$$y(x,0)=\sum_{n=1}^{x} B_n$$

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Intro Calculus

微积分初步

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内 容 简 介

本书主要介绍一元微积分和多元微积分的基本内容，其特点是按国内现行教学大纲，全部采用英文编写。全书共分 11 章，主要内容包括：变量、函数；极限和连续；导数与微分；中值定理及导数的应用；不定积分；定积分及应用；微分方程；空间向量与平面、直线；多元函数的微分学；多元函数的积分学；级数。本书每章末均附有习题，最后附有习题答案，以供学生复习巩固书中所学内容。

本书可作为高等院校各专业本科生《高等数学》双语教学的教材，还可作为职教、函授大学双语教学的教材及工程技术人员提高科技英语能力的阅读材料。

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前　言

本书《Intro Calculus》是一本全部采用英文编写的微积分双语教学教材。

国家教育部提出了加强大学本科教学的 12 项措施，其中要求各高校在三年内开设 5% ~ 10% 的双语课程，并引进原版教材和提高师资水平，各高校纷纷响应，根据不同条件开设双语课程。

《微积分》作为各类高等院校的基础课，虽然是舶来品，但在我国已经形成了独特、成熟的体系；与国外的《微积分》教材相比，国内教材在章节结构、内容侧重上大相径庭。我们进行双语教学时，考虑到学生将来继续攻读研究生的问题，许多时候，依据国内现行的《高等数学》教学大纲，并参考原版教材，这也正是我们编写本书的目的。

本书中我们安排了 11 章内容，包括一元和多元微积分的基本内容：Chapter 1 variable, function; Chapter 2 the limit and continuity of functions; Chapter 3 a derivative and differentia; Chapter 4 some theorems on differentiable functions、investigating functions; Chapter 5 the indefinite integral; Chapter 6 the definite integral and computing the area and the arc length of a curve; Chapter 7 differential equations; Chapter 8 vectors in three dimensions、planes and lines in space; Chapter 9 functions of several variables; Chapter 10 multiple integrals; Chapter 11 series.

本书由刘利华编写第 1 章 ~ 第 7 章，蔺小林编写第 8 章、第 10 章、第 11 章，王莉编写第 9 章。由于我们水平有限，书中存在缺点和疏漏之处，恳请读者批评指正。

作者

2004 年 8 月

What is calculus ?

Calculus was invented in the seventeenth century to provide a tool for solving problems involving motion. The subject matter to geometry, algebra, and trigonometry is applicable to objects which move at constant speeds; however methods introduced in calculus are required to study the orbits of planets, to calculate the flight of a rocket, to predict the path of a charged particle through an electromagnetic field and, for that matter, to deal with all aspects of motion.

In order to discuss objects in motion it is essential first to define what is meant by velocity and acceleration. Roughly speaking, the velocity of an object is a measure of the rate at which the distance traveled changes with respect to time. Acceleration is a measure of the rate at which velocity changes. Velocity may vary considerably, as is evident from the motion of a drag-strip racer or the descent of a space capsule as it reenters the Earth's atmosphere. In order to give precise meanings to the notions of velocity and acceleration it is necessary to use one of the fundamental concepts of calculus, the derivative.

Although calculus was introduced to help solve problems in physics, it has been applied to many different fields. One of the reasons for its versatility is the fact that the derivative is useful in the study of rates of change of many entities other than objects in motion. For example, a chemist may use derivatives to forecast the outcome of various chemical reactions. A biologist may employ it in the investigation of the rate of growth of bacteria in a culture. An electrical engineer uses the derivative to describe the change in current in an electric circuit. Economists have applied it to problems involving corporate profits and losses.

The derivative is also used to find tangent lines to curves. Although this has some independent geometric interest, the significance of tangent lines is of major importance in physical problems. For example, if a particle moves along a cure, then the tangent line indicates the direction of motion. If we restrict our attention to a sufficiently small portion of the curve, then in a certain sense the tangent line may be used to approximate the position of the particle.

Many problems involving maximum and minimum values may be attacked with the aid of the derivative. Some typical questions that can be answered are: At what angle of elevation should a projectile be fired in order to achieve its maximum range? If a tin can is to hold one gallon of a liquid, what dimensions require the least amount of tin? At what point between two light sources will the illumination be greatest? How can certain corporations maximize their revenue? How can a manufacturer minimize the cost of producing a given article?

Another fundamental concept of calculus is known as the definite integral. It, too, has many applications in the sciences. A physicist uses it to find the work required to stretch or compress a spring. An engineer may use it to find the center of mass or moment of inertia of a solid. An economist may employ it to estimate depreciation of equipment a manufacturing plant. Mathematicians use definite integrals to investigate such concepts as areas of surfaces, volumes of geometric solids, and lengths of curves.

The derivative and the definite integral are defined in terms of certain limiting processes. The notion of limit is the initial idea which separates calculus from the more elementary branches of mathematics. Sir Isaac Newton and Gottfried Wilhelm Leibniz discovered the connection between derivatives and integral. Because of this, and their other contributions to the subject, they are credited with the invention of calculus. Many other mathematicians have added a great deal to its development.

The preceding discussion has not answered the question " what is calculus ?" Actually, there is no simple answer. Calculus could be called the study of limits, derivatives, and integrals; however, this statement is meaningless if definition of the terms are unknown.

Contents

CHAPTER 1 NUMBER, VARIABLE, FUNCTION

1.1 REAL NUMBERS. REAL NUMBERS AS POINTS ON A NUMBER SCALE

Number is one of the basic concepts of mathematics. Whole numbers and fraction, together with the number zero are called *rational numbers*. Every rational number may be represented in the form of a ratio, $\frac{p}{q}$, of two integers p and q; for example, $\frac{4}{5}$, $0.5 = \frac{1}{2}$.

Rational numbers may be represented in the form of periodic terminating or nonterminating fractions. Numbers represented by nonterminating, but nonperiodic, decimal fractions are called *irrational numbers*; such are numbers $\sqrt{2}$, $\sqrt[4]{3}$, $3-\sqrt{2}$, π, etc.

The collection of all rational and irrational numbers makes up the set of real numbers. The real numbers are ordered in magnitude; that is to say, for each pair of real numbers x and y there is one, and only one, of the following relations: $x < y$, $x = y$, $x > y$

Real numbers may be depicted as points on a number scale. A number scale is an infinite straight line on which are chosen: (1) a certain point o called the origin, (2) a positive direction indicated by an arrow, and (3) a suitable unit of length. We shall usually make the number scale horizontal and take the positive direction to be from left to right.

If the number x_1 is positive, it is depicted as a pointat M_1 at a distance $OM_1 = x_1$ to the right of the origin o; if the number x_2 is negative, it is represented by a point M_2 to the left of o at a distance $OM_2 = -x_2$. It is obvious that every real number is represented by a definite point on the number scale. Two different real numbers are represented by different points on the number scale.

The following assertion is also true: each point on the number scale represents only one real number (rational or irrational).

To summarize, all real numbers and all points on the number scale are in one-to-one correspondence: to each number there corresponds only one point, and conversely, to each point there corresponds only one number. This frequently enables us to regard "the number x" and "the point x" as, in a certain sense, equivalent expressions.

We state without proof the following important property of the set of real numbers: both rational and irrational numbers may be found between any two arbitrary real numbers. In geometrical terms, this proposition reads thus: both rational and irrational points may be found between any two arbitrary points on the number scale.

The integer consists of all positive and negative integers together with the real number 0.

Throughout our work **R** will denote the set of real numbers, **N** the set of positive integers, and **Z** the integers. Of major importance in calculus are certain subsets of **R** called intervals.

1.2 THE ABSOLUTE VALUE OF A REAL NUMBER

Let us introduce a concept which we shall need later on: the absolute value of a real number.

Definition1 If $|a| = \begin{cases} a & a \geqslant 0 \\ -a & a < 0 \end{cases}$, then the nonnegative number $|a|$ is called the *absolute value of a*.

For all real numbers a and b, some of the properties of absolute values:

(1) $-|a| \leqslant a \leqslant |a|$

(2) $|a| - |b| \leqslant |a \pm b| \leqslant |a| + |b|$

Proof From $-|a| \leqslant a \leqslant |a|$ and $-|b| \leqslant b \leqslant |b|$. Adding corresponding sides we obtain

$$-(|a| + |b|) \leqslant a \pm b \leqslant |a| + |b|$$

and hence

$$|a \pm b| \leqslant |a| + |b|$$

1.3 VARIABLES AND CONSTANTS

The numerical values of such physical quantities as time, length, area, volume, mass, velocity, pressure, temperature, etc. are determined by measurement. Mathematics deals with quantities divested of any specific content. From now on, when speaking of quantities, we shall have in view their numerical values. In various phenomena, the numerical values of certain quantities vary, while the numerical values of others remain fixed. For instance, in the uniform motion of a point, time and distance change, while the velocity remains constant.

A variable is a quantity that takes on various numerical values. *A constant* is a quantity whose numerical values remain fixed. We shall use the letters $x, y, z, u, \cdots$, etc. to designate variables and the letters $a, b, c, \cdots$, etc. to designate constants.

Note In mathematics, a constant is frequently regarded as a special case of variable whose numerical are the same.

It should be noted that when considering specific physical phenomena it may happen that one and the same quantity in one phenomenon is a constant while in another it is a variable. For example, the velocity of uniform motion is a constant, while the velocity of uniformly accelerated motion is a variable. Quantities that have the same value under all circumstances are called *absolute constants*. For example, the ratio of the circumference of a circle to its diameter is an absolute constant: $\pi = 3.14159\ldots$.

As we shall see throughout this course, the concept of a variable quantity is the basic concept of differential and integral calculus.

1.4 THE RANGE OF A VARIABLE

A variable takes on a series of numerical values. The values of a variable are geometrically depicted as points on a number scale.

Definition1 The set of all numerical values of a variable quantity is called the range of the variable.

We shall now define the following ranges of a variable that will be frequently used later on.

An *interval* is the set of all numbers x lying between the given points a and b (*the end points*) and is called closed or open accordingly as it does or does not include its end points.

An *open interval* is the collection of all numbers x lying between and excluding the given numbers a and $b(a < b)$; it is denoted (a, b) or, by means of the inequalities $a < x < b$.

A *closed interval* is the set of all numbers x lying between and including the two given numbers a and b; it is denoted $[a, b]$ or, by means of the inequalities $a \leqslant x \leqslant b$.

If one of the numbers a or b (say, a) belongs to the interval, while the other does not, we have a partly closed (*half-closed*) interval, which may be given by the inequalities $a \leqslant x < b$ and is denoted $[a, b)$. If the number b belongs to the set and a does not, we have the *half-closed* interval $(a, b]$, which may be given by the inequalities $a < x \leqslant b$.

If the variable x assumes all possible values greater than a, such an interval is denoted $(a, +\infty)$ and is represented by the conditional inequalities $a < x < +\infty$. In the same way regard the infinite intervals and half-closed infinite intervals represented by the conditional inequalities

$$a \leqslant x < +\infty, \quad -\infty < x < c, \quad -\infty < x \leqslant c, \quad -\infty < x < +\infty$$

The neighbourhood $(x_0 - \varepsilon, x_0 + \varepsilon)$ of the point x_0 with radius ε is an open interval; one often considers that the point x_0 is the midpoint and the quantity ε, the radius of the neighbourhood.

1.5 FUNCTION

In the study of natural phenomena and the solution of technical and mathematical problems, one finds it necessary to consider the variation of one quantity as dependent on the variation of another.

For example, we know that the area of circle, in terms of the radius, is $A = \pi R^2$. If the radius R takes on a variety of numerical values, the area A will also assume various numerical values. Thus, the variation of one variable brings about a variation in the other. Here, the area of a circle A is a function of the radius R. Let us formulate a definition of the concept "function".

Definition1 If to each value of a variable x (within a certain range) there corresponds one definite value of another variable y, then y is a function of x or, in functional notation, $y = f(x), y = \phi(x)$, and so forth.

The variable x is called the *independent variable* or *argument*. The relation between the variables x and y is called *a functional relation*. The letter f in the functional notation $y = f(x)$ indicates that some kind of operations must be performed on the value of x in order to obtain the value of y. In place of the notation $y = f(x), y = \phi(x)$ etc. one occasionally finds $y = y(x), u = u(x)$, etc. the letters y、u designating both the dependent variable and the symbol of the operations to be performed on x.

The notation $y = C$, where C is a constant, denotes a function whose value for any value of x

is the same and is equal to C.

Definition2 The set of values of x for which the values of the function y are determined by the rule $f(x)$ is called the *domain of definition of the function*.

Example1 The function $y = \sin x$ is defined for all values of x. Therefore, its domain of definition is the infinite interval $-\infty < x < +\infty$.

Note The definition of a function is sometimes broadened so that to each value of x, within a certain range, there corresponds not one but several values of y or even an infinitude of values of y. In this case we have a *multiple-valued* function in contrast to the one defined above, which is called a *single-valued* function. Henceforward, when speaking of a function, we shall have in view only *single-valued* functions. If it becomes necessary to deal with multiple-valued function we shall specify this fact.

Definition3 Two functions f and g from X to Y are said to be equal, and we write $f = g$, provided $f(x) = g(x)$ for every x in X.

For example, if $g(x) = (1/2)(2x^2 - 6) + 3$ and $f(x) = x^2$ for all x in $\mathbf{R}$, then $f = g$.

The *graph of a function f* is defined as the set of all points $(x, f(x))$ in a coordinate plane, where x is in the domain of f. Graphs are very useful for describing the behavior of $f(x)$ as x varies. if $P(x, y)$ is on the graph of f, then the ordinate y is the functional value $f(x)$.

1.6 ALGEBRAIC FUNCTION

Algebraic function include elementary functions of the following kind.

1.6.1 Power function

$$y = x^{\alpha}$$

1. α is a positive integer. The function is defined in the infinite interval $-\infty < x < +\infty$. For example: $y = x^2, y = x^3$. Their graphs are shown in Fig.1.1 and Fig.1.2.

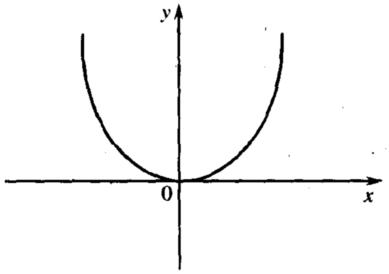


Fig.1.1

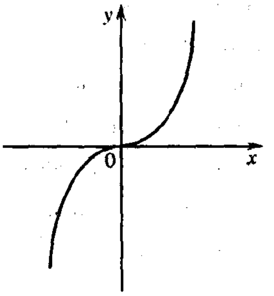


Fig.1.2

2. α is a negative integer. In this case, the function is defined for all values of x with the exception of $x = 0$. For example: $y = x^{-2}$, $y = x^{-3}$. Their graphs are shown in Fig.1.3 and Fig. 1.4.

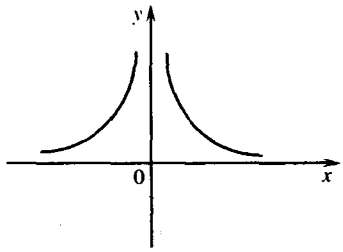


Fig.1.3

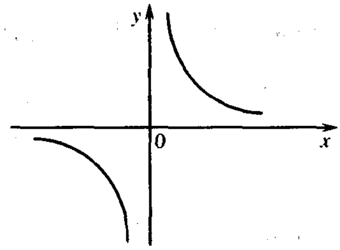


Fig.1.4

3. α is a fractional rational values. For example $y = x^{\frac{2}{3}}$, $y = x^{\frac{1}{3}}$. Their graphs are shown in Fig.1.5 and Fig.1.6.

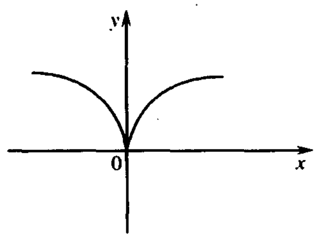


Fig.1.5

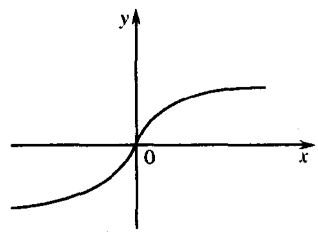


Fig.1.6

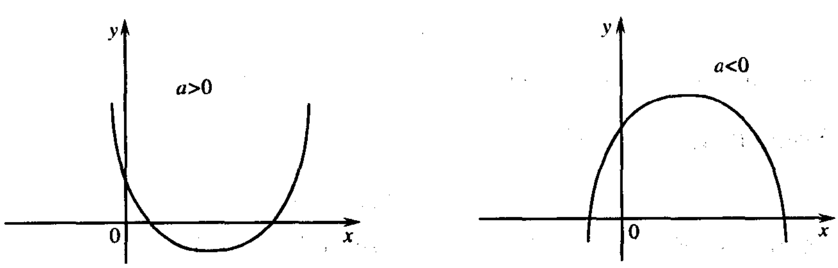
1.6.2 The rational integral function or polynomial

$$y = a_0x^n + a_1x^{n-1} + \cdots + a_n$$

where $a_0, a_1, \cdots, a_n$ are constants called *coefficients* and n is a nonnegative integer called the *degree of the polynomial*. It is obvious that this function is defined for all values of x, that is, it is defined in an infinite interval.

Example1 $y = ax + b$ is a linear function. When $a = 0$, $y = b$, the function is a constant.

Example2 $y = ax^2 + bx + c$ is a *quadratic function*. The graph of a quadratic function is a parabola (Fig.1.7)



(a) (b)

Fig.1.7