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KERNREAKTIONEN II: THEORIE

MIT 61 FIGUREN

SPRINGER-VERLAG
BERLIN · GÖTTINGEN · HEIDELBERG
1959

Druck der Universitätsdruckerei H. Stürtz AG., Würzburg

Contents.

Theory of Resonance Reactions and Allied Topics.

By

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With 49 Figures.

A. Introductory survey.

1. Introductory survey. *α) Early developments.* Alpha, beta and gamma emissions have been well known to early workers in the field of radioactivity. Spontaneous alpha and gamma emissions are closely related to processes referred to as nuclear reactions even though they are hardly describable as bona fide reactions since no incidence of a particle is directly involved. Beta emissions are considered on the other hand to be outside the scope of this review. The early work of Rutherford[1] and collaborators on (α, p) reactions which was followed by some confirmatory observations by Petterson and Kirsch provided the first experimental material on nuclear reactions in the ordinary sense of these words. All of this work was carried out by means of natural radioactive source of alpha particles.

A great impetus to the understanding of alpha emission came from the invention of wave mechanics and the associated realization that Schrödinger waves can pass through regions of space within which the classical mechanics value of the kinetic energy is negative. This realization led to the well known explanation of the Geiger-Nuttall relation by Gamow[2] and independently by Condon and Gurney[3]. The method of complex eigenvalues introduced in this connection by Gamow has a close relationship to some of the methods of nuclear reaction theory and will be discussed later in this article. The penetrability of a region of negative kinetic energy will also enter some phases of nuclear reaction theory to be discussed below.

As a follow up to his alpha-decay theory Gamow published[4] a related consideration which made use of barrier penetrability to explain the general fall off of reaction cross sections for the then known (α, p) reactions.

Making use of the general idea of barrier penetrability d'E. Atkinson and Houtermans[5] soon afterwards have pointed out that in stellar atmospheres protons are likely to have an advantage over alpha particles in causing nuclear

[1] It appears desirable to quote some of the early experimental papers because the large output of experimental work produced by means of modern techniques tends to obliterate the memory of the beautifully simple and significant early experiments. A few of the early papers from other schools and countries are also listed. The same policy will be followed in some of the other footnotes of the present introductory section. E. Rutherford: Phil. Mag. **37**, 581 (1919); E. Rutherford and J. Chadwick: Phil. Mag. **42**, 809 (1921); **44**, 417 (1922) and numerous other papers of the Cambridge school. Kirsch, Petterson *et al.*: Z. Physik **42**, 641 (1927); G. Kirsch and H. Pettersson: Atomzertrümmerung. Leipzig: Akademische Verlagsgesellschaft 1926; W. Bothe and H. Fränz: Z. Physik **43**, 456 (1927); **49**, 1 (1928).

[2] G. Gamow: Z. Physik **51**, 204 (1928).

[3] R. W. Gurney and E. U. Condon: Nature, Lond. **122**, 439 (1928).

[4] G. Gamow: Z. Physik **52**, 510 (1928).

[5] R. d'E. Atkinson and F. G. Houtermans: Z. Physik **54**, 656 (1929).

reactions, the barrier penetrability being much in favor of protons if the energies of the bombarding particles are the same. The energies under consideration were very small corresponding to negligible cross sections in a laboratory experiment. The possibility of obtaining nuclear reactions with the accelerators then under construction was pointed out by BREIT[1], whose estimates showed that rather modest accelerating potentials should suffice in the case of proton bombardment. Soon afterwards the first nuclear reaction produced by means of a man-made accelerating machine was observed by COCKROFT and WALTON[2].

While work leading to the first observation of a nuclear reaction by means of artificially accelerated particles was being carried on at the Cavendish Laboratory, several efforts[3] in the same direction were in progress elsewhere and have also furnished early information regarding nuclear reactions with charged particle. The discovery of the neutron enriched the experimental material manyfold[4]. The absence of Coulomb barrier penetration effects for neutrons made it possible to obtain reactions with the heaviest nuclei. Bombardment with slow neutrons proved especially successful in the discovery of many reactions and in revealing the existence of resonances in the capture of slow neutrons.

The interpretation of data was also making progress and the rate of appearance of theoretical papers concerned with the theory of nuclear reactions increased greatly. No attempt will be made here to give even an approximately complete bibliography of nuclear reaction theory through this and later periods. The developments which are usually considered to be the most fruitful will be discussed and only occasional facts concerning the history of the developments will be mentioned.

It is, of course, not altogether clear what the definition of nuclear reaction theory should be. This subject is usually understood to include any consideration dealing with reactions of the nuclear transmutation type, excluding β-decay and γ-internal conversion processes. Elastic and inelastic scattering are usually considered to be a part of nuclear reaction theory. The reasons for these some what arbitrary divisions are more matters of custom than of logic.

β) Objectives. The objectives of nuclear reaction theory have varied with the time and are bound to do so in the future. In most cases the object has been to make use of observations on nuclear reactions in order to derive conclusions concerning nuclear structure. Thus the theory of alpha-decay has as a partial object the determination of nuclear radii, the theory of stripping and pick-up has similarly to do with the assignment of quantum numbers to individual nucleons in the independent particle nuclear model. In connection with such applications

[1] G. BREIT: Phys. Rev. **34**, 817 (1929).

[2] J.D. COCKROFT and E.T.S. WALTON: Proc. Roy. Soc. Lond., Ser. A **137**, 229 (1932).

[3] M. WOLFKE: Phys. Z. **24**, 249 (1923). — G. BREIT and M.A. TUVE: Nature, Lond. **121**, 535 (1928). — G. BREIT, M.A. TUVE and O. DAHL: Phys. Rev. **35**, 51 (1930). — M.A. TUVE, G. BREIT and L.R. HAFSTAD: Phys. Rev. **35**, 66 (1930). — R. J. VAN DE GRAAFF: Phys. Rev. **38**, 1919 (1931). — R. J. VAN DE GRAAFF, K.T. COMPTON and L.C. VAN ATTA: Phys. Rev. **43**, 149 (1933). — M.A. TUVE, L.R. HAFSTAD and O. DAHL: Phys. Rev. **48**, 241, 315 (1935). — R.G. HERB, D.B. PARKINSON and D.W. KERST: Rev. Sci. Instrum. **6**, 261 (1935). — Phys. Rev. **51**, 75 (1937). — E.O. LAWRENCE and N.E. EDLEFSON: Science, Lancaster, Pa. **72**, 376 (1930). — E.O. LAWRENCE and M.S. LIVINGSTON: Phys. Rev. **40**, 19 (1932); **45**, 608 (1934). — E.O. LAWRENCE, E. MCMILLAN and R.L. THORNTON: Phys. Rev. **48**, 495 (1935). — E.O. LAWRENCE and D. COOKSEY: Phys. Rev. **50**, 1131 (1936).

[4] J. CHADWICK: Proc. Roy. Soc. Lond., Ser. A **136**, 692 (1932). — I. CURIE-JOLIOT and F. JOLIOT: C. R. Acad. Sci., Paris **194**, 273 (1932). — P.B. MOON and J.R. TILLMAN: Nature, Lond. **135**, 904 (1935). — L. SZILARD: Nature, Lond. **136**, 150 (1935). — T. BJERGE and C.H. WESTCOTT: Proc. Roy. Soc. Lond., Ser. A **150**, 709 (1935). — E. AMALDI and E. FERMI: Ric. Sci. **1**, 310 (1936). — J.R. DUNNING, G.B. PEGRAM, G.A. FINK and D.P. MITCHELL: Phys. Rev. **48**, 265 (1935).

of nuclear reaction theory there is the natural question of the separation of features of a model which may be considered as essential to the explanation of the reactions from unessential ones. The substitution of the scattering matrix for the Hamiltonian in a treatment of a quantum mechanical problem such as has been attempted by HEISENBERG is a rather extreme example of this viewpoint. WIGNER's $\mathcal{R}$-Matrix theory is a development of a partially related type. It is less abstract since it presupposes the existence of a Hamiltonian and has been the basis of many developments in nuclear reaction theory. An account of the $\mathcal{R}$-matrix formalism will be given in the present article. One of the gains in concreteness achieved in the $\mathcal{R}$-matrix theory arises in the employment of a definite nuclear radius, a circumstance which may perhaps also be considered as a disadvantage from another viewpoint. A development along more abstract and even more general lines is the analysis of nuclear reaction theory in terms of the principle of causality and dispersion relations. The causality considerations and to a partial degree the $\mathcal{R}$-matrix theory are in a good position to separate those features of a reaction which do not depend on detailed assumptions regarding nuclear structure from features which depend strongly on special circumstances. In most cases there is no clear distinction between this type of consideration and the preceding one the practical execution of the task usually resulting in a partial submergence of motivation. It should be pointed out that even from the viewpoint of applying nuclear reaction theory to investigations of nuclear structure it should be very useful to know which features of nuclear reactions may be expected to hold independently of detailed assumptions, and it is probable therefore that the more abstract theories will prove to be of great value.

The engineering applications of nuclear physics have made it desirable to have formulas for cross sections independently of their physical significance. The soundness of theoretical considerations used in arriving at a formula for a cross section affects such applications only to a partial degree. The problem of representing a cross section to a certain accuracy by a formula independently of the correctness of the underlying physical picture will be considered as outside the scope of the present considerations.

γ) *Resonance formulas and the compound nucleus.* Nuclear reaction cross sections are frequently represented by means of resonance formulas. The earliest significant introduction of the idea of resonance in nuclear interactions is probably to be found in WIGNER's work on neutron-proton scattering[1]. This matter has also come up in connection with charged particle reactions[2] and soon afterwards the capture of slow neutrons required an explanation in terms of resonance theory. The earlier attempts to explain the large slow neutron capture were made by BETHE[3], FERMI[4], PERRIN and ELSASSER[5], BECK and HORSLEY[6].

These papers have correctly attributed the phenomenon to the operation of the s part of the incident neutron wave. The employment of ideas of central field resonances has given however too much scattering as has been particularly clearly brought out by BETHE. The relatively high importance of γ-emission in depressing the scattering has been brought out nearly simultaneously and independently

[1] E. WIGNER: Z. Physik **83**, 253 (1933).

[2] G. BREIT: Phys. Rev. **40**, 127 (1932). — G. BREIT and F.L. YOST: Phys. Rev. **47**, 508 (1935). — Ref. [*1*]. — L.R. HAFSTAD and M.A. TUVE: Phys. Rev. **47**, 506, 507 (1935).

[3] H.A. BETHE: Phys. Rev. **47**, 747 (1935).

[4] E. FERMI, B. PONTECORVO, F. RASETTI and E. SEGRÈ: Proc. Roy. Soc. Lond., Ser. A **149**, 552 (1935).

[5] F. PERRIN and W.M. ELSASSER: C. R. Acad. Sci., Paris **200**, 450 (1935).

[6] G. BECK and L.H. HORSLEY: Phys. Rev. **47**, 510 (1935).

by Bohr[1] as well as by Wigner and the writer[2]. The feature common to both papers is the employment of quasi-stationary levels and of the competition between different modes of disintegration. It may be mentioned that, although the title of the paper by Breit and Wigner has been concerned only with the capture of slow neutrons, the paper itself points out the applicability of the scheme to charged particles; the specializations made in the calculations were stated in the paper quoted, and it was brought out that the interaction of two continua through a resonance level is the essential circumstance so that the resonance feature is more general than the special model used. The similarity to the Weisskopf-Wigner theory of light emission indicated the possibility of extensions of the resonance formula to the interaction between several continua and the well-known optical dispersion formula showed the possibility of employing formulas containing the combined action of several resonance levels.

It became clear through comparison with experiment that some nuclear reactions can be pictured rather well by means of the compound nucleus picture. Bohr's paper has done a great deal in stimulating work on applications of this picture. It was soon followed by Bethe and Placzek's and Bethe's contributions[3] which have shown that the experimental material lends itself to this type of interpretation. This work also contains extensions of the single-level formula to the case of many levels and shows how interference between different levels may be expected to take place. These early formulations of the "dispersion theory" of nuclear reactions have since been supplanted by more rigorous treatments. The value of Bohr's and Bethe's contributions to the development of the subject has been very great and even now many of the considerations introduced by Bethe appear in other or modified forms in terms of the more rigorous $\mathcal{R}$-matrix theory. The subject also owes much to the papers of Bohr and Kalckar[4] and those of Kalckar, Oppenheimer and Serber[5].

The early experimental work on resonances owes much to the development of van de Graaff machines. References to early work on these and to other instrumentation developments have already been made at an earlier stage in this article. Their application to vacuum tubes has been accomplished successfully in time to make observations on charged particles contribute significantly to the accumulation of data on nuclear levels. The development of the modification of this machine which operates in a gaseous atmosphere under pressure has been accomplished under the leadership of Herb and has led to the most accurate and significant data on charged particles and nuclear levels. Among the early significant observations of resonances in addition to those quoted here earlier one may mention those of Herb, Kerst and McKibben[6] through whose work it became clear that there are many nuclear resonances in the proton bombardment of Li, Be, B, F, Al when nuclei of these elements are bombarded with protons in the energy range of 2 Mev. These data have been improved on since, partly by the same and partly by other workers, and they have been extended to many other nuclei. There has been a healthful mutual stimulation between the experimental

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[1] N. Bohr: Nature, Lond. **137**, 344 (1936). — Science, Lancaster, Pa. **86**, 161 (1937).

[2] Cf. Ref. [*1*]. A preliminary account of this paper was read at the New York meeting of the American Physical Society and its abstract appeared as below. E. Wigner and G. Breit: Phys. Rev. **49**, 642 (1936).

[3] H. Bethe and G. Placzek: Phys. Rev. **51**, 450 (1937). — H. Bethe: Rev. Mod. Phys. **9**, 69 (1937).

[4] N. Bohr and F. Kalckar: Kgl. danske Vidensk. Selskab., mat.-fys. Medd. **14**, 10 (1937).

[5] F. Kalckar, J. R. Oppenheimer and R. Serber: Phys. Rev. **52**, 273, 279 (1937).

[6] R. G. Herb, D. W. Kerst and J. L. McKibben: Phys. Rev. **51**, 691 (1937).

work and the theoretical speculations regarding dispersion formalisms for nuclear reactions which is only partly accounted for by the published literature[1].

Independently of the development of mathematical theories of resonances in nuclear reactions the observation of resonances has great value since when they are sharp they may be interpreted as levels of a nucleus and since such information contributes directly to studies of nuclear structure. In cases of very sharp resonances with spacing between levels appreciably greater than the level width, the complete dispersion theory of nuclear reactions is hardly needed, a very elementary form of it being sufficient. In such cases the experimental result is usually clear enough to indicate the position of the level and in these applications the many-level formulae are therefore not needed. Even so essential nuclear data are lost if one does not make use of the observed level width to derive some conclusion regarding the size of the nucleus or the internal normalization of the wave function if it is adjusted to give unit flux at an infinite distance. It is thus difficult to interpret data without the foundation of adequate theories of nuclear resonances.

As has been previously mentioned, the early attempts to formulate many-level dispersion theories of nuclear reactions have not been rigorous. The assumptions made in the derivations gave rise to a simple superposition of effects of separate levels in formulas for amplitudes, the squares of whose absolute values give the reaction cross sections. Consideration of special models [*6*], [*10*] indicated however that the effects of different levels do not combine in this simple manner except in cases of weak coupling between parts of the system responsible for defining the position of the level and the continuum. The same situation has been treated soon afterwards by WIGNER[2] by means of the $\mathcal{R}$-matrix approach with the same result regarding the way in which different levels interfere. WIGNER'S $\mathcal{R}$-matrix method gives the most general and most completely worked out approach to the understanding of nuclear reactions. It will be discussed more fully in the third chapter of this article. A few general remarks regarding it and the compound nucleus picture appear to be appropriate now. The $\mathcal{R}$-matrix method avoids detailed discussion of the many-dimensional wave equation in the nuclear interior and substitutes for it the specification of quantities known as energy levels and of the reduced widths. The energy levels in the $\mathcal{R}$-matrix theory have a different meaning from the naive one to which one is accustomed in the treatment of discrete levels. Thus the energy values depend on the choice of channel radii and the identification of an experimental peak in a measured nuclear cross section with a level of the $\mathcal{R}$-matrix theory has to be made with caution. The $\mathcal{R}$-matrix approach is sometimes referred to as a "black box treatment" because the nuclear interior enters the theory only through the levels and the reduced widths. One is therefore not in an especially good position for drawing conclusions regarding the interior in terms of a nuclear model. At all events if a conclusion is to be derived it has to be made *via* the intermediate step of going through the assignment of level energies and reduced widths. The situation is similar to that of describing the action of a coil of wire which is enclosed in a box and connected to two binding posts intended for electrical connection. For most purposes in ordinary electrical measurements the action of

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[1] E. CREUTZ [Phys. Rev. **55**, 819 (1939)] demonstrated for the first time the existence of resonance scattering of charged particles associated with a resonant γ-ray capture. He showed that a resonance occurs in scattering of protons by Li^7 attributable to an excited state of Be^{8*}. Many cases of resonance reactions have been investigated since then [*5*].

[2] E.P. WIGNER: Phys. Rev. **70**, 606 (1946); **73**, 1002 (1948). — Amer. J. Phys. **17**, 99 (1949). — Ref. [*2*] to [*4*].

the coil is represented completely by its resistance and one need not look into the box if one is only interested in the results of such measurements. The resistance of the coil plays a role somewhat like that of the parameters of the $\mathcal{R}$-matrix theory. A complete characterization of the action of a physical coil of wire requires the knowledge of the impedance presented at the two terminals. On account of skin effect and distributed capacitance effects the reactance and the resistance are both functions of the frequency so that a complete specification requires an infinite number of parameters somewhat similarly to the necessity of using an infinite number of level energies and reduced widths in the $\mathcal{R}$-matrix theory. In both cases it is not necessary to open the box in order to have a mathematical description of its action either in the case of a nuclear reaction or in the case of a coil of wire.

In the latter case one sees that, while the specification of the complex impedance at all frequencies describes the action of the coil completely for many purposes, it is insufficient in other instances and that, furthermore, it does not reveal in a direct way the physical nature of the object inside the black box. In the nuclear case the ultimate object is still that of ascertaining the constitution of the nuclear system rather than a mere description of its reaction to external impulses. Part of the object of the present article is to clarify the extent to which the nuclear interior can be clearly characterized by the black box treatment. An additional object is to evaluate the chance that the $\mathcal{R}$-matrix theory will give a sufficiently simple connection with the internal constitution to make a determination of the parameters illuminating in terms of the physical constitution of the nuclear interior.

In this connection the electrical analogy may be used to illustrate the way in which the mere possibility of describing the action of a black box does not necessarily assure one of dealing with the simplest or most illuminating description. If, for example, the interior of the box consisted of an inductor in series with a capacitor, its behaviour could be described approximately as an impedance

$$R + \sqrt{-1}\left(L\omega - \frac{1}{C\omega}\right)$$

where R, L, ω are respectively the resistance, inductance and capacitance while $\omega/2\pi$ is the frequency. This is often a very good description in certain frequency ranges even though strictly speaking R, L, ω are not constants. The emphasis on strict energy independence of the parameters of the $\mathcal{R}$-matrix theory does not appear therefore to be necessarily desirable and theories in which the nuclear parameters are allowed to have some variation need not be considered as necessarily inferior to the $\mathcal{R}$-matrix theory. It will be seen in the first chapter of the present article and partly in the second that from some viewpoints such theories have a closer connection with the elementary ideas regarding resonances than the $\mathcal{R}$-matrix and that the connection of levels with stationary states is simpler for them. A study of the relationship of these two viewpoints was also one of the objects in writing the present article.

The compound nucleus model in nuclear reaction theory is proved as well as disproved by the theoretical developments. This is possible because there is no clear meaning in the words "compound nucleus". Their interpretation differs in fact quite widely. To some these words imply a system each particle of which is located within a certain radius as measured from their common center of mass. The radius is generally agreed on as having to be smaller than say $1.5\times10^{-13} A^{\frac{1}{3}}$ cm with A standing for the mass number. There is no exact reason for preferring one radius to another one in such a definition and the language generally employed in this connection lacks precision. The distinction between compound nucleus type reactions and other types such as stripping, pick-up, Coulomb excitation is made primarily on a roughly geometrical basis of the largest distance at which the reaction takes place with appreciable probability. The exact value of the radius to be used in distinguishing compound nucleus formation from other processes is usually not important. In fact while the usual descriptions are put in terms of the radius, the distinctions are made in practice on the basis of applicability of approximations which may be expected to hold, provided two nuclear aggregates are never too close together. The criterion is that of working out the consequences of a simplified mechanism such as Coulomb excitation which cannot be expected to hold if the colliding particles come too close to each other. When the results are compared with experiment a lack of agreement is taken to indicate the participation of processes other than the relatively simple one of Coulomb excitation. Usually any such lack of agreement is likely to be referred to as being caused by compound nucleus formation. In such cases the words "compound nucleus" stand at least partly for matters believed to be too complicated to be treated more than qualitatively.

In some cases the words "compound nucleus" are meant to bring to mind a state consisting of all the particles which, once formed, decays in a statistically predetermined manner. This is often the intended connotation when the words are used in connection with the one-level formula. The aspect of the theory which makes the words applicable is the *factorization of the numerator* of the one-level formula which results in the appearance of such numerators as Γ_n^2, $\Gamma_n\Gamma_\gamma$, Γ_γ^2 in the consideration of neutron scattering, radiative scattering, and gamma-ray scattering. Similarly, the compound nucleus is sometimes considered as emitting particles as though they were in thermal equilibrium before their emission. The feature of definite division of the process into two stages, the first of which consists in the formation of the compound nucleus while the second consists in the disintegration of the system so formed, is characteristic of the use of the words "compound nucleus" in these instances.

In the course of the development of the subject there has grown a generalization of this meaning of the term "compound nucleus" to systems which are describable by means of the $\mathcal{R}$-matrix theory. Such systems are rather general and a description of a nuclear reaction as taking place "after" compound nucleus formation in this sense is only a weak restriction on the character of the reaction. The mathematical feature of $\mathcal{R}$-matrix theory which suggests this employment of the words "compound nucleus" is the occurrence of products

$$\gamma_{\lambda s}\,\gamma_{\lambda t}$$

in formulas for probability amplitudes. The $\gamma_{\lambda s}$ are quantities characteristic of a level E_λ and a channel s. The occurrence of $\gamma_{\lambda s}\gamma_{\lambda t}$ in a reaction which takes place when incidence in channel s gives rise to disintegration by way of channel t brings to mind the formation of the energy level E_λ. The resultant probability amplitude consists of a sum over λ. The theory thus does not correspond to the formation of a compound nucleus in an ordinary sense but rather to a linear superposition of amplitudes which originate in separate levels. The possibility of representing each term as a product of factors attributable to incident and emergent channels is the common link with the more naive employment of the "compound nucleus" terminology. Since the $\mathcal{R}$-matrix formalism can be applied to a great variety of systems, a great many reactions would have to be classified as of the compound nucleus type if the mere applicability of the $\mathcal{R}$-matrix were the only criterion. In practice, however, the criterion of a reasonable nuclear radius is usually combined with the applicability of the $\mathcal{R}$-matrix. If a conservative nuclear radius is employed it is seen that the non-compound nucleus reactions such as stripping are not describable by the $\mathcal{R}$-matrix. These distinctions are seen to be to some degree matters of convention. It appears rational therefore not to be making sharp distinctions between reactions involving compound nucleus formation and those that do not. If such distinctions will be made it will be understood that the nuclear radius is of the conservative type and is not much larger than $1.4 \times 10^{-13} A^{\frac{1}{3}}$ cm.

Even before the formulation of the $\mathcal{R}$-matrix theory KAPUR and PEIERLS[1] have developed a mathematical formalism of the black box type. Their theory employs complex energy eigenvalues in the expansion of the wave function for the interior region. This complication has detracted from its popularity for many years. The representation of the scattering matrix is on the other hand somewhat simpler in the Peierls-Kapur theory than in the Wigner formalism. The two approaches have much in common both making use of the black-box viewpoint and the associated possibility of employing many formally equivalent level systems which can be varied by increasing the nuclear radius. Since in the Wigner approach one also has available the answer for the scattering matrix which can be expanded in partial fractions in terms of the poles, there is no difference in substance between the results. C. BLOCH[2] has shown recently how both formalisms can be obtained from a common starting point employing a general GREEN's function statement of the problem. The presentation of resonance theory in the present article is carried out mainly in terms of the $\mathcal{R}$-matrix because many mathematical results regarding it are available and partly because of its greater popularity.

δ) Stripping theories. Even in its early history nuclear physics has received a great impetus through the pioneer work of LAWRENCE and collaborators[3] with the cyclotron which had led among other things to the discovery that deuterons are more effective in producing nuclear reactions than could be expected from barrier penetrabilities employing conservative nuclear radii. The process involved was explained by OPPENHEIMER and PHILLIPS[4] who pointed out that it is not necessary for the deuteron as a whole to come close to the bombarded nucleus and that it suffices for the neutron to leave the deuteron and to attach itself to the target particle. The (d, p) reactions can thus take place with relatively high probability even at moderate bombarding energies. A revival of interest in these stripping reactions has taken place after it was found by BUTLER[5] at the general

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[1] P.L. KAPUR and R. PEIERLS: Proc. Roy. Soc. Lond., Ser. A **166**, 277 (1938). — R. PEIERLS: Proc. Cambridge Phil. Soc. **44**, 242 (1947). — J. BOWCOCK: Compound nuclear theory and the optical model. Phys. Rev. (to be published). — G.E. BROWN: Direct interaction and nuclear dispersion theory (to be published). — G.E. BROWN: Resonances in a complex well (to be published). — G.E. BROWN and C.T. DE DOMINICIS: Elastic scattering of several Mev nucleons by complex potential wells (to be published). — In these developments the part of the scattering matrix corresponding to the optical model is singled out and the remainder is expanded as a sum of resonance type terms. In this way the hard core scattering characteristic of the Wigner-Eisenbud formulation does not appear in the discussion and instead there is present a potential scattering term somewhat as in BETHE's older treatment.

[2] CLAUDE BLOCH: Une formulation unifiée de la théorie des réactions nucléaires (Saclay report No. 250) (to be published). The contents of this paper have been read in abridged form at the 1957 Pittsburgh conference on nuclear physics.

[3] E.O. LAWRENCE, E. MCMILLAN, R.L. THORNTON: Phys. Rev. **48**, 493 (1935). E. MCMILLAN and E.O. LAWRENCE: Phys. Rev. **47**, 17 (1935).

[4] J.R. OPPENHEIMER and M. PHILLIPS: Phys. Rev. **48**, 500 (1935).

[5] S.T. BUTLER: Phys. Rev. **80**, 1095 (1950); **88**, 685 (1952). — Proc. Roy. Soc. Lond., Ser. A **208**, 559 (1951); and other references in Sect. 45 below.

suggestion of PEIERLS that the angular distribution of protons in a (d, p) reaction can be used to determine the orbital angular momentum of the neutron after its transfer. There has been much related research involving also the inverse of stripping, viz. the pick-up reactions. These subjects are of considerable importance for the collection of information regarding nuclear levels but they do not appear to involve quite the same kind of generality of method as the $\mathcal{R}$-matrix theory. Their treatment will be presented only as incidental to the general one of the collision matrix and of the $\mathcal{R}$-matrix theory. An earlier approach to stripping theory has been made by SERBER[1] in connection with the high energy observations of HELMHOLZ, MCMILLAN and SEWELL[2]. This work is significant in indicating the qualitative correctness of theoretical views regarding the structure of the deuteron and of the bombarded nucleus. The general viewpoint is related to that of SERBER's high energy optical model and the growth of the two sets of ideas has been mutually helpful.

ε) Direct interactions. The low energy stripping theory is related to the development of the direct interaction theory of nuclear reactions[3] which has been very successful in explaining angular distributions of reaction products on the basis of interactions taking place on the nuclear surface. Limitations of time will unfortunately make it impossible to cover this interesting development in the detail that it deserves. The last mentioned paper of BUTLER'S has been very successful in accounting for the experimental material on angular distributions for reactions with residual nuclei in ground states. One may hope that as in the case of low and medium energy stripping specific information regarding nuclear structure will be the result of these studies, the calculated distributions depending on wave function configuration assignments for the nucleons.

WÄFFLER has found that (n, p) cross sections in elements between Fe and La with neutrons having maximum energies of about 14 Mev are higher than those predicted by the statistical theory of WEISSKOPF. This work was suggested by the well known similar discrepancy for the (γ, p) reaction. On the other hand, the $(n, 2n)$ reaction showed no such marked discrepancy. In an attempt to account for the large (γ, p) cross sections COURANT suggested the mechanism of direct interaction for the (n, p) reaction as well. In the work of MCMANUS and SHARP as well as in that of AUSTERN, BUTLER and MCMANUS special attention is paid to (n, p) reactions. The incident neutron is supposed to interact with the tail of the proton wave function in the region of space within which the proton has a negative kinetic energy. The work of MCMANUS and SHARP was motivated by the attempt to explain observations[4] on (n, α) and (n, p) cross sections at 14 Mev for a number of elements with $A > 80$. These cross sections were 70 to 1000 times larger than predicted by a literal application of WEISSKOPF's statistical theory of nuclear reactions while the $(n, 2n)$ reactions were in approximate agreement with the theory. MCMANUS and SHARP were able to account for the discrepancy by employing a square-well model for individual nucleons and a perturbation calculation in the manner of COURANT, making reasonable assumptions regarding the probability of direct interaction. The calculations of AUSTERN, BUTLER and MCMANUS are carried out in the impulse approximation of CHEW[5] which

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[1] R. SERBER: Phys. Rev. **72**, 1008 (1947).

[2] A.C. HELMHOLZ, E.M. MCMILLAN and D.C. SEWELL: Phys. Rev. **72**, 1003 (1947).

[3] H. WÄFFLER: Helv. phys. Acta **23**, 238 (1950). — E.D. COURANT: Phys. Rev. **82**, 703 (1951). — H. MCMANUS and W.T. SHARP: Phys. Rev. **87**, 188 (1952). — N. AUSTERN, S.T. BUTLER and H. MCMANUS: Phys. Rev. **92**, 350 (1953). — R.M. EISBERG and G. IGO: Phys. Rev. **93**, 1039 (1954). — S.T. BUTLER: Phys. Rev. **106**, 272 (1957).

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appears to be justifiable because neither particle interacts with the nucleus in the region of configuration space in which they interact with each other. The calculation yields distribution curves showing peaks in the proton distribution which are characteristic of the orbital angular momenta l_p, l_n of the proton and neutron in their bound states. Thus if $l_p = l_n = 0$ the peak in the differential cross section is in the incident direction, the cross section dropping to half value for neutron and proton energies of 14 Mev at an angle of $\sim 20°$ and becoming zero at $\sim 35°$ for mass number $A = 27$. If $l_p = 0$ and $l_n = 1$ the peak is at about 30° for the same energies and the same value of A, the cross section becoming very small at an angle of $\sim 60°$. The origin of the angular distribution is the occurrence in the transition matrix element of the factor

$$\int_{r \geq b} u_p(\boldsymbol{r})\, u_n^*(\boldsymbol{r}) \exp\left[i\left(\boldsymbol{k}_n - \boldsymbol{k}_p\right)\boldsymbol{r}\right] d\boldsymbol{r}$$

where n, p refer to the two bound states while $\boldsymbol{k}_n$, $\boldsymbol{k}_p$ are the propagation vectors of the neutron and proton when they are free. This factor is arrived at as a result of approximations one of which involves the relative smallness of the variation of the free neutron-proton scattering amplitude in the relevant range and another the approximate isotropy of the free neutron-proton scattering in the center-of-mass system of these two particles. The angular dependence of the bound state functions singles out from the exponential parts, with an angular dependence on the angle between $\boldsymbol{k}_n$ and $\boldsymbol{k}_p$, containing Legendre functions of an order l which can be obtained by quantum composition of $\boldsymbol{l}_n$ and $\boldsymbol{l}_p$. In the expansion of the exponential there occur the well-known Bessel functions of order $l + \frac{1}{2}$ and argument $|\boldsymbol{k}_n - \boldsymbol{k}_p|\, r$. When the integration over r is performed this leaves, on account of the exponential character of the dependence of u_n and u_p on r, essentially the values of the Bessel functions of argument $|\boldsymbol{k}_n - \boldsymbol{k}_p|\, b$ which introduce a dependence on the angle between $\boldsymbol{k}_n$ and $\boldsymbol{k}_p$.

Employing a more quantitative consideration, BUTLER accounts with marked success for several observed angular distributions such as that of $Mg^{24}(\alpha, \alpha')\, Mg^{24*}$ proceeding to the 1.37 Mev excited level at an incident energy of 31.5 Mev and for $C^{12}(\alpha, p)\, N^{15}$ proceeding to the ground state of N^{15} at an incident energy of 30.5 Mev. Whenever possible he determines the value of l in the order $l + \frac{1}{2}$ of the Bessel function by a generalization of the simple condition corresponding to the matrix element considered above, *viz.*

$$|\boldsymbol{J}_i + \boldsymbol{J}_f| \leqq l \leqq J_i + J_f$$

where $\boldsymbol{J}_i$, $\boldsymbol{J}_f$ are the total angular momenta of the initial and final nuclei disregarding spin flip, in the inequality.

At the 1957 Pittsburgh Conference on Nuclear Physics a number of experiments having direct interaction as one of the possible explanations have been reported by R. SHERR and a report on the theory of the $C^{12}(p, p')\, C^{12*}$ reaction from this viewpoint was read by C. LEVINSON. There is little doubt regarding there being a wide field of application for the direct interaction approximation to experimental data. The way in which direct interaction effects had best be combined with typical compound nucleus formation processes has not been sufficiently thoroughly worked out so far to make much comment desirable. It is probable however that the approach used by R. G. THOMAS in formulating the combined treatment of compound nucleus and pick-up processes can be used for a similar purpose in the theory of direct interaction. The essential step may be seen in Eq. (45.15) below. It appears probable that the formulation of a more quantitative theory of this type may prove useful and may even be more essential to further progress than the clearer understanding of the way in which direct interaction is contained in the Wigner $\mathcal{R}$-matrix or the Kapur-Peierls formalisms. It appears in fact likely that studies of the nuclear charge distribution by electron and muon scattering and of the optical model by nucleon and pion scattering will make the nuclear radius a more definite quantity, the preponderance of evidence being for the Fermi-type charge distribution discussed in Sect. 40 below. While these advances cannot be expected to remove the mathematical possibility of increasing the nuclear radius indefinitely without violating usual formal requirements, the choice of the smallest practical nuclear radius may be expected to become more definite than it has been. The region inside this radius will probably continue to be best manageable by the $\mathcal{R}$-matrix formalism and the reaction amplitude thus obtained will have to be corrected by an additional amplitude which by definition will be the direct interaction amplitude.

ζ) *Miscellaneous topics.* The *continuum theory* of nuclear reactions, disagreement with which has stimulated the theory of direct interaction as mentioned above, will not be reported on below and the evaporation model will also not be discussed, there being many books in which these matters are fully and adequately covered. These theoretical developments have been a most important stimulus to the subject as a whole. In addition to the paper of Bohr already quoted one may mention especially the work of Weisskopf and Ewing[1] and of Bethe[2]. The subject has benefited greatly by the personal stimulus which it has received through the influence of Bohr, Weisskopf and Bethe. The development and application of the continuum theory has had a strong and beneficial influence on nuclear physics.

One may refer to the article by Kinsey[3] for an account of various experimental evidence concerning the validity of the statistical theory as well as its failures, the experimental and partly the theoretical aspects of stripping and pick-up theory, the main theoretical considerations as well as the experimental evidence regarding theories of photonuclear reactions, level densities and a brief summary of theoretical and experimental results on Coulomb excitation.

Similar reference is made to the article by Burcham in this Encyclopedia [5] for a general introduction to the theory of nuclear reactions, the *statistical theory*, *elastic scattering*, *radiative transitions*, a summary of experimental material on the reactions of protons, neutrons and alpha particles with light nuclei, elementary deuteron stripping theory, a summary of experimental material on stripping and pick up reactions with special attention to the indications concerning nuclear levels derivable from it.

The theory of *angular correlations* of nuclear reaction products is treated in an elegant manner by Devons and Goldfarb[4]. It will be barely touched on here in the form of a discussion of the angular distribution of reaction products so as to provide a connection with the general nuclear reaction theory.

The *theory of fission* is so closely connected with the collective particle model that a treatment of it here was felt to be out of place.

η) *Selection rules.* Among the subjects which limitations of time will not permit to cover in any detail is that of selection rules in general and of isotopic spin selection rules in particular. The first of these is adequately covered in general reference texts. The second has a bearing on the validity of *charge independence of nuclear forces.* A brief résumé of the advances appears therefore in order at this place.

Oppenheimer and Serber[5] have apparently been the first to suggest that conservation of isotopic spin should be the basis of selection rules in nuclear reactions. Adair[6] and Kroll and Foldy[7] have discussed the effect of isotopic

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spin in heavy particle reactions. Trainor[1] has pointed out the existence of isotopic spin selection rules in the emission of electric dipole radiation. A general statement of the rules has been given by Radicati[2] and by Gell-Mann and Telegdi[3] who showed that for any multipole the selection rule $\Delta T = 0, \pm 1$ must hold and that besides for electric dipole transitions in self-conjugate nuclei the more restricted rule $\Delta T = \pm 1$ obtains. These rules are a consequence of the possibility of expressing the charge operator as a vector in isotopic spin space, the problem becoming formally similar therefore to the well-known one of dipole emission in atomic spectroscopy. The isotopic spin quantum number T is expected to be a good quantum number if the nuclear Hamiltonian is symmetric in neutrons and protons, i.e. if charge independence of nuclear forces is valid. On account of the presence of Coulomb effects the nuclear Hamiltonian is not exactly symmetric but for the lighter nuclei the effect of these perturbations is not expected to be large. It has been pointed out by Kroll and Foldy[4] that tests of the selection rules for self-conjugate nuclei are not sensitive to the full content of charge independence, the observations regarding absence of transitions depending only on the conservation of charge parity which is an expression of the charge symmetry of nuclear forces rather than of the more general principle of charge independence. A number of experiments such as that of Bockelman, Browne, Buechner and Sperduto[5] on inelastic scattering of protons and deuterons by B^{10} and N^{14} have contributed to the development. The most systematic investigations have been carried out by Wilkinson and collaborators[6]. In the last of the above series of papers there is a summary of the then available evidence in the form of two tables. The first of these deals with energy differences of corresponding states in mirror nuclei. These are used in order to ascertain an empirical correction for Coulomb energy which is applied in the second table to the test of charge independence for isobaric triplets. Thus for example the pair of mirror nuclei $Be^7 - Li^7$ has a mass difference of about 0.00093 which must be corrected for the mass difference $n - H^1 = 0.00084$ to give $0.00177 = 1.65$ Mev as the nuclear mass difference that would exist were the masses of neutrons and protons the same and which is therefore the Coulomb energy resulting from a change of a proton into a neutron. This empirically determined Coulomb energy is expected to be somewhat smaller for nuclei of mass 8. On the basis of a uniform charge distribution it is expected to be $(\frac{7}{8})^{\frac{1}{3}} \times 1.65$ Mev $= 1.57(2)$ Mev. The ground state mass difference $Li^8 - Be^8 = 0.017184 = 16.00$ Mev. This again is not a correct measure of the difference in nucleon binding because of the extra neutron in Li^8. If the nuclear forces were the same but the neutron had the same mass as the proton the energy difference would be $16.00 - 0.78 = 15.22$ Mev. With Wilkinson's more careful consideration of the masses this number is (15.20 ± 0.04) Mev. The change in Coulomb energy in going from Li^8 to Be^8 has been estimated to be 1.57 Mev. The energy of Be^8 which enters the experimental determination of the 15.20 Mev is thus relatively too high by 1.57 Mev

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