Tables of the Cumulative Binomial Probability Distribution* (Harvard University Press 1955)

Hollander, M. & Wolfe, D. A. *Nonparametric Statistical Methods* (Wiley, New York 1973)

Miller, J. C. P. & Powell, F. C. *The Cambridge Elementary Mathematical Tables* (Cambridge University Press 1979)

Molina, E. C. *Poisson's Exponential Binomial Limit* (van Nostrand, New York 1942)

Owen, D. B. *Handbook of Statistical Tables* (Addison-Wesley, Reading, Mass. 1962)

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CONTENTS

EXPLANATION OF STATISTICAL TERMS AND PROCEDURES

Tail probabilities. The *lower tail probability* P and the *upper tail probability* Q of a random variable X are defined as:

$$P(x) = \text{Prob}(X \leqslant x) \qquad Q(x) = \text{Prob}(X \geqslant x)$$

$P(x)$ is also called the *cumulative probability distribution function.* If $P(x)$ and $Q(x)$ are continuous functions of x:

$$P(x)+Q(x) = 1$$

If X is a discrete variable taking integer values:

$$P(x)+Q(x+1) = 1$$

Quantiles. There is no generally agreed definition or notation for these. Here the following definitions are adopted. For $P \leqslant \frac{1}{2}$ the *lower quantile* $x_{[P]}$ is the smallest x such that $\text{Prob}(X \leqslant x) \geqslant P$. Equivalently:

$$\text{Prob}(X \leqslant x_{[P]}) \geqslant P \qquad \text{Prob}(X \leqslant x) < P \quad \text{if} \quad x < x_{[P]}$$

For $P \geqslant \frac{1}{2}$ the *upper quantile* $x_{[P]}$ is the largest x such that $\text{Prob}(X \geqslant x) \geqslant 1-P$ or Q. Equivalently:

$$\text{Prob}(X \geqslant x_{[P]}) \geqslant 1-P \qquad \text{Prob}(X \geqslant x) < 1-P \quad \text{if} \quad x > x_{[P]}$$

For a continuous distribution (more precisely a distribution having a continuous and strictly increasing cumulative distribution function) these definitions reduce to:

$$\text{Prob}(X < x_{[P]}) = P \quad \text{or equivalently} \quad \text{Prob}(X > x_{[P]}) = 1-P \text{ or } Q$$

Graphical illustrations. (1) Consider first a variable X having the standard normal probability distribution. In fig. 1 the ordinate represents the probability density; the areas under the graph represent tail probabilities. The functions $P(x)$ and $Q(x)$ are graphed in fig. 2. In fig. 3 $x_{[P]}$ is graphed as a function of P.

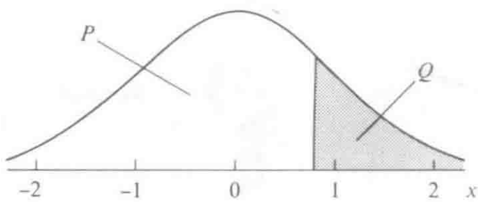

Fig. 1

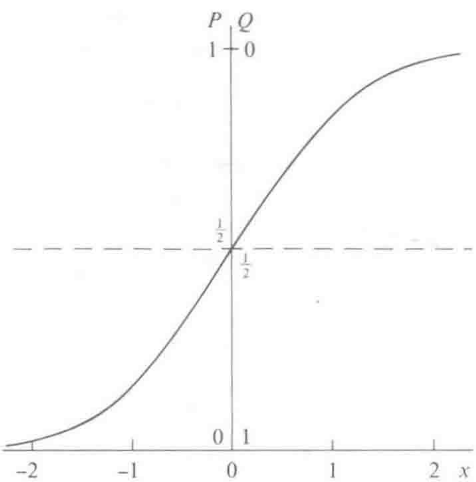

Fig. 2

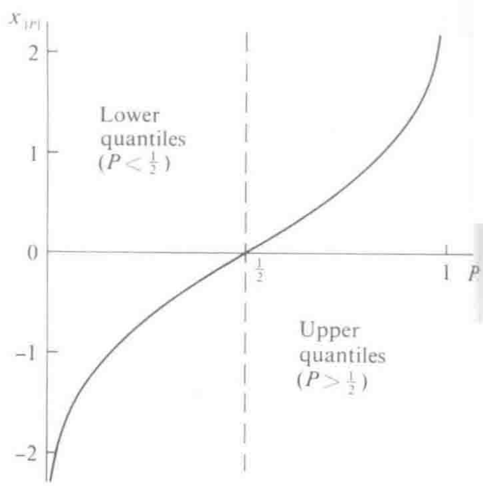

Fig. 3

(2) Next consider a variable X having the binomial distribution $\text{Bi}(2, \frac{1}{3})$.† It takes values 0, 1, 2 with probabilities $\frac{4}{9}$, $\frac{4}{9}$, $\frac{1}{9}$ respectively. The tail probabilities are now step functions of x, as shown in fig. 4; the values of $P(x)$ and $Q(x)$ for $x = 0, 1, 2$ are indicated by crosses and circles respectively.

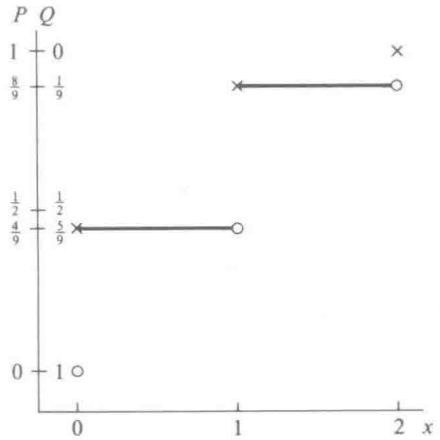

Fig. 4

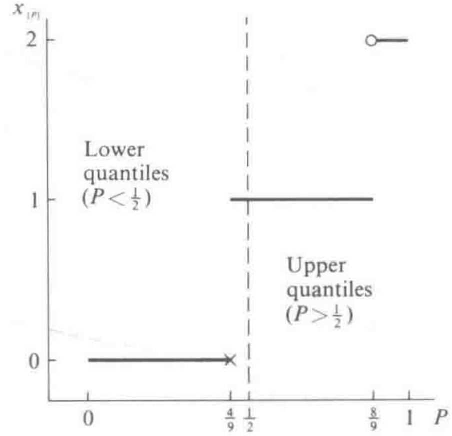

Fig. 5

In fig. 5 $x_{[P]}$ is graphed as a function of P. For $P = \frac{4}{9}$ and $\frac{8}{9}$ the values indicated by the cross and the circle are to be taken.

Interpolation. Many of the tables give probabilities or quantiles for a family of distributions specified by the values of one or more parameters, e.g. $t(\nu)$, $\chi^2(\nu)$, $\text{Bi}(n, \pi)$. It is not usually necessary or possible to include all values of the parameters and use must be made of interpolation. These tables have been constructed with *linear interpolation* in mind; tabular intervals and the number of significant figures have been chosen accordingly.

Linear interpolation is based on the assumption that the tabulated function changes at a constant rate between one tabulated value and the next. If successive tabular entries y_1 and $y_2 = y_1 + \Delta$ correspond to values x_1 and $x_2 = x_1 + h$ of the independent variable, the value of the function corresponding to an intermediate value $x = x_1 + \theta h$ (where $0 < \theta < 1$) is taken to be $y = y_1 + \theta\Delta$ or, equivalently, $(1-\theta)y_1 + \theta y_2$. For example, suppose we have to find the upper tail probability $Q(1)$ for the binomial distribution $\text{Bi}(2, \pi)$ with $\pi = 0.408$. Now for $\pi = 0.40$ the value of $Q(1)$ is 0.6400, for $\pi = 0.42$ it is 0.6636 (see p. 15), and so $\Delta = 0.0236$. For $\pi = 0.408$, $\theta = 0.4$, and so $Q(1) \approx 0.6400 + 0.4 \times 0.0236 \approx 0.6494$. Alternatively we may calculate $Q(1)$ as $0.6 \times 0.6400 + 0.4 \times 0.6636$ with the same result.

The errors introduced by linear interpolation should be borne in mind. Where tabulation is at equal intervals of a parameter, these errors cannot exceed $1 + S/8$ units in the last decimal place, where S is the absolute value of the second difference of the tabular values (i.e. the difference between successive values of the first difference Δ). In the example above, $S = 8$ and so the error cannot exceed 2 units. The true value is in fact 0.649 536 and the error is 0.000 136.

In a number of tables interpolation should be linear in an inverse power of the parameter (usually n or ν). Suppose, for example, we wish to find $t_{[.999]}(50)$. From the table on p. 39 we have $t_{[.999]}(40) = 3.307$ and $t_{[.999]}(60) = 3.232$; interpolation should be linear in $120/\nu$. Now $120/40 = 3$, $120/60 = 2$ and $120/50 = 2.4$, so

$$t_{[.999]}(50) \approx 3.307 + \frac{2.4-3}{2-3} \times (3.232 - 3.307)$$

$$= 3.307 - 0.6 \times 0.075$$

$$= 3.262$$

The true value is 3.261...

† We write $X \sim \text{Bi}(2, \frac{1}{3})$.

Approximations. It may be that the probability distribution of a discrete random variable r approximates to the known distribution of a continuous random variable, e.g. $N(\mu, \sigma^2)$. Then $N(0, 1)$ is an approximation to the distribution of $(r-\mu)/\sigma$. This means that the tail probabilities $P(r)$ and $Q(r)$ of the r-distribution are nearly equal to the corresponding tail probabilities $P(z)$ and $Q(z)$ of $N(0, 1)$ with $z = (r-\mu)/\sigma$.

A *correction for continuity* should however be made. If r takes integer values, $P(r)$ is given more correctly by $P(z)$ with $z = (r-\mu+\frac{1}{2})/\sigma$ and $Q(r)$ is given by $Q(z)$ with $z = (r-\mu-\frac{1}{2})/\sigma$. Values of $Q(z)$ are given on p. 31. We can also use $z = (r-\mu-\frac{1}{2})/\sigma$ as a test statistic by referring it to the table of upper quantiles on p. 30.

Lower quantiles of r are given approximately by:

$$r_{[P]} \approx \mu + \sigma z_{[P]} - \tfrac{1}{2}$$

and upper quantiles by:

$$r_{[P]} \approx \mu + \sigma z_{[P]} + \tfrac{1}{2}$$

These corrections for continuity are not usually recommended if the approximation is conservative (i.e. if it overestimates tail probabilities).

In some tables the variable r takes even values only or odd values only. Corrections of ± 1 should then be made instead of $\pm\frac{1}{2}$.

Tests of hypotheses. The procedure and terminology used in many tests can be explained by means of an example. Suppose that a theory predicts that the value x of a physical quantity is 10 units, the alternative being $x < 10$. To test the theory, a series of twelve measurements is made by a method that is liable to a normally distributed error with zero mean and standard deviation 6 units. The average of the twelve measurements is 6.3 units. Does this result conflict with the theory?

The twelve measurements can be regarded as a *random sample* from an *infinite population* of measurements having a normal probability distribution with unknown mean μ and variance 6^2, $N(\mu, 6^2)$. The object is to decide between the *null hypothesis* H_0 that $\mu = 10$ and the *alternative hypothesis* H_1 that $\mu < 10$. As *test statistic* we take the sample mean $\bar{x}$. If we repeatedly take samples of size twelve from the population we build up the *sampling distribution* of $\bar{x}$; this is a normal distribution with mean μ and variance $6^2/12$ (see p. 46). Under H_0 the sampling distribution of $\bar{x}$ is thus $N(10, 3)$; this is the *null distribution* of $\bar{x}$.

If m is a value of $\bar{x}$ in the extreme lower tail of the null distribution, Prob $(\bar{x} \leqslant m)$ is smaller for the null distribution than for a distribution with $\mu < 10$ (see fig. 6). If the observed value of $\bar{x}$ is less

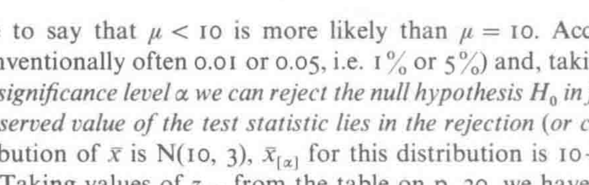

Fig. 6

than m it is plausible to say that $\mu < 10$ is more likely than $\mu = 10$. Accordingly we choose a *significance level* α (conventionally often 0.01 or 0.05, i.e. 1% or 5%) and, taking m to be the quantile $\bar{x}_{[\alpha]}$, we declare that *at significance level* α *we can reject the null hypothesis* H_0 *in favour of the alternative hypothesis* H_1 *if the observed value of the test statistic lies in the rejection* (*or critical*) *region* $\bar{x} < \bar{x}_{[\alpha]}$.

Since the null distribution of $\bar{x}$ is $N(10, 3)$, $\bar{x}_{[\alpha]}$ for this distribution is $10+\sqrt{3}\, z_{[\alpha]}$, where $z_{[\alpha]}$ is a quantile of $N(0, 1)$. Taking values of $z_{[\alpha]}$ from the table on p. 30, we have:

$$\bar{x}_{[.01]} = 10 - \sqrt{3} \times 2.326 = 5.97 \quad \bar{x}_{[.05]} = 10 - \sqrt{3} \times 1.645 = 7.15$$

The observed value of $\bar{x}$, 6.3, lies between these quantiles. Accordingly we reject the null hypothesis (that $\mu = 10$) in favour of the alternative hypothesis (that $\mu < 10$) at significance level 0.05 but not at 0.01.

One-sided and two-sided tests. In the example discussed above the alternative hypothesis is that $\mu < 10$; the test is thus *one-sided.* The rejection region of the null distribution at significance level α is in the lower tail of the null distribution; thus the test is also a *one-tail* test.

If, instead, the alternative hypothesis is taken to be $\mu > 10$, the test is again one-sided. The appropriate rejection region at significance level α is defined by $\bar{x} > \bar{x}_{[1-\alpha]}$, and again the test is a one-tail test.

If the alternative hypothesis is taken to be $\mu \neq 10$ we have a *two-sided* test. We now reject H_0 in favour of H_1 at significance level α if $\bar{x} < \bar{x}_{[\alpha/2]}$ or $\bar{x} > \bar{x}_{[1-\alpha/2]}$. The rejection region lies partly in the lower tail and partly in the upper tail of the null distribution; thus we have a *two-tail* test.

Two-sided tests are not necessarily two-tailed. For instance, if a random sample has been taken from a normal population with unknown mean and variance, the null hypothesis $\mu = \mu_0$ can be tested against the alternative hypothesis $\mu \neq \mu_0$ either by a two-tail t-test or by a one-tail F-test; see p. 46.

Critical values. Consider first a test statistic U taking *discrete values.* For a rejection region $U < u_{[\alpha]}$, the *critical value* $u_{(\alpha)}$ is defined as the largest value of U in the region; it is the next value of U below $u_{[\alpha]}$. If U takes integer values:

$$u_{(\alpha)} = u_{[\alpha]} - 1$$

The rejection region is $U \leqslant u_{(\alpha)}$.

For a rejection region $U > u_{[1-\alpha]}$, the *critical value* $u_{(1-\alpha)}$ is the smallest value of u in the region; it is the next value of U above $u_{[1-\alpha]}$. If U takes integer values:

$$u_{(1-\alpha)} = u_{[1-\alpha]} + 1$$

The rejection region is $U \geqslant u_{(1-\alpha)}$.

Note that a rejection region contains the corresponding critical value but not the corresponding quantile.

For a test statistic U with a *continuous range of values*, critical values are identical with the corresponding quantiles:

$$u_{(\alpha)} = u_{[\alpha]} \qquad u_{(1-\alpha)} = u_{[1-\alpha]}$$

Ties. In some tests involving the ranking of observations, according to their magnitudes perhaps, special procedures are required for assessing significance when ties occur; these are not described here. It is however often sufficient to assign to each of a tied group of observations their 'average rank', i.e. the average of the ranks they would have been given if their magnitudes had been unequal. If, for instance, four observations are tied 'equal fifth', they are each given the rank $(5+6+7+8)/4 = 6\frac{1}{2}$. In some tests tied observations can safely be discarded provided that they are not too numerous.

Confidence intervals. Consider again the example discussed above. Suppose that the average of twelve measurements of a physical quantity, by a method liable to a normally distributed error with zero mean and standard deviation 6 units, is 6.3 units. If the true value is μ, the probability distribution of the measured value x is $N(\mu, 36)$ and the distribution of the sample mean $\bar{x}$ is $N(\mu, 3)$. Therefore, with probability $1-\alpha$,

$$\bar{x} - \mu > \sqrt{3}\, z_{[\alpha]}$$

where $z_{[\alpha]}$ is a quantile of $N(0, 1)$, so that

$$\mu < \bar{x} - \sqrt{3}\, z_{[\alpha]}$$

For given $\bar{x}$ this defines a *one-sided confidence interval* for μ. At confidence level $1-\alpha$ we may expect μ to lie in the interval (but note that we cannot say that μ lies in the interval with probability $1-\alpha$).

For $\alpha = 0.01$ and $\bar{x} = 6.3$, the upper bound of the confidence interval is $6.3 + \sqrt{3} \times 2.326$ or 10.33; for $\alpha = 0.05$ it is $6.3 + \sqrt{3} \times 1.645$ or 9.15. Therefore the predicted value of 10 units does not conflict with the measurements at confidence level 0.99, but it does conflict at level 0.95. This conclusion agrees with that reached earlier by a different procedure.

Upper and lower bounds of *two-sided confidence intervals* can be found in a similar way. An example is given on p. 47.

FACTORIALS AND LOGARITHMS OF FACTORIALS (1–199)

n	†$n!$	lg $n!$	n	†$n!$	lg $n!$	n	†$n!$	lg $n!$	n	†$n!$	lg $n!$
0	1	0.0000	50	3.0414	64.4831	100	9.3326	157.9700	150	5.7134	262.7569
1	1	0.0000	51	1.5511	66.1906	101	9.4259	159.9743	151	8.6272	264.9359
2	2	0.3010	52	8.0658	67.9066	102	9.6145	161.9829	152	1.3113	267.1177
3	6	0.7782	53	4.2749	69.6309	103	9.9029	163.9958	153	2.0063	269.3024
4	24	1.3802	54	2.3084	71.3633	104	1.0299	166.0128	154	3.0898	271.4899
5	120	2.0792	55	1.2696	73.1037	105	1.0814	168.0340	155	4.7891	273.6803
6	720	2.8573	56	7.1100	74.8519	106	1.1463	170.0593	156	7.4711	275.8734
7	5040	3.7024	57	4.0527	76.6077	107	1.2265	172.0887	157	1.1730	278.0692
8	40320	4.6055	58	2.3506	78.3712	108	1.3246	174.1221	158	1.8533	280.2679
9	3.6288	5.5598	59	1.3868	80.1420	109	1.4439	176.1595	159	2.9467	282.4693
10	3.6288	6.5598	60	8.3210	81.9202	110	1.5882	178.2009	160	4.7147	284.6734
11	3.9917	7.6012	61	5.0758	83.7055	111	1.7630	180.2462	161	7.5907	286.8802
12	4.7900	8.6803	62	3.1470	85.4979	112	1.9745	182.2955	162	1.2297	289.0898
13	6.2270	9.7943	63	1.9826	87.2972	113	2.2312	184.3485	163	2.0044	291.3020
14	8.7178	10.9404	64	1.2689	89.1034	114	2.5436	186.4054	164	3.2872	293.5168
15	1.3077	12.1165	65	8.2477	90.9163	115	2.9251	188.4661	165	5.4239	295.7343
16	2.0923	13.3206	66	5.4434	92.7359	116	3.3931	190.5306	166	9.0037	297.9544
17	3.5569	14.5511	67	3.6471	94.5619	117	3.9699	192.5988	167	1.5036	300.1771
18	6.4024	15.8063	68	2.4800	96.3945	118	4.6845	194.6707	168	2.5261	302.4024
19	1.2165	17.0851	69	1.7112	98.2333	119	5.5746	196.7462	169	4.2691	304.6302
20	2.4329	18.3861	70	1.1979	100.0784	120	6.6895	198.8254	170	7.2574	306.8606
21	5.1091	19.7083	71	8.5048	101.9297	121	8.0943	200.9082	171	1.2410	309.0938
22	1.1240	21.0508	72	6.1234	103.7870	122	9.8750	202.9945	172	2.1346	311.3293
23	2.5852	22.4125	73	4.4701	105.6503	123	1.2146	205.0844	173	3.6928	313.5674
24	6.2045	23.7927	74	3.3079	107.5196	124	1.5061	207.1779	174	6.4254	315.8079
25	1.5511	25.1906	75	2.4809	109.3946	125	1.8827	209.2748	175	1.1244	318.0509
26	4.0329	26.6056	76	1.8855	111.2754	126	2.3722	211.3751	176	1.9790	320.2965
27	1.0889	28.0370	77	1.4518	113.1619	127	3.0127	213.4790	177	3.5029	322.5444
28	3.0489	29.4841	78	1.1324	115.0540	128	3.8562	215.5862	178	6.2351	324.7948
29	8.8418	30.9465	79	8.9462	116.9516	129	4.9745	217.6967	179	1.1161	327.0477
30	2.6525	32.4237	80	7.1569	118.8547	130	6.4669	219.8107	180	2.0090	329.3030
31	8.2228	33.9150	81	5.7971	120.7632	131	8.4716	221.9280	181	3.6362	331.5606
32	2.6313	35.4202	82	4.7536	122.6770	132	1.1182	224.0485	182	6.6179	333.8207
33	8.6833	36.9387	83	3.9455	124.5961	133	1.4873	226.1724	183	1.2111	336.0832
34	2.9523	38.4702	84	3.3142	126.5204	134	1.9929	228.2995	184	2.2284	338.3480
35	1.0333	40.0142	85	2.8171	128.4498	135	2.6905	230.4298	185	4.1225	340.6152
36	3.7199	41.5705	86	2.4227	130.3843	136	3.6590	232.5634	186	7.6679	342.8847
37	1.3764	43.1387	87	2.1078	132.3238	137	5.0129	234.7001	187	1.4339	345.1565
38	5.2302	44.7185	88	1.8548	134.2683	138	6.9178	236.8400	188	2.6957	347.4307
39	2.0398	46.3096	89	1.6508	136.2177	139	9.6157	238.9830	189	5.0949	349.7071
40	8.1592	47.9116	90	1.4857	138.1719	140	1.3462	241.1291	190	9.6803	351.9859
41	3.3453	49.5244	91	1.3520	140.1310	141	1.8981	243.2783	191	1.8489	354.2669
42	1.4050	51.1477	92	1.2438	142.0948	142	2.6954	245.4306	192	3.5500	356.5502
43	6.0415	52.7811	93	1.1568	144.0632	143	3.8544	247.5860	193	6.8514	358.8358
44	2.6583	54.4246	94	1.0874	146.0364	144	5.5503	249.7443	194	1.3292	361.1236
45	1.1962	56.0778	95	1.0330	148.0141	145	8.0479	251.9057	195	2.5919	363.4136
46	5.5026	57.7406	96	9.9168	149.9964	146	1.1750	254.0700	196	5.0801	365.7059
47	2.5862	59.4127	97	9.6193	151.9831	147	1.7272	256.2374	197	1.0008	368.0003
48	1.2414	61.0939	98	9.4269	153.9744	148	2.5563	258.4076	198	1.9816	370.2971
49	6.0828	62.7841	99	9.3326	155.9700	149	3.8089	260.5808	199	3.9433	372.5959

† If n is greater than 8, multiply the tabulated value by 10^c, where c is the integer part of lg $n!$. Thus $99! = 9.3326 \times 10^{155}$.

The definition of $n!$ is:

$$n! = n(n-1)(n-2)\dots 1 \text{ for positive integers } n$$
$$0! = 1$$

FACTORIALS AND LOGARITHMS OF FACTORIALS (200–399)

n	†$n!$	lg $n!$	n	†$n!$	lg $n!$	n	†$n!$	lg $n!$	n	†$n!$	lg $n!$
200	7.8866	374.8969	250	3.2329	492.5096	300	3.0606	614.4858	350	1.2359	740.0920
201	1.5852	377.2001	251	8.1145	494.9093	301	9.2123	616.9644	351	4.3379	742.6373
202	3.2021	379.5054	252	2.0448	497.3107	302	2.7821	619.4444	352	1.5269	745.1838
203	6.5003	381.8129	253	5.1735	499.7138	303	8.4298	621.9258	353	5.3901	747.7316
204	1.3261	384.1226	254	1.3141	502.1186	304	2.5627	624.4087	354	1.9081	750.2806
205	2.7184	386.4343	255	3.3509	504.5252	305	7.8161	626.8930	355	6.7738	752.8308
206	5.5999	388.7482	256	8.5782	506.9334	306	2.3917	629.3787	356	2.4115	755.3823
207	1.1592	391.0642	257	2.2046	509.3433	307	7.3426	631.8659	357	8.6089	757.9349
208	2.4111	393.3822	258	5.6878	511.7549	308	2.2615	634.3544	358	3.0820	760.4888
209	5.0392	395.7024	259	1.4732	514.1682	309	6.9881	636.8444	359	1.1064	763.0439
210	1.0582	398.0246	260	3.8302	516.5832	310	2.1663	639.3357	360	3.9832	765.6002
211	2.2329	400.3489	261	9.9968	518.9999	311	6.7373	641.8285	361	1.4379	768.1577
212	4.7337	402.6752	262	2.6192	521.4182	312	2.1020	644.3226	362	5.2053	770.7164
213	1.0083	405.0036	263	6.8884	523.8381	313	6.5793	646.8182	363	1.8895	773.2764
214	2.1577	407.3340	264	1.8185	526.2597	314	2.0659	649.3151	364	6.8778	775.8375
215	4.6391	409.6664	265	4.8191	528.6830	315	6.5076	651.8134	365	2.5104	778.3997
216	1.0020	412.0009	266	1.2819	531.1078	316	2.0564	654.3131	366	9.1881	780.9632
217	2.1744	414.3373	267	3.4226	533.5344	317	6.5188	656.8142	367	3.3720	783.5279
218	4.7403	416.6758	268	9.1727	535.9625	318	2.0730	659.3166	368	1.2409	786.0937
219	1.0381	419.0162	269	2.4674	538.3922	319	6.6128	661.8204	369	4.5790	788.6608
220	2.2839	421.3587	270	6.6621	540.8236	320	2.1161	664.3255	370	1.6942	791.2290
221	5.0473	423.7031	271	1.8054	543.2566	321	6.7927	666.8320	371	6.2855	793.7983
222	1.1205	426.0494	272	4.9108	545.6912	322	2.1872	669.3399	372	2.3382	796.3689
223	2.4987	428.3977	273	1.3406	548.1273	323	7.0648	671.8491	373	8.7216	798.9406
224	5.5972	430.7480	274	3.6734	550.5651	324	2.2890	674.3596	374	3.2619	801.5135
225	1.2594	433.1002	275	1.0102	553.0044	325	7.4392	676.8715	375	1.2232	804.0875
226	2.8462	435.4543	276	2.7881	555.4453	326	2.4252	679.3847	376	4.5992	806.6627
227	6.4608	437.8103	277	7.7230	557.8878	327	7.9304	681.8993	377	1.7339	809.2390
228	1.4731	440.1682	278	2.1470	560.3318	328	2.6012	684.4152	378	6.5542	811.8165
229	3.3733	442.5281	279	5.9901	562.7774	329	8.5578	686.9324	379	2.4840	814.3952
230	7.7586	444.8898	280	1.6772	565.2246	330	2.8241	689.4509	380	9.4393	816.9749
231	1.7922	447.2534	281	4.7130	567.6733	331	9.3477	691.9707	381	3.5964	819.5559
232	4.1580	449.6189	282	1.3291	570.1235	332	3.1034	694.4918	382	1.3738	822.1379
233	9.6881	451.9862	283	3.7613	572.5753	333	1.0334	697.0143	383	5.2617	824.7211
234	2.2670	454.3555	284	1.0682	575.0287	334	3.4517	699.5380	384	2.0205	827.3055
235	5.3275	456.7265	285	3.0444	577.4835	335	1.1563	702.0631	385	7.7789	829.8909
236	1.2573	459.0994	286	8.7069	579.9399	336	3.8852	704.5894	386	3.0027	832.4775
237	2.9798	461.4742	287	2.4989	582.3977	337	1.3093	707.1170	387	1.1620	835.0652
238	7.0918	463.8508	288	7.1968	584.8571	338	4.4255	709.6460	388	4.5087	837.6540
239	1.6950	466.2292	289	2.0799	587.3180	339	1.5003	712.1762	389	1.7539	840.2440
240	4.0679	468.6094	290	6.0316	589.7804	340	5.1009	714.7076	390	6.8401	842.8351
241	9.8036	470.9914	291	1.7552	592.2443	341	1.7394	717.2404	391	2.6745	845.4272
242	2.3725	473.3752	292	5.1252	594.7097	342	5.9487	719.7744	392	1.0484	848.0205
243	5.7651	475.7608	293	1.5017	597.1766	343	2.0404	722.3097	393	4.1202	850.6149
244	1.4067	478.1482	294	4.4149	599.6449	344	7.0190	724.8463	394	1.6234	853.2104
245	3.4464	480.5374	295	1.3024	602.1147	345	2.4216	727.3841	395	6.4123	855.8070
246	8.4781	482.9283	296	3.8551	604.5860	346	8.3786	729.9232	396	2.5393	858.4047
247	2.0941	485.3210	297	1.1450	607.0588	347	2.9074	732.4635	397	1.0081	861.0035
248	5.1933	487.7155	298	3.4120	609.5330	348	1.0118	735.0051	398	4.0122	863.6034
249	1.2931	490.1116	299	1.0202	612.0087	349	3.5311	737.5479	399	1.6009	866.2044

† Multiply the tabulated value by 10^c, where c is the integer part of lg $n!$. Thus $390! = 6.8401 \times 10^{842}$.

Source: For the tables on pp. 10–12, *Chambers's Shorter Six-Figure Mathematical Tables* (L. J. Comrie).

FACTORIALS AND LOGARITHMS OF FACTORIALS (400–600)

n	†$n!$	lg $n!$	n	†$n!$	lg $n!$	n	†$n!$	lg $n!$	n	†$n!$	lg $n!$
400	6.4035	868.8064	450	1.7334	1000.2389	500	1.2201	1134.0864	550	1.2789	1270.1069
401	2.5678	871.4096	451	7.8175	1002.8931	501	6.1129	1136.7862	551	7.0470	1272.8480
402	1.0322	874.0138	452	3.5335	1005.5482	502	3.0687	1139.4870	552	3.8899	1275.5899
403	4.1600	876.6191	453	1.6007	1008.2043	503	1.5435	1142.1885	553	2.1511	1278.3327
404	1.6806	879.2255	454	7.2671	1010.8614	504	7.7794	1144.8909	554	1.1917	1281.0762
405	6.8065	881.8329	455	3.3065	1013.5194	505	3.9286	1147.5942	555	6.6141	1283.8205
406	2.7635	884.4415	456	1.5078	1016.1783	506	1.9879	1150.2984	556	3.6774	1286.5655
407	1.1247	887.0510	457	6.8905	1018.8383	507	1.0079	1153.0034	557	2.0483	1289.3114
408	4.5889	889.6617	458	3.1559	1021.4991	508	5.1199	1155.7093	558	1.1430	1292.0580
409	1.8769	892.2734	459	1.4485	1024.1609	509	2.6060	1158.4160	559	6.3892	1294.8054
410	7.6951	894.8862	460	6.6633	1026.8237	510	1.3291	1161.1236	560	3.5779	1297.5536
411	3.1627	897.5001	461	3.0718	1029.4874	511	6.7916	1163.8320	561	2.0072	1300.3026
412	1.3030	900.1150	462	1.4192	1032.1520	512	3.4773	1166.5412	562	1.1281	1303.0523
413	5.3815	902.7309	463	6.5707	1034.8176	513	1.7838	1169.2514	563	6.3510	1305.8028
414	2.2279	905.3479	464	3.0488	1037.4841	514	9.1690	1171.9623	564	3.5820	1308.5541
415	9.2459	907.9660	465	1.4177	1040.1516	515	4.7220	1174.6741	565	2.0238	1311.3062
416	3.8463	910.5850	466	6.6065	1042.8200	516	2.4366	1177.3868	566	1.1455	1314.0590
417	1.6039	913.2052	467	3.0852	1045.4893	517	1.2597	1180.1003	567	6.4948	1316.8126
418	6.7044	915.8264	468	1.4439	1048.1595	518	6.5253	1182.8146	568	3.6891	1319.5669
419	2.8091	918.4486	469	6.7718	1050.8307	519	3.3866	1185.5298	569	2.0991	1322.3220
420	1.1798	921.0718	470	3.1828	1053.5028	520	1.7610	1188.2458	570	1.1965	1325.0779
421	4.9671	923.6961	471	1.4991	1056.1758	521	9.1750	1190.9626	571	6.8319	1327.8345
422	2.0961	926.3214	472	7.0756	1058.8498	522	4.7894	1193.6803	572	3.9078	1330.5919
423	8.8666	928.9478	473	3.3468	1061.5246	523	2.5048	1196.3988	573	2.2392	1333.3501
424	3.7594	931.5751	474	1.5864	1064.2004	524	1.3125	1199.1181	574	1.2853	1336.1090
425	1.5978	934.2035	475	7.5353	1066.8771	525	6.8908	1201.8383	575	7.3904	1338.8687
426	6.8064	936.8329	476	3.5868	1069.5547	526	3.6246	1204.5593	576	4.2569	1341.6291
427	2.9063	939.4633	477	1.7109	1072.2332	527	1.9101	1207.2811	577	2.4562	1344.3903
428	1.2439	942.0948	478	8.1781	1074.9127	528	1.0086	1210.0037	578	1.4197	1347.1522
429	5.3364	944.7272	479	3.9173	1077.5930	529	5.3353	1212.7272	579	8.2201	1349.9149
430	2.2947	947.3607	480	1.8803	1080.2742	530	2.8277	1215.4514	580	4.7676	1352.6783
431	9.8900	949.9952	481	9.0443	1082.9564	531	1.5015	1218.1765	581	2.7700	1355.4425
432	4.2725	952.6307	482	4.3593	1085.6394	532	7.9880	1220.9024	582	1.6121	1358.2074
433	1.8500	955.2672	483	2.1056	1088.3234	533	4.2576	1223.6292	583	9.3988	1360.9731
434	8.0289	957.9047	484	1.0191	1091.0082	534	2.2736	1226.3567	584	5.4889	1363.7395
435	3.4926	960.5431	485	4.9426	1093.6940	535	1.2164	1229.0851	585	3.2110	1366.5066
436	1.5228	963.1826	486	2.4021	1096.3806	536	6.5197	1231.8142	586	1.8816	1369.2745
437	6.6545	965.8231	487	1.1698	1099.0681	537	3.5011	1234.5442	587	1.1045	1372.0432
438	2.9147	968.4646	488	5.7087	1101.7565	538	1.8836	1237.2750	588	6.4946	1374.8126
439	1.2795	971.1071	489	2.7916	1104.4458	539	1.0152	1240.0066	589	3.8253	1377.5827
440	5.6299	973.7505	490	1.3679	1107.1360	540	5.4823	1242.7390	590	2.2569	1380.3535
441	2.4828	976.3949	491	6.7162	1109.8271	541	2.9659	1245.4722	591	1.3339	1383.1251
442	1.0974	979.0404	492	3.3044	1112.5191	542	1.6075	1248.2062	592	7.8964	1385.8974
443	4.8615	981.6868	493	1.6291	1115.2119	543	8.7289	1250.9410	593	4.6826	1388.6705
444	2.1585	984.3342	494	8.0476	1117.9057	544	4.7485	1253.6766	594	2.7814	1391.4443
445	9.6053	986.9825	495	3.9836	1120.6003	545	2.5879	1256.4130	595	1.6550	1394.2188
446	4.2840	989.6318	496	1.9758	1123.2958	546	1.4130	1259.1501	596	9.8636	1396.9940
447	1.9149	992.2822	497	9.8199	1125.9921	547	7.7292	1261.8881	597	5.8885	1399.7700
448	8.5789	994.9334	498	4.8903	1128.6893	548	4.2356	1264.6269	598	3.5213	1402.5467
449	3.8519	997.5857	499	2.4403	1131.3874	549	2.3254	1267.3665	599	2.1093	1405.3241
450	1.7334	1000.2389	500	1.2201	1134.0864	550	1.2789	1270.1069	600	1.2656	1408.1023

† Multiply the tabulated value by 10^c, where c is the integer part of lg $n!$. Thus $450! = 1.7334 \times 10^{1000}$.

For large n,
$$\ln n! = \ln \sqrt{(2\pi)} + (n+\tfrac{1}{2}) \ln n - \left(n - \frac{1}{12n} + \dots\right)$$

which leads to:
$$\lg n! \approx 0.3991 + (n+\tfrac{1}{2}) \lg n - 0.4342945\, n + 0.036/n$$