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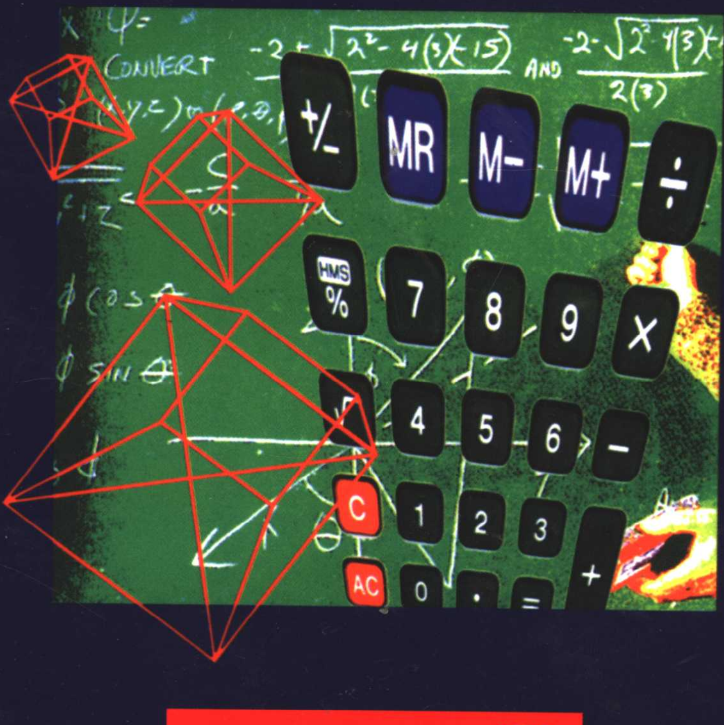

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Oxford Concise Dictionary of

Mathematics

牛 津 数 学 词 典

CHRISTOPHER CLAPHAM

外教社

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Preface

This dictionary is intended to be a reference book that gives reliable definitions or clear and precise explanations of mathematical terms. The level is such that it will suit, among others, sixth-form pupils, college students and first-year university students who are taking mathematics as one of their courses. Such students will be able to look up any term they may meet and be led on to other entries by following up cross-references or by browsing more generally.

The concepts and terminology of all those topics that feature in pure and applied mathematics and statistics courses at this level today are covered. There are also entries on mathematicians of the past and important mathematics of more general interest. Computing is not included. The reader's attention is drawn to the appendices which give useful tables for ready reference.

Some entries give a straight definition in an opening phrase. Others give the definition in the form of a complete sentence, sometimes following an explanation of the context. In this case, the keyword appears again in bold type at the point where it is defined. Other keywords in bold type may also appear if this is the most appropriate context in which to define or explain them. Italic is used to indicate words with their own entry, to which cross-reference can be made if required.

This edition is more than half as large again as the first edition. A significant change has been the inclusion of entries covering applied mathematics and statistics. In these areas, I am very much indebted to the contributors, whose names are given on page v. I am most grateful to these colleagues for their specialist advice and drafting work. They are not, however, to be held responsible for the final form of the entries on their subjects. There has also been a considerable increase in the number of short biographies, so that all the major names are included. Other additional entries have greatly increased the comprehensiveness of the dictionary.

The text has benefited from the comments of colleagues who have read different parts of it. Even though the names of all of them will not be given, I should like to acknowledge here their help and express my thanks.

Christopher Clapham

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A

Abel, Niels Henrik (1802–1829) Norwegian mathematician who, at the age of 19, proved that the general equation of degree greater than 4 cannot be solved algebraically. In other words, there can be no formula for the roots of such an equation similar to the familiar formula for a quadratic equation. He was also responsible for fundamental developments in the theory of algebraic functions. He died in some poverty at the age of 26, just a few days before he would have received a letter announcing his appointment to a professorship in Berlin.

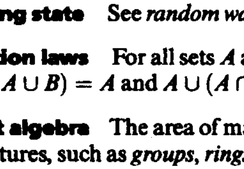
abelian group Suppose that G is a *group* with the operation $\circ$. Then G is **abelian** if the operation $\circ$ is commutative; that is, if, for all elements a and b in G, $a \circ b = b \circ a$.

abscissa The x-coordinate in a Cartesian coordinate system in the plane.

absolute error See *error*.

absolute value For any real number a, the **absolute value** (also called the *modulus*) of a, denoted by $|a|$, is a itself if $a \geq 0$, and $-a$ if $a < 0$. Thus $|a|$ is positive except when $a = 0$. The following properties hold:

(i) $|ab| = |a||b|$.
(ii) $|a + b| \leq |a| + |b|$.
(iii) $|a - b| \geq ||a| - |b||$.
(iv) For $a > 0$, $|x| \leq a$ if and only if $-a \leq x < a$.



absorbing state See *random walk*.

absorption laws For all sets A and B (subsets of some *universal set*), $A \cap (A \cup B) = A$ and $A \cup (A \cap B) = A$. These are the **absorption laws**.

abstract algebra The area of mathematics concerned with algebraic structures, such as *groups*, *rings* and *fields*, involving sets of elements with particular operations satisfying certain axioms. The purpose is to derive, from the set of axioms, general results that are then applicable to any particular example of the algebraic structure in question. The theory of certain algebraic structures is highly developed; in particular, the theory of vector spaces is so extensive that its study, known as *linear algebra*, would probably no longer be classified as abstract algebra.

acceleration Suppose that a particle is moving in a straight line, with a point O on the line taken as origin and one direction taken as positive. Let x be the *displacement* of the particle at time t. The **acceleration** of the particle is equal to $\ddot{x}$ or d^2x/dt^2, the *rate of change* of the *velocity* with respect to t. If the velocity is positive (that is, if the particle is moving in the positive direction), the acceleration is positive when the particle is speeding up and negative when it is slowing down. However, if the velocity is negative, a positive acceleration means that the particle is slowing down and a negative acceleration means that it is speeding up.

In the preceding paragraph, a common convention has been followed, in which the unit vector $\mathbf{i}$ in the positive direction along the line has been suppressed. Acceleration is in fact a vector quantity, and in the one-dimensional case above it is equal to $\ddot{x}\mathbf{i}$.

When the motion is in two or three dimensions, vectors are used explicitly. The acceleration $\mathbf{a}$ of a particle is a vector equal to the rate of change of the velocity $\mathbf{v}$ with respect to t. Thus $\mathbf{a} = d\mathbf{v}/dt$. If the particle has *position vector* $\mathbf{r}$, then $\mathbf{a} = d^2\mathbf{r}/dt^2 = \ddot{\mathbf{r}}$. When Cartesian coordinates are used, $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and then $\ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k}$.

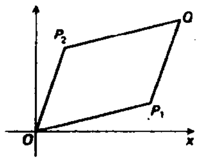
Acceleration has the dimensions LT^{-2}, and the SI unit of measurement is the metre per second per second, abbreviated to '$m\,s^{-2}$'.

acceleration–time graph A graph that shows acceleration plotted against time for a particle moving in a straight line. Let $v(t)$ and $a(t)$ be the velocity and acceleration, respectively, of the particle at time t. The acceleration–time graph is the graph $y = a(t)$, where the t-axis is horizontal and the y-axis is vertical with the positive direction upwards. With the convention that any area below the horizontal axis is negative, the area under the graph between $t = t_1$ and $t = t_2$ is equal to $v(t_2) - v(t_1)$. (Here a common convention has been followed, in which the unit vector $\mathbf{i}$ in the positive direction along the line has been suppressed. The velocity and acceleration of the particle are in fact vector quantities equal to $v(t)\mathbf{i}$ and $a(t)\mathbf{i}$, respectively.)

acceptance region See *hypothesis testing*.

acute angle An angle that is less than a *right angle*. An **acute-angled** triangle is one all of whose angles are acute.

addition (of complex numbers) Let the complex numbers z_1 and z_2, where $z_1 = a + bi$ and $z_2 = c + di$, be represented by the points P_1 and P_2 in the *complex plane*. Then $z_1 + z_2 = (a + c) + (b + d)i$, and $z_1 + z_2$ is represented in the complex plane by the point Q such that OP_1QP_2 is a parallelogram; that is, such that $\overrightarrow{OQ} = \overrightarrow{OP_1} + \overrightarrow{OP_2}$. Thus, if the complex number z is associated with the *directed line-segment* $\overrightarrow{OP}$, where P



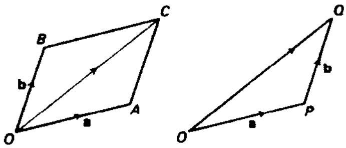
represents z, then the addition of complex numbers corresponds exactly to the addition of the directed line-segments.

addition (of directed line-segments) See *addition* (of vectors).

addition (of matrices) Let **A** and **B** be $m \times n$ matrices, with $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$. The operation of **addition** is defined by taking the **sum** $\mathbf{A} + \mathbf{B}$ to be the $m \times n$ matrix **C**, where $\mathbf{C} = [c_{ij}]$ and $c_{ij} = a_{ij} + b_{ij}$. The sum $\mathbf{A} + \mathbf{B}$ is not defined if **A** and **B** are not of the same order. This operation + of addition on the set of all $m \times n$ matrices is *associative* and *commutative*.

addition (of vectors) Given vectors **a** and **b**, let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be *directed line-segments* that represent **a** and **b**, with the same initial point O. The sum of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is the directed line-segment $\overrightarrow{OC}$, where $OACB$ is a parallelogram, and the **sum** $\mathbf{a} + \mathbf{b}$ is defined to be the vector **c** represented by $\overrightarrow{OC}$. This is called the **parallelogram law**. Alternatively, the sum of vectors **a** and **b** can be defined by representing **a** by a directed line-segment $\overrightarrow{OP}$ and **b** by $\overrightarrow{PQ}$, where the final point of the first directed line-segment is the initial point of the second. Then $\mathbf{a} + \mathbf{b}$ is the vector represented by $\overrightarrow{OQ}$. This is called the **triangle law**. Addition of vectors has the following properties, which hold for all **a**, **b** and **c**:

(i) $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$, the commutative law.
(ii) $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$, the associative law.
(iii) $\mathbf{a} + \mathbf{0} = \mathbf{0} + \mathbf{a} = \mathbf{a}$, where $\mathbf{0}$ is the zero vector.
(iv) $\mathbf{a} + (-\mathbf{a}) = (-\mathbf{a}) + \mathbf{a} = \mathbf{0}$, where $-\mathbf{a}$ is the negative of **a**.



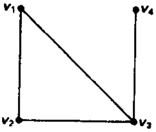
The parallelogram law The triangle law

addition modulo *n* See *modulo n, addition* and *multiplication*.

additive group A *group* with the operation +, called addition, may be called an **additive group**. The operation in a group is normally denoted by addition only if it is *commutative*, so an additive group is usually *abelian*.

additive inverse See *inverse element*.

adjacency matrix For a *simple graph G*, with n vertices $v_1, v_2, \ldots, v_n$, the **adjacency matrix** $\mathbf{A}$ is the $n \times n$ matrix $[a_{ij}]$ with $a_{ij} = 1$, if v_i is joined to v_j, and $a_{ij} = 0$, otherwise. The matrix $\mathbf{A}$ is *symmetric* and the diagonal entries are zero. The number of ones in any row (or column) is equal to the *degree* of the corresponding vertex. An example of a graph and its adjacency matrix $\mathbf{A}$ is shown below.



$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

adjoint The **adjoint** of a square matrix $\mathbf{A}$, denoted by adj $\mathbf{A}$, is the transpose of the matrix of cofactors of $\mathbf{A}$. For $\mathbf{A} = [a_{ij}]$, let A_{ij} denote the *cofactor* of the entry a_{ij}. Then the matrix of cofactors is the matrix $[A_{ij}]$ and adj $\mathbf{A} = [A_{ij}]^T$. For example, a 3×3 matrix $\mathbf{A}$ and its adjoint can be written

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \qquad \operatorname{adj} \mathbf{A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

In the 2×2 case, a matrix $\mathbf{A}$ and its adjoint have the form

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad \operatorname{adj} \mathbf{A} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

The adjoint is important because it can be used to find the *inverse* of a matrix. From the properties of cofactors, it can be shown that $\mathbf{A}$ adj $\mathbf{A} = (\det \mathbf{A})\mathbf{I}$. It follows that, when $\det \mathbf{A} \neq 0$, the inverse of $\mathbf{A}$ is $(1/\det \mathbf{A})$ adj $\mathbf{A}$.

adjugate = *adjoint*.

aerodynamic drag A body moving through the air, such as an aeroplane flying in the Earth's atmosphere, experiences a force due to the flow of air over the surface of the body. The force is the sum of the **aerodynamic drag**, which is tangential to the flight path, and the **lift**, which is normal to the flight path.

air resistance The resistance to motion experienced by an object moving through the air caused by the flow of air over the surface of the object. It is a force that affects, for example, the speed of a drop of rain or of a parachutist falling towards the Earth's surface. As well as depending on the nature of the object, air resistance depends on the speed of the object. Possible *mathematical models* are to assume that the magnitude of the air resistance is proportional to the speed or to the square of the speed.

Algebra, Fundamental Theorem of See *Fundamental Theorem of Algebra*.

algebra of sets The set of all subsets of a *universal set* E is closed under the binary operations $\cup$ (*union*) and $\cap$ (*intersection*) and the unary operation $'$ (*complementation*). The following are some of the properties, or laws, that hold for subsets A, B and C of E:

(i) $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$, the associative properties.
(ii) $A \cup B = B \cup A$ and $A \cap B = B \cap A$, the commutative properties.
(iii) $A \cup \varnothing = A$ and $A \cap \varnothing = \varnothing$, where $\varnothing$ is the *empty set*.
(iv) $A \cup E = E$ and $A \cap E = A$.
(v) $A \cup A = A$ and $A \cap A = A$.
(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, the distributive properties.
(vii) $A \cup A' = E$ and $A \cap A' = \varnothing$.
(viii) $E' = \varnothing$ and $\varnothing' = E$.
(ix) $(A')' = A$.
(x) $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$, De Morgan's laws.

The application of these laws to subsets of E is known as the **algebra of sets**. Despite some similarities with the algebra of numbers, there are important and striking differences.

algebraic number A real number that is the root of a *polynomial equation* with integer coefficients. All *rational numbers* are algebraic, since a/b is the root of the equation $bx - a = 0$. Some *irrational numbers* are algebraic; for example, $\sqrt{2}$ is the root of the equation $x^2 - 2 = 0$. An irrational number that is not algebraic (such as π) is called a *transcendental number*.

algebraic structure The term used to describe an abstract concept defined as consisting of certain elements with operations satisfying given axioms. Thus, a *group* or a *ring* or a *field* is an algebraic structure. The purpose of the definition is to recognize similarities that appear in different contexts within mathematics and to encapsulate these by means of a set of axioms.

algorithm A precisely described routine procedure that can be applied and systematically followed through to a conclusion.

al-Khwārizmī See under K.

alternate angles See *transversal*.

alternative hypothesis See *hypothesis testing*.

altitude A line through one vertex of a triangle and perpendicular to the opposite side. The three altitudes of a triangle are concurrent at the *orthocentre*.

amicable numbers A pair of numbers with the property that each is equal to the sum of the positive divisors of the other. (For the purposes of this definition, a number is not included as one of its own divisors.) For example, 220 and 284 are amicable numbers because the positive divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, whose sum is 284, and the positive divisors of 284 are 1, 2, 4, 71 and 142, whose sum is 220.

These numbers, known to the Pythagoreans, were used as symbols of friendship. The amicable numbers 17 296 and 18 416 were found by Fermat, and a list of 64 pairs was produced by Euler. In 1867, a sixteen-year-old Italian boy found the second smallest pair, 1184 and 1210, overlooked by Euler. More than 600 pairs are now known. It has not been shown whether or not there are infinitely many pairs of amicable numbers.

amplitude Suppose that $x = A\sin(\omega t + \alpha)$, where $A\ (> 0)$, ω and α are constants. This may, for example, give the displacement x of a particle, moving in a straight line, at time t. The particle is thus oscillating about the origin O. The constant A is the **amplitude**, and gives the maximum distance in each direction from O that the particle attains.

The term may also be used in the case of *damped oscillations* to mean the corresponding coefficient, even though it is not constant. For example, if $x = 5e^{-2t}\sin 3t$, the oscillations are said to have amplitude $5e^{-2t}$, which tends to zero as t tends to infinity.

analysis The area of mathematics generally taken to include those topics that involve the use of limiting processes. Thus *differential calculus* and *integral calculus* certainly come under this heading. Besides these, there are other topics, such as the summation of infinite series, which involve 'infinite' processes of this sort. The *Binomial Theorem*, a theorem of algebra, leads on into analysis when the index is no longer a positive integer, and the study of sine and cosine, which begins as trigonometry, becomes analysis when the power series for the functions are derived. The term 'analysis' has also come to be used to indicate a rather more rigorous approach to the topics of calculus, and to the foundations of the real number system.

analysis of variance A general procedure for partitioning the overall variability in a set of data into components due to specified causes and random variation. It involves calculating such quantities as the 'between-groups sum of squares' and the 'residual sum of squares', and dividing by the *degrees of freedom* to give so-called 'mean squares'. The results are usually presented in an **ANOVA** table, the name being derived from the opening letters of the words 'analysis of variance'. Such a table provides a concise summary from which the influence of the *explanatory variables* can be estimated and hypotheses can be tested, usually by means of *F-tests*.

anchor ring = *torus*.

and See *conjunction*.

angle (between lines in space) Given two lines in space, let $\mathbf{u}_1$ and $\mathbf{u}_2$ be vectors with directions along the lines. Then the **angle** between the lines, even if they do not meet, is equal to the angle between the vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ (see *angle* (between vectors)), with the directions of $\mathbf{u}_1$ and $\mathbf{u}_2$ chosen so that the angle θ satisfies $0 \le \theta \le \pi/2$ (θ in radians), or $0 \le \theta \le 90$ (θ in degrees). If l_1, m_1, n_1 and l_2, m_2, n_2 are direction ratios for directions along the lines, the angle θ between the lines is given by

$$\cos\theta = \frac{|l_1 l_2 + m_1 m_2 + n_1 n_2|}{\sqrt{l_1^2 + m_1^2 + n_1^2}\sqrt{l_2^2 + m_2^2 + n_2^2}}.$$

angle (between lines in the plane) In coordinate geometry of the plane, the angle α between two lines with gradients m_1 and m_2 is given by

$$\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

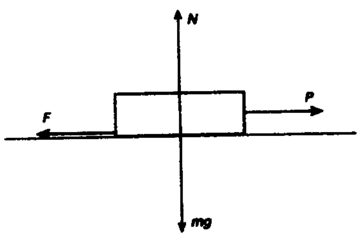
This is obtained from the formula for $\tan(A - B)$. In the special cases when $m_1 m_2 = -1$ or when m_1 or m_2 is infinite, it has to be interpreted appropriately.

angle (between planes) Given two planes, let $\mathbf{n}_1$ and $\mathbf{n}_2$ be vectors *normal* to the two planes. Then a method of obtaining the **angle** between the planes is to take the angle between $\mathbf{n}_1$ and $\mathbf{n}_2$ (see *angle* (between vectors)), with the directions of $\mathbf{n}_1$ and $\mathbf{n}_2$ chosen so that the angle θ satisfies $0 \le \theta \le \pi/2$ (θ in radians), or $0 \le \theta \le 90$ (θ in degrees).

angle (between vectors) Given vectors $\mathbf{a}$ and $\mathbf{b}$, let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be *directed line-segments* representing $\mathbf{a}$ and $\mathbf{b}$. Then the **angle** θ between the vectors $\mathbf{a}$ and $\mathbf{b}$ is the angle $\angle AOB$, where θ is taken to satisfy $0 \le \theta \le \pi$ (θ in radians), or $0 \le \theta \le 180$ (θ in degrees). It is given by

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

angle of friction The angle λ such that $\tan\lambda = \mu_s$, where μ_s is the *coefficient of static friction*. Consider a block resting on a horizontal plane, as shown in the figure. In the limiting case when the block is about to move to the right on account of an applied force of magnitude P, $N = mg$, $P = F$ and $F = \mu_s N$. Then the *contact force*, whose components are N and F, makes an angle λ with the vertical.



angle of inclination See *inclined plane.*

angle of projection The angle that the direction in which a particle is projected makes with the horizontal. Thus it is the angle that the initial velocity makes with the horizontal.

angular acceleration Suppose that the particle P is moving in the plane, in a circle with centre at the origin O and radius r_0. Let (r_0, θ) be the polar coordinates of P. At an elementary level, the **angular acceleration** may be defined to be $\ddot{\theta}$.

At a more advanced level, the **angular acceleration** $\boldsymbol{\alpha}$ of the particle P is the vector defined by $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$, where $\boldsymbol{\omega}$ is the *angular velocity.* Let $\mathbf{i}$ and $\mathbf{j}$ be unit vectors in the directions of the positive x- and y-axes and let $\mathbf{k} = \mathbf{i} \times \mathbf{j}$. Then, in the case above of a particle moving along a circular path, $\boldsymbol{\omega} = \dot{\theta}\mathbf{k}$ and $\boldsymbol{\alpha} = \ddot{\theta}\mathbf{k}$. If $\mathbf{r}$, $\mathbf{v}$ and $\mathbf{a}$ are the position vector, velocity and acceleration of P, then

$$\mathbf{r} = r_0\mathbf{e}_r, \qquad \mathbf{v} = \dot{\mathbf{r}} = r_0\dot{\theta}\mathbf{e}_\theta, \qquad \mathbf{a} = \ddot{\mathbf{r}} = -r_0\dot{\theta}^2\mathbf{e}_r + r_0\ddot{\theta}\mathbf{e}_\theta,$$

where $\mathbf{e}_r = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$ and $\mathbf{e}_\theta = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta$ (see *circular motion*). Using the fact that $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$, it follows that the acceleration $\mathbf{a}$ is given by $\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$.

angular frequency The constant ω in the equation $\ddot{x} = -\omega^2 x$ for *simple harmonic motion.* In certain respects ωt, where t is the time, acts like an angle. The angular frequency ω is usually measured in radians per second. The *frequency* of the oscillations is equal to $\omega/2\pi$.

angular measure There are two principal ways of measuring angles: by using *degrees*, in more elementary work, and by using *radians*, essential in more advanced work.

angular momentum Suppose that the particle P of mass m has position vector $\mathbf{r}$ and is moving with velocity $\mathbf{v}$. Then the **angular momentum** $\mathbf{L}$ of P about the point A with position vector $\mathbf{r}_A$ is the vector defined by $\mathbf{L} = (\mathbf{r} - \mathbf{r}_A) \times m\mathbf{v}$. It is the *moment* of the *linear momentum* about the point A. See also *conservation of angular momentum.*

Consider a rigid body rotating with angular velocity $\boldsymbol{\omega}$ about a fixed axis, and let $\mathbf{L}$ be the angular momentum of the rigid body about a point on the fixed axis. Then $\mathbf{L} = I\boldsymbol{\omega}$, where I is the *moment of inertia* of the rigid body about the fixed axis.

To consider the general case, let $\boldsymbol{\omega}$ and $\mathbf{L}$ now be column vectors representing the angular velocity of a rigid body and the angular momentum of the rigid body about a fixed point (or the centre of mass). Then $\mathbf{L} = \mathbf{I}\boldsymbol{\omega}$, where $\mathbf{I}$ is a 3×3 matrix, called the **inertia matrix**, whose elements involve the *moments of inertia* and the *products of inertia* of the rigid body relative to axes through the fixed point (or centre of mass).

The rotational motion of a rigid body depends on the angular momentum of the rigid body. In particular, the rate of change of the angular momentum about a fixed point (or centre of mass) equals the sum of the moments of the forces acting on the rigid body about the fixed point (or centre of mass).

angular speed The magnitude of the *angular velocity*.

angular velocity Suppose that the particle P is moving in the plane, in a circle with centre at the origin O and radius r_0. Let (r_0, θ) be the polar coordinates of P. At an elementary level, the **angular velocity** may be defined to be $\dot{\theta}$.

At a more advanced level, the **angular velocity** $\boldsymbol{\omega}$ of the particle P is the vector defined by $\boldsymbol{\omega} = \dot{\theta}\mathbf{k}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the directions of the positive x- and y-axes, and $\mathbf{k} = \mathbf{i} \times \mathbf{j}$. If $\mathbf{r}$ and $\mathbf{v}$ are the position vector and velocity of P, then

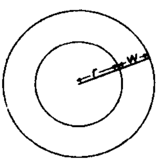
$$\mathbf{r} = r_0\mathbf{e}_r, \qquad \mathbf{v} = \dot{\mathbf{r}} = r_0\dot{\theta}\mathbf{e}_\theta,$$

where $\mathbf{e}_r = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$ and $\mathbf{e}_\theta = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta$ (see *circular motion*). By using the fact that $\mathbf{k} = \mathbf{e}_r \times \mathbf{e}_\theta$, it follows that the velocity $\mathbf{v}$ is given by $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$.

Consider a rigid body rotating about a fixed axis, and take coordinate axes so that the z-axis is along the fixed axis. Let (r_0, θ) be the polar coordinates of some point of the rigid body, not on the axis, lying in the plane $z = 0$. Then the angular velocity $\boldsymbol{\omega}$ of the rigid body is defined by $\boldsymbol{\omega} = \dot{\theta}\mathbf{k}$.

In general, for a rigid body that is rotating, such as a top spinning about a fixed point, the rigid body possesses an angular velocity $\boldsymbol{\omega}$ whose magnitude and direction depend on time.

annulus (plural: annuli) The region between two concentric circles. If the circles have radii r and $r + w$, the area of the annulus is equal to $\pi(r + w)^2 - \pi r^2$, which equals $w \times 2\pi(r + \frac{1}{2}w)$. It is therefore the same as the area of a rectangle of width w and length equal to the circumference of the circle midway in size between the two original circles.



ANOVA See *analysis of variance*.

antiderivative Given a *real function* f, any function ϕ such that $\phi'(x) = f(x)$, for all x (in the domain of f), is an **antiderivative** of f. If ϕ_1 and ϕ_2 are both antiderivatives of a *continuous function* f, then $\phi_1(x)$ and $\phi_2(x)$ differ by a constant. In that case, the notation

$$\int f(x)\, dx$$

may be used for an antiderivative of f, with the understanding that an arbitrary constant can be added to any antiderivative. Thus,

$$\int f(x)\, dx + c,$$

where c is an arbitrary constant, is an expression that gives all the antiderivatives.

antilogarithm The **antilogarithm** of x, denoted by antilog x, is the number whose *logarithm* is equal to x. For example, suppose that common logarithm tables are used to calculate 2.75×3.12. Then, approximately, $\log 2.75 = 0.4393$ and $\log 3.12 = 0.4942$ and $0.4393 + 0.4942 = 0.9335$. Now antilog 0.9335 is required and, from tables, the answer 8.58 is obtained. Now that logarithm tables have been superseded by calculators, the term 'antilog' is little used. If y is the number whose logarithm is x, then $\log_a y = x$. This is equivalent to $y = a^x$ (from the definition of logarithm). So, if base a is being used, $\text{antilog}_a\, x$ is identical with a^x; for common logarithms, $\text{antilog}_{10}\, x$ is just 10^x, and this notation is preferable.

antipodal points Two points on a sphere that are at opposite ends of a diameter.

antiprism Normally, a convex *polyhedron* with two 'end' faces that are congruent regular polygons lying in parallel planes in such a way that, with each vertex of one polygon joined by an edge to two vertices of the other polygon, the remaining faces are isosceles triangles. The term could be used for a polyhedron of a similar sort in which the end faces are not regular and the triangular faces are not isosceles, in which case the first definition would be said to give a right-regular antiprism. If the end faces are regular and the triangular faces are equilateral, the antiprism is a *semi-regular polygon*.

antisymmetric matrix = *skew-symmetric matrix*.

antisymmetric relation A binary relation $\sim$ on a set S is **antisymmetric** if, for all a and b in S, whenever $a \sim b$ and $b \sim a$, then $a = b$. For example, the relation $\leq$ on the set of integers is antisymmetric. (Compare this with the definition of an *asymmetric* relation.)

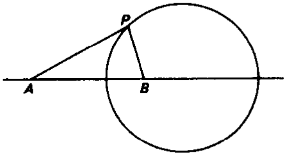
apex (plural: apices) See *base* (of a triangle) and *pyramid*.

aphelion See *apse*.

apogee See *apse*.

Apollonius of Perga (about 262–190 BC) Greek mathematician whose most famous work *The Conics* was, until modern times, the definitive work on the *conic sections*: the ellipse, parabola and hyperbola. He proposed the idea of epicyclic motion for the planets. Euclid, Archimedes and Apollonius were pre-eminent in the period covering the third century BC known as the Golden Age of Greek mathematics.

Apollonius' circle Given two points A and B in the plane and a constant k, the locus of all points P such that $AP/PB = k$ is a circle. A circle obtained like this is an **Apollonius' circle**. Taking $k = 1$ gives a straight line, so either this value must be excluded or, in this context, a straight line must be considered to be a special case of a circle. In the figure, $k = 2$.



approximation When two quantities X and x are approximately equal, written $X \approx x$, one of them may be used in suitable circumstances in place of, or as an **approximation** for, the other. For example, $\pi \approx \frac{22}{7}$ and $\sqrt{2} \approx 1.414$.

apse A point in an *orbit* at which the body is moving in a direction perpendicular to the radius vector. In an elliptical orbit in which the centre of attraction is at one focus there are two apses, the points at which the body is at its nearest and its furthest from the centre of attraction. When the centre of attraction is the Sun, the points at which the body is nearest and furthest are the **perihelion** and the **aphelion**. When the centre of attraction is the Earth, they are the **perigee** and the **apogee**.

apsis (plural: apses) = *apse*.

Arabic numeral See *numeral*.

arc (of a curve) The part of a curve between two given points on the curve. If A and B are two points on a circle, there are two arcs AB. When A and B are not at opposite ends of a diameter, it is possible to distinguish between the longer and shorter arcs by referring to the **major arc** AB and the **minor arc** AB.

arc (of a digraph) See *digraph*.

arccos, arccosec, arccot, arcsin, arcsec, arctan See *inverse trigonometric function*.

arccosh, arccosech, arccoth, arcsinh, arcsech, arctanh See *inverse hyperbolic function*.

arc length Let $y = f(x)$ be the graph of a function f such that f' is *continuous* on $[a, b]$. The length of the arc, or **arc length**, of the curve $y = f(x)$ between $x = a$ and $x = b$ equals