Nonlinear Computational Solid Mechanics

Jamshid Ghaboussi
David A. Pecknold
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Nonlinear Computational Solid Mechanics presents the fundamentals of nonlinear mechanics within a modern computational approach based mainly on finite element methods. Both material and geometric nonlinearities are treated. The topics build up from the mechanics of finite deformation of solid bodies through to nonlinear structural behaviour including buckling, bifurcation, and snap-through. The principles are illustrated with a series of solved problems. This book serves as a text book for a second year graduate course and as a reference for practitioners using nonlinear analysis in engineering and design.

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Nonlinear Computational Solid Mechanics

To Our Families

Preface

Computational mechanics is well established and widely used in diverse fields of engineering and science. Nevertheless, it is a comparatively recent field that is still rapidly developing. Vast improvements in computational capabilities have made possible the routine computational simulation of mechanical and structural systems that would have been unimaginable only a few decades ago. Recent advances in information technology and bioscience have also led to the development of soft computing systems that provide new directions in computational mechanics.

There are now a number of excellent and widely used commercially available general-purpose software packages for simulating well into the nonlinear range the response of diverse types of structural and mechanical systems. Extensive material model and finite element libraries are a typical feature of these packages. In addition, in-house proprietary software systems for nonlinear analysis remain in use in many industries.

The justification and verification of analysis result from large-scale simulations that have become even more critical than the modeling and analysis itself. Engineering practitioners need a thorough understanding of the underlying fundamentals on which these computational tools are based in order to utilize them most effectively in their work. This book is intended to provide much of that necessary background in an accessible form.

The book can serve as a reference for engineers, analysts, and software developers in practice, as well as a graduate course text. Graduate students in various engineering and science disciplines can benefit from the relatively broad coverage of topics, including an introduction to information-based material modeling (in Chapter 12, "Soft Computing in Computational Mechanics").

We assume that the reader is familiar with the basics of linear finite element analysis; that of course requires a working knowledge of the fundamentals of mechanics and of linear elasticity theory. We do not therefore cover in detail basic linear finite element theory; that would require an entire book in itself, and there are many excellent references available.

We start with an overview of linear and nonlinear mechanics and some simple examples of nonlinear behavior in Chapter 1. This is followed by three chapters (Chapters 2 through 4) discussing the fundamentals of nonlinear continuum mechanics, including the treatment of large displacements and strains (geometric nonlinearity), definitions of stresses and strains and their rates, and the principle of virtual work. Chapters 5 through 7 describe constitutive laws governing stress–strain relations, including material nonlinearity: Chapter 5 covers linear and nonlinear elastic material properties, Chapter 6 discusses stress invariants and material testing, and Chapter 7 covers elastoplastic material models. In Chapter 8, we discuss applications involving solid continuum mechanics, specifically the total Lagrangian formulation and the updated Lagrangian formulation.

The treatment of large rotations in three dimensions, required for nonlinear applications involving structural finite elements (i.e., beams, plates, and shells) with nodal rotational degrees of freedom, is presented in Chapter 9. Formulations for structural elements, including beams, plates, and shells, are covered in Chapter 10, before the discussion of incremental-iterative numerical solution methods for computational simulation in Chapter 11.

With the exception of this latter presentation of incremental-iterative Newton–Raphson methods, we do not attempt to delve deeply into the large inventory of numerical algorithms that are the cornerstones of numerical computational mechanics. That also requires an entire book in itself (see Ghaboussi and Wu, *Numerical Methods in Computational Mechanics*, CRC Press, Taylor & Francis Group 2016).

Throughout the book, we consistently present simple examples to illustrate and clarify the basic concepts and mechanics fundamentals that are being discussed.

Authors

Jamshid Ghaboussi is Emeritus Professor in the Department of Civil and Environmental Engineering at the University of Illinois at Urbana–Champaign. He received his doctoral degree from the University of California at Berkeley. He has more than 40 years of teaching and research experience in computational mechanics and soft computing with applications in structural engineering, geomechanics, and biomedical engineering. He has published extensively in these areas and is the inventor of five patents, mainly in the application of soft computing and computational mechanics. He is the coauthor of the book *Numerical Methods in Computational Mechanics* (CRC Press, Taylor & Francis Group) and author of the book *Soft Computing in Engineering* (CRC Press, Taylor & Francis Group). In recent years, he has been conducting research on complex systems and has coauthored the book *Understanding Systems: A Grand Challenge for 21st Century Engineering* (World Scientific Publishing).

David A. Pecknold is Emeritus Professor in the Department of Civil and Environmental Engineering at the University of Illinois at Urbana–Champaign. He received his PhD degree in structural engineering from the University of Illinois.

He has more than 40 years of teaching, consulting, and university and industrial research experience in structural mechanics and dynamics. He has conducted research and published technical papers on the strength of tubular joints in offshore structures, fatigue, and fracture in railway tank cars, modeling of damage and failure in high-performance structural composites, inelastic response of structures to earthquakes and other dynamic loadings, and nonlinear constitutive models for reinforced concrete and other engineering materials.

He has served as an industrial consultant on many projects, including studies of progressive collapse of offshore oil production platforms under wave and wind loads; investigations of structural vibrations in long-span floors, sports facilities, and other large structures; behavior of railway and subway tunnels; and failure in pressure vessels and containments, large-diameter flexible piping systems, and long-span roof systems.

Xiping Steven Wu received his PhD degree from the University of Illinois at Urbana–Champaign in 1991 in structural engineering. His professional career includes academic teaching and research, applied research for frontier development of oil and gas resources, and project delivery and management. He is a principal engineer in civil and marine engineering with Shell International Exploration & Production, Inc. He has more than 25 years of experience in onshore and offshore structural engineering, arctic engineering, deepwater floating systems, plant design, liquefied natural gas development, and capital project management. He has published more than 40 research papers in computational mechanics, numerical methods, arctic engineering, structural integrity and reliability, and applications of soft computing to civil engineering. He is the coauthor of the book *Numerical Methods in Computational Mechanics* (CRC Press, Taylor & Francis Group).

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