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材料力学（含光盘）

Mechanics of Materials

Pvtel • Kiusalaas

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Preface

This textbook is intended for use in a first course in mechanics of materials. Programs of instruction relating to the mechanical sciences, such as mechanical, civil, and aerospace engineering, often require that students take this course in the second or third year of studies. Because of the fundamental nature of the subject matter, mechanics of materials is often a required course, or an acceptable technical elective in many other curricula. Students must have completed courses in statics of rigid bodies and mathematics through integral calculus as prerequisites to the study of mechanics of materials.

To place this book in context for engineering education, the user should know that it is an extensive revision of the fourth edition of *Strength of Materials* by Pytel and Singer. The contents have been thoroughly modernized to reflect the realities and trends in contemporary engineering education. In addition to eliminating a few of the specialized topics normally taught in more advanced civil engineering courses, the coverage of fundamental topics has been expanded. All of the illustrations have been redrawn and improved, with the addition of a second color for clarity and as an aid to understanding complex structures. Many new diagrams aid the visualization of concepts and improve the comprehension of derivations. Fully 60% of the homework problems are new or modified versions of previous problems. A new feature is the computer problems found at the end of each chapter.

Every effort has been made to maintain the conciseness and organization that were the hallmarks of the earlier editions of Pytel and Singer. In the first eight chapters, emphasis is placed exclusively on elastic analysis through the presentation of stress, strain, torsion, bending, and combined loading. An instructor can easily teach these topics within the time constraints of a two- or three-credit course. The remaining five chapters of the text cover material that can be omitted from an introductory course. Because these more advanced topics are not interwoven in the early chapters on the basic theory, the core material can efficiently be taught without skipping over topics within chapters. Once the instructor has covered the material on elastic analysis, he or she can freely choose topics from the more advanced later chapters, as time permits. Organizing the material in this manner has created a significant savings in the number of pages without sacrificing topics that are usually found in an introductory text.

Features The most notable features of the organization of this text include the following:

- Chapter 1 introduces the concept of stress (including stresses acting on inclined planes). However, the general stress transformation equations and Mohr's circle are deferred until Chapter 8. Engineering instructors often hold off teaching the concept of state of stress at a point due to combined loading until students have gained sufficient experience analyzing axial, torsional, and bending loads. However, if instructors wish to teach the general transformation equations and Mohr's circle at the beginning of the course, they may go to the freestanding discussion in Chapter 8 and use it whenever they see fit.
- Advanced beam topics, such as composite and curved beams, unsymmetrical bending, and shear center appear in chapters that are distinct from the basic beam theory. This makes it convenient for instructors to choose only those topics that they wish to present in their course.
- Chapter 12, entitled "Special Topics," consolidates topics that are important but not essential to an introductory course, including energy methods, theories of failure, stress concentrations, and fatigue. Some, but not all, of this material is commonly covered in a three-credit course at the discretion of the instructor.
- Chapter 13, the final chapter of the text, discusses the fundamentals of inelastic analysis. Positioning this topic at the end of the book enables the instructor to present an efficient and coordinated treatment of elastoplastic deformation, residual stress, and limit analysis after students have learned the basics of elastic analysis.

The text contains an equal number of problems using SI and U.S. Customary units. Homework problems strive to present a balance between directly relevant engineering-type problems and "teaching" problems that illustrate the principles in a straightforward manner. An outline of the applicable problem-solving procedure is included in the text to help students make the sometimes difficult transition from theory to problem analysis. Throughout the text and the sample problems, free-body diagrams are used to identify the unknown quantities and to recognize the number of independent equations. The three basic concepts of mechanics—equilibrium, compatibility, and constitutive equations—are continually reinforced in statically indeterminate problems. The problems are arranged in the following manner:

- Virtually every article in the text is followed by sample problems and homework problems that illustrate the principles and the problem-solving procedure introduced in the article.
- Every chapter contains review problems, with the exception of optional topics. In this way, the review problems test the students' comprehension of the material presented in the entire chapter, since it is not always obvious which of the principles presented in the chapter apply to the problem at hand.
- Most chapters conclude with computer problems, the majority of which are design oriented. Students should solve these problems using a high-level language, such as MathCad® or MATLAB®, which minimizes the programming effort and permits them to concentrate on the organization and presentation of the solution.

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We are indebted to Dr. Christine Masters for checking solutions to the homework problems.

Andrew Pytel
Jaan Kiusalaas

List of Symbols

A	area
A'	partial area of beam cross section
b	width; distance from origin to center of Mohr's circle
c	distance from neutral axis to extreme fiber
C	centroid of area; couple
C_c	critical slenderness ratio of column
D, d	diameter
d	distance
E	modulus of elasticity
e	eccentricity of load; spacing of connectors
f	frequency
F	force
G	shear modulus
g	gravitational acceleration
H	horizontal force
h	height; depth of beam
I	moment of inertia of area
$\bar{I}$	centroidal moment of inertia of area
I_1, I_2	principal moments of inertia of area
J	polar moment of inertia of area
$\bar{J}$	centroidal polar moment of inertia of area
k	stress concentration factor; radius of gyration of area; spring stiffness
L	length
L_e	effective length of column
M	bending moment
M_L	limit moment
M_{yp}	yield moment
m	mass
N	factor of safety; normal force; number of load cycles
n	impact factor; ratio of moduli of elasticity
P	force; axial force in bar
P_{cr}	critical (buckling) load of column
$\mathscr{P}$	power
p	pressure
Q	first moment of area; dummy load
q	shear flow
R	radius; reactive force; resultant force
r	radius; least radius of gyration of cross-sectional area of column
S	section modulus; length of median line
s	distance
T	kinetic energy; temperature; tensile force; torque

T_L	limit torque
T_{yp}	yield torque
t	thickness; tangential deviation; torque per unit length
$\mathbf{t}$	stress vector
U	strain energy; work
u, v	rectangular coordinates
v	deflection of beam; velocity
V	vertical shear force
W	weight or load
w	load intensity
x, y, z	rectangular coordinates
$\bar{x}, \bar{y}, \bar{z}$	coordinates of centroid of area or center of gravity
α	coefficient of thermal expansion
α, β	angles
γ	shear strain; weight density
δ	elongation or contraction of bar; displacement
δ_s	static displacement
Δ	prescribed displacement
ϵ	normal strain
$\epsilon_1, \epsilon_2, \epsilon_3$	principal strains
θ	angle; slope angle of elastic curve
θ_1, θ_2	angles between x-axis and principal directions
ν	Poisson's ratio
ρ	radius of curvature; variable radius; mass density
σ	normal stress
$\sigma_1, \sigma_2, \sigma_3$	principal stresses
σ_a	stress amplitude in cyclic loading
σ_b	bearing stress
σ_c	circumferential stress
σ_ℓ	longitudinal stress
σ_{pl}	normal stress at proportional limit
σ_{ult}	ultimate stress
σ_w	working (allowable) normal stress
σ_{yp}	normal stress at yield point
τ	shear stress
τ_w	working (allowable) shear stress
τ_{yp}	shear stress at yield point
ω	angular velocity

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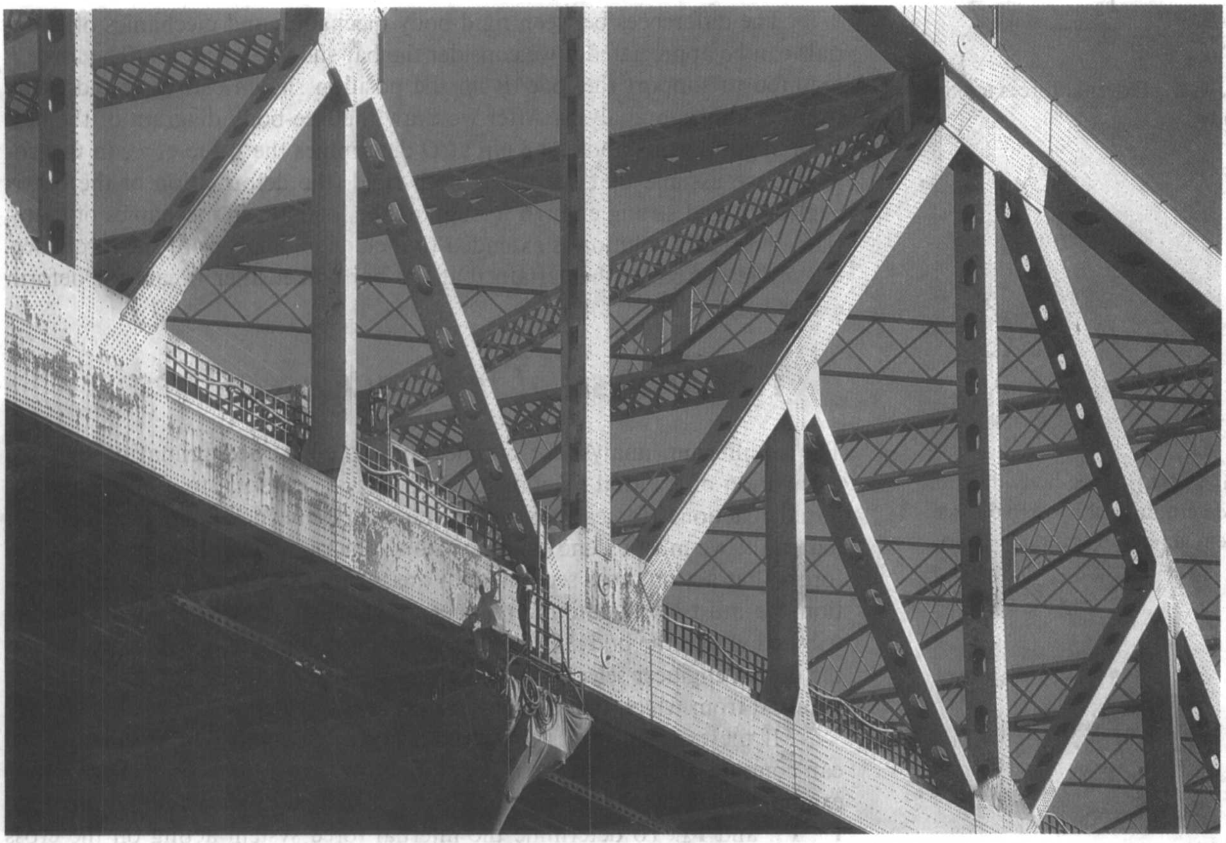
1 Stress



Truss of a highway bridge. The members of a truss carry loading by direct tension or compression; there is very little bending. A truss is an efficient structure in the sense that it has a high load/structural weight ratio.

1.1 Introduction

The three fundamental areas of engineering mechanics are statics, dynamics, and mechanics of materials. Statics and dynamics are devoted primarily to the study of the *external effects upon rigid bodies*—that is, bodies for which the change in shape (deformation) can be neglected. In contrast, *mechanics of materials* deals with the *internal effects and deformations* that are caused by the applied loads. Both considerations are of paramount importance in design. A machine part or structure must be strong enough to carry the applied load without breaking and, at the same time, the deformations must not be excessive.

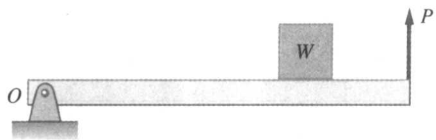


FIG. 1.1 Equilibrium analysis will determine the force P, but not the strength or the rigidity of the bar.

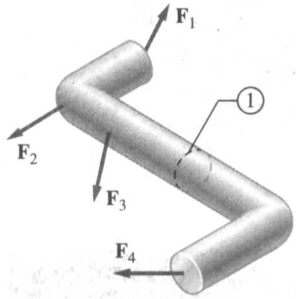


FIG. 1.2 External forces acting on a body.

The differences between rigid-body mechanics and mechanics of materials can be appreciated if we consider the bar shown in Fig. 1.1. The force P required to support the load W in the position shown can be found easily from equilibrium analysis. After we draw the free-body diagram of the bar, summing moments about the pin at O determines the value of P. In this solution, we assume that the bar is both rigid (the deformation of the bar is neglected) and strong enough to support the load W. In mechanics of materials, the statics solution is extended to include an analysis of the forces acting *inside* the bar to be certain that the bar will neither break nor deform excessively.

1.2 *Analysis of Internal Forces; Stress*

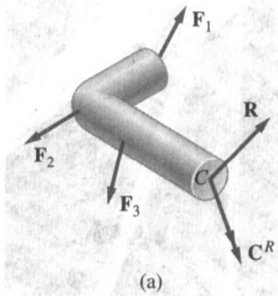


FIG. 1.3(a) Free-body diagram for determining the internal force system acting on section ①.

The equilibrium analysis of a rigid body is concerned primarily with the calculation of external reactions (forces that act external to a body) and internal reactions (forces that act at internal connections). In mechanics of materials, we must extend this analysis to determine *internal forces*—that is, forces that act on cross sections that are *internal* to the body itself. In addition, we must investigate the manner in which these internal forces are distributed within the body. Only after these computations have been made can the design engineer select the proper dimensions for a member and select the material from which the member should be fabricated.

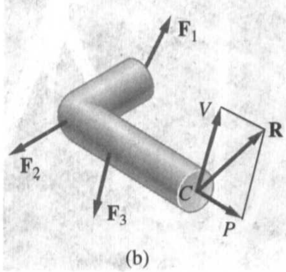


FIG. 1.3(b) Resolving the internal force $\mathbf{R}$ into the axial force P and the shear force V.

If the external forces that hold a body in equilibrium are known, we can compute the internal forces by straightforward equilibrium analysis. For example, consider the bar in Fig. 1.2 that is loaded by the external forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_4$. To determine the internal force system acting on the cross section labeled ①, we must first isolate the segments of the bar lying on either side of section ①. The free-body diagram of the segment to the left of section ① is shown in Fig. 1.3(a). In addition to the external forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, this free-body diagram shows the resultant force-couple system of the internal forces that are distributed over the cross section: the resultant force $\mathbf{R}$, acting at the centroid C of the cross section, and $\mathbf{C}^R$, the resultant couple[1] (we use double-headed arrows to represent couple-vectors). If the external forces are known, the equilibrium equations $\Sigma\mathbf{F} = \mathbf{0}$ and $\Sigma\mathbf{M}_C = \mathbf{0}$ can be used to compute $\mathbf{R}$ and $\mathbf{C}^R$.

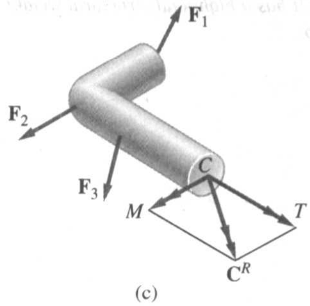


FIG. 1.3(c) Resolving the internal couple $\mathbf{C}^R$ into the torque T and the bending moment M.

It is conventional to represent both $\mathbf{R}$ and $\mathbf{C}^R$ in terms of two components: one perpendicular to the cross section and the other lying in the cross section, as shown in Figs. 1.3(b) and (c). These components are given the

[1] The resultant force $\mathbf{R}$ can be located at any point, provided that we introduce the correct resultant couple. The reason for locating $\mathbf{R}$ at the centroid of the cross section will be explained shortly.

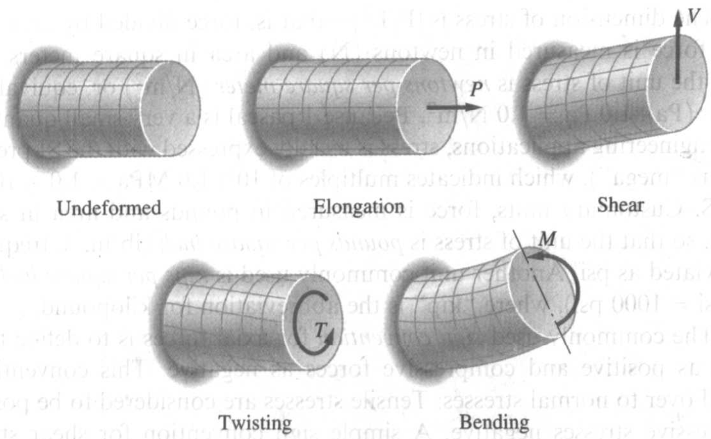


FIG. 1.4 Deformations produced by the components of internal forces and couples.

following physically meaningful names:

P: The force component that is perpendicular to the cross section, tending to elongate or shorten the bar, is called the *normal force*.

V: The force component lying in the plane of the cross section, tending to shear (slide) one segment of the bar relative to the other segment, is called the *shear force*.

T: The component of the resultant couple that tends to twist (rotate) the bar is called the *twisting moment* or *torque*.

M: The component of the resultant couple that tends to bend the bar is called the *bending moment*.

The deformations produced by these internal forces and internal couples are shown in Fig. 1.4.

Up to this point, we have been concerned only with the resultant of the internal force system. However, in design, the manner in which the internal forces are distributed is equally important. This consideration leads us to introduce the force intensity at a point, called *stress*, which plays a central role in the design of load-bearing members.

Figure 1.5(a) shows a small area element ΔA of the cross section located at the arbitrary point O. We assume that $\Delta \mathbf{R}$ is that part of the resultant force that is transmitted across ΔA, with its normal and shear components being ΔP and ΔV, respectively. The *stress vector* acting on the cross section at point O is defined as

$$\mathbf{t} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{R}}{\Delta A} \tag{1.1}$$

Its normal component σ (lowercase Greek *sigma*) and shear component τ (lowercase Greek *tau*), shown in Fig. 1.5(b), are

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A} = \frac{dP}{dA} \qquad \tau = \lim_{\Delta A \to 0} \frac{\Delta V}{\Delta A} = \frac{dV}{dA} \tag{1.2}$$

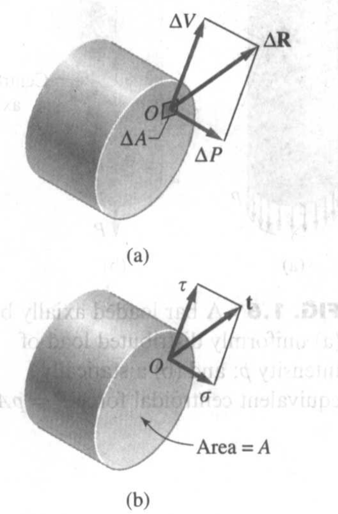


FIG. 1.5 Normal and shear stresses acting on the section at point O are defined in Eq. (2).

The dimension of stress is $[F/L^2]$—that is, force divided by area. In SI units, force is measured in newtons (N) and area in square meters, from which the unit of stress is *newtons per square meter* (N/m^2) or, equivalently, *pascals* (Pa): 1.0 Pa = 1.0 N/m^2. Because 1 pascal is a very small quantity in most engineering applications, stress is usually expressed with the SI prefix M (read as "mega"), which indicates multiples of 10^6: 1.0 MPa = 1.0×10^6 Pa. In U.S. Customary units, force is measured in pounds and area in square inches, so that the unit of stress is *pounds per square inch* (lb/in.^{2}), frequently abbreviated as psi. Another unit commonly used is *kips per square inch* (ksi) (1.0 ksi = 1000 psi), where "kip" is the abbreviation for kilopound.

The commonly used *sign convention* for axial forces is to define tensile forces as positive and compressive forces as negative. This convention is carried over to normal stresses: Tensile stresses are considered to be positive, compressive stresses negative. A simple sign convention for shear stresses does not exist; a convention that depends on a coordinate system will be introduced later in the text. If the stresses are *uniformly distributed*, they can be computed from

$$\sigma = \frac{P}{A} \qquad \tau = \frac{V}{A} \tag{1.3}$$

where A is the area of the cross section. If the stress distribution is not uniform, then Eqs. (1.3) should be viewed as the *average stress* acting on the cross section.

1.3 Axially Loaded Bars

a. Centroidal (axial) loading

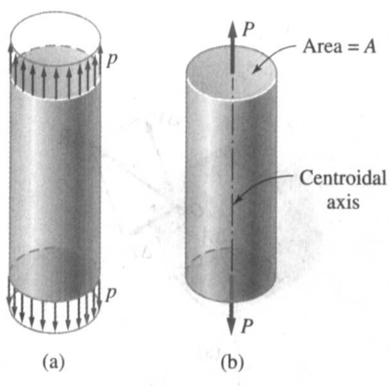


FIG. 1.6 A bar loaded axially by (a) uniformly distributed load of intensity p; and (b) a statically equivalent centroidal force $P = pA$.

Figure 1.6(a) shows a bar of constant cross-sectional area A. The ends of the bar carry uniformly distributed normal loads of intensity p (units: Pa or psi). We know from statics that when the loading is uniform, its resultant passes through the centroid of the loaded area. Therefore, the resultant $P = pA$ of each end load acts along the centroidal axis (the line connecting the centroids of cross sections) of the bar, as shown in Fig. 1.6(b). The loads shown in Fig. 1.6 are called *axial* or *centroidal loads*.

Although the loads in Figs. 1.6(a) and (b) are statically equivalent, they do not result in the same stress distribution in the bar. In the case of the uniform loading in Fig. 1.6(a), the internal forces acting on all cross sections are also uniformly distributed. Therefore, the normal stress acting at any point on a cross section is

$$\boxed{\sigma = \frac{P}{A}} \tag{1.4}$$

The stress distribution caused by the concentrated loading in Fig. 1.6(b) is more complicated. Advanced methods of analysis show that on cross sections close to the ends, the maximum stress is considerably higher than the average stress P/A. As we move away from the ends, the stress becomes more uniform, reaching the constant value P/A in a relatively short

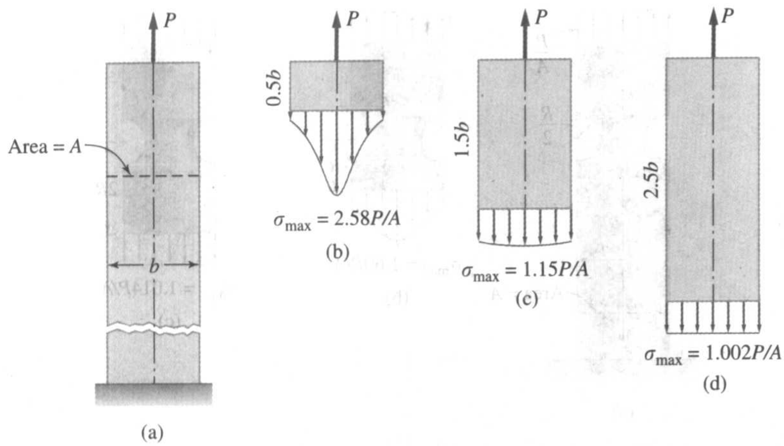


FIG. 1.7 Normal stress distribution in a strip caused by a concentrated load.

distance from the ends. In other words, the stress distribution is approximately uniform in the bar, except in the regions close to the ends.

As an example of concentrated loading, consider the thin strip of width b shown in Fig. 1.7(a). The strip is loaded by the centroidal force P. Figures 1.7(b)–(d) show the stress distribution on three different cross sections. Note that at a distance $2.5b$ from the loaded end, the maximum stress differs by only 0.2% from the average stress P/A.

b. *Saint Venant's principle*

About 150 years ago the French mathematician Saint Venant studied the effects of statically equivalent loads on the twisting of bars. His results led to the following observation, called *Saint Venant's principle*:

> *The difference between the effects of two different but statically equivalent loads becomes very small at sufficiently large distances from the load.*

The example in Fig. 1.7 is an illustration of Saint Venant's principle. The principle also applies to the effects caused by abrupt changes in the cross section. Consider, as an example, the grooved cylindrical bar of radius R shown in Fig. 1.8(a). The loading consists of the force P that is uniformly distributed over the end of the bar. If the groove were not present, the normal stress acting at all points on a cross section would be P/A. Introduction of the groove disturbs the uniformity of the stress, but this effect is confined to the vicinity of the groove, as seen in Figs. 1.8(b) and (c).

Most analysis in mechanics of materials is based on simplifications that can be justified with Saint Venant's principle. We often replace loads (including support reactions) by their resultants and ignore the effects of holes, grooves, and fillets on stresses and deformations. Many of the simplifications are not only justified but necessary. Without simplifying assumptions, analysis would be exceedingly difficult. However, we must always keep in mind the approximations that were made, and make allowances for them in the final design.