

# FREE AND INTERACTING QUANTUM FIELDS

*First Order Processes on Electromagnetic  
and Gravitational Backgrounds*



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*“There is nothing more practical  
than a good theory”*

Kurt Lewin

## Preface

The study of interactions between various categories of particles belonging to the atomic and sub-atomic world is of quantum nature *par excellence*; it became possible due to several fundamental theoretical discoveries in modern physics. Usually, by “modern physics” one understands the period of physics’ development beginning with the revolutionary Plank’s and Einstein’s theories and continuing up to today. Making abstraction for the progress performed by the empirical investigation, modern physics is characterized by two great theories, which make it fundamentally distinguishable from the whole of its previous development: *quantum theory* and *theory of relativity*.

In this book we want to bring to the reader’s attention several solutions to problems connected to the quantum-relativistic interaction of particles. Remarkably, such solutions furnished rigorous and pertinent explanations of a large set of phenomena, both in microscopic universe and galactic world.

One of the most important targets of today’s physics is to give a dynamic model capable to explain the complex properties of particles, their interactions, and symmetry relations existent in the microcosmos. Basically, the procedure consists in a simultaneous approach – both quantum and relativistic – of these interactions. The quantum nature of interactions emerges from similitude between the structure of particle systems and atomic structures. But the energies characterizing processes that involve subatomic particles are very high, so that they cannot be investigated only from the quantum point of view. Under these circumstances, only a

quantum-relativistic model proved to be able to correctly explain the particles' properties and their interaction in the microcosmos.

The discovery of such models began with the Quantum Field Theory. The first step was performed by Planck (1900) by quantization of the energy. The theory was then developed by Jordan and Wigner, while in 1927 P.A.M. Dirac formulated the first quantum theory of the electromagnetic field. Quantum electrodynamics soon became a theory with impressive experimental checks (for example, the fine-structure constant shows an exact concordance between its experimental and theoretical values up to the eighth decimal digit). Despite of these initial performances, some difficulties appeared connected to the physical interpretation of some divergent expressions obtained for several concrete processes. The problem was solved by Heisenberg who, in 1943, elaborated the  $S$ -matrix theory. Subsequently, this theory became a mathematical complex structure, ideal for description of quantum interactions between particles. According to the  $S$ -matrix theory, the investigator's attention is displaced from objects to events; it mostly deals with the processes of particle interactions, rather than the particles themselves.

Investigation is oriented towards events (processes), this being demanded by both quantum and relativistic theories. As it was shown by quantum theory, a subatomic particle must be understood as an interaction between various measurement processes. Instead of an isolated object, this time we deal with an event responsible for the correlation of some other events. As Heisenberg [202] used to say, "Modern physics divides the world not in different groups of objects, but in various groups of correlations... What we can distinguish is the type of connection, which is of the first importance for a phenomenon. This way, the world appears as a complicated net of events, with alternating, overlapping, and combining connections of various types, determining the appearance of the whole".

On the other side, the theory of relativity displayed the space-time four-dimensional characteristics of microparticles as processes rather than as objects. Approaching these matters within the  $S$ -matrix formalism unifies both points of view. Using the mathematical tools of the Theory of Relativity, this formalism describes all the properties

of particles in terms of interactions (*i.e.* in terms of transition probabilities between various quantum states), setting up a relationship between particles and processes. Any interaction implies particles connecting that interaction with some other interacting processes; this way, an entire system of interdependent processes is configured.

Among the multitude of processes and particles implied in these processes, in the book we investigate only a narrow (but demonstrating an increasing interest) domain, namely the class of high (*i.e.* superior to the conventional one) spin particles. Most recent investigations showed that a great amount of still unsolved problems of the contemporary physics could receive an answer by using high-spin quantum fields.

A considerable part of this book is dedicated to a set of research papers concerning interaction between various types of particles (mostly those of high-spin) and the gravitational and electromagnetic fields, considered as background fields within the modern formalism of the Quantum Field Theory. Our endeavor concerning both the study of high-spin fields, in general, and the scattering and photogeneration of several kinds of particles (both fermions and bosons, massive and zero-rest mass), in particular, is justified by necessity of clarifying some aspects of the fundamental theory of vector and tensor fields, especially those implying the mechanisms of interaction with some other fields (with applications in Quantum Chromodynamics and Cosmology). It is also determined by a considerable number of concrete practical applications, and, last but not least, by the permanent subjective necessity of the scientist to clarify any “mystery” of theoretical physics.

The importance of our investigation is proved by the great amount of research papers on this subject which appeared during the last three decades and still continue to appear in prestigious periodicals like *Classical and Quantum Gravity*, *Physical Review*, *Nuclear Physics*, *Physics Letters*, etc.

This book is mostly addressed to the senior undergraduates, MSc, and PhD students of the Faculties of Physics, their instructors, but it can also be used by those studying mathematics, physics researchers, and the high school teachers.



This volume is dedicated to the memory of our mentor and adviser Professor Emeritus Ioan Gottlieb, the founder of the theoretical physics school in our faculty, Alexandru Ioan Cuza University of Iasi, and profound investigator in the Quantum Mechanics, Quantum Field Theory and Theory of General Relativity. We also want to express our deep gratitude to Professor Gheorghe Zet, from the Technical University Gheorghe Asachi of Iasi, who read the manuscript and made several pertinent and useful observations.

Last but not least, we are grateful to our colleague, Dr. Ioana-Laura Velicu, for imagining and drawing up the nice design of the book cover.

*The Authors*  
Iasi, June 2016

# Introduction

The research results exposed in this volume fall in the general international effort of investigation of the interaction processes of particles in gravitational and electromagnetic fields. They also concern the transformation process of particles on various space-time structures, the applicative chapters being mostly dedicated to processes implying the *high-spin particles*.

One must first explain the significance of the expression “high-spin” associated to a particle. There is no definite convention in this respect, but most frequently the values 0 and  $1/2$  (in  $\hbar$  units) of the spin of a particle are considered small (or *low*), while the values  $3/2$ , 2,  $5/2$ , 3, etc. are considered large (or *high*). These two domains are separated by the value 1 of the spin, which value is being considered as *intermediate*. Therefore, according to this classification, any quantum field whose quanta have at least value  $3/2$  (in  $\hbar$  units) for its spin is considered as being a *high-spin field*.

The importance and necessity of the study of quantum fields (in general) and those with high spin (in particular) is justified by numerous reasons, and here are only some of them:

- First, the spin  $3/2$ , 2, and  $5/2$  fields play a fundamental role in the theory of quantum gravity and supergravity.
- Second, somewhat recently have been discovered several quasiparticles (in fact, nuclear resonances) whose spin exceeds the value 2 (getting even value  $7/2$ ). These quasiparticles can be described by high-spin fields, even if they are non-fundamental fields (like the multiplet or supermultiplet fields). In addition, it has been recently

shown that for the massless fields with spin higher than 2 is possible to give an interpretation in terms of gauge-type potentials; such an interpretation was not obvious within previous investigations involving local (both massive and non-massive) fields having spin smaller or equal to 3 and massive local fields of an arbitrary spin as well. These new interpretations stand for advanced steps in understanding the high-spin fields, being capable to lead to the usage of these fields in phenomenological investigations.

- Third, there exists a more or less subjective reason: even if stable particles<sup>1</sup> having the spin higher than  $7/2$  have not been experimentally discovered yet, the inquisitive spirit of the human being cannot leave unexplored any domain of Reality. Furthermore, at the present we already have a complete and consequent theory – both in Lagrangian and Hamiltonian formalisms – involving classical and quantum fields of arbitrary spin, but deep investigations were performed only up to value 4 of spin, inclusive.

- Fourth, the string and superstring theories show that, at least up to the levels where the generation of particles is still well-defined, one must be considered massive excitations of *any spin*.

- Fifth, even if until now have not been observed particles with no internal structure having spin higher<sup>2</sup> than 2, particles whose

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<sup>1</sup>Being aware of the fact that even the life of proton, within the Standard Model, has been predicted as being finite, the notion of *stable particle* became today at least questionable. That is why, in the present volume, by “stable particle” we shall understand that particle which has an average lifetime long enough to make it able to effectively participate in at least one observable and measurable physical process.

<sup>2</sup>Related to this value of the spin, *i.e.* 2, Paul van Nieuwenhuizen states in his notorious study published in Physics Reports, *Supergravity* [287], that “*It seems, at this moment, that Nature stops at spin 2*”. Nevertheless, even within the extended supergravity theory with at least 8 gravitino (*i.e.* for the supergravity theories with  $N > 8$ ) one appears the necessity of introduction of a  $5/2$  spin field and consideration of more than one graviton. The main argument of van Nieuwenhuizen is that only up to value 2 of the spin of a particle the mass discontinuity appearing in the Lagrangian of the theory (the so-called *van Dam–Veltman discontinuity*) is finite, for any other value superior to 2 this discontinuity being insurmountable. Nevertheless, the papers published in this field during the last decades determined the authors to approach such a problem. Recently [194] a procedure of construction of a gauge theory for the massive spin-2 tensor field, having a smooth limit for  $m \rightarrow 0$  has been elaborated by Shinji Hamamoto, by using the Batalin–Fradkin algorithm for the massive spin-2 tensor field theory, with straight application to renormalization theory of quantum gravitation. This way, the solutions to infrared divergences met in quantum gravitation can be obtained in a manner similar

description demand fundamental fields of superior spin, within the frame of ToE (Theory of Everything) type theories, there are clear theoretical evidences showing that the already known interactions (including gravitation) can be incorporated into a supersymmetric scheme only by taking into account the *high-spin fields*. Thus, it is well-known the fact that an irreducible supermultiplet having spin lower than or equal to 2 cannot unify a gauge theory based on local symmetry  $SU(3)_{color} \times SU(2) \times U(1)_{electroweak}$  with the fundamental fields of the necessary vector bosons. The simplest extension to a single supermultiplet able to incorporate both the gauge electromagnetic and strong standard interactions, together with the isospin doublets of the weak nuclear interactions, requires the use of both spin-5/2 and spin-3 fields.

- Sixth, there is an increasing importance of the non-massive Rarita–Schwinger field within the modern cosmological models, due to the role played in various scenarios by its quanta, *i.e.* the *gravitino*.

- Seventh – last but not least – the importance of the research involving classical and quantum high-spin fields comes also from some concrete applications in quantum chromodynamics. So, according to numerous studies performed by researches from CERN (Geneva) and DESY (Hamburg), the vector and tensor mesons play an essential role in various theoretical models on interaction involving strongly interacting particles.

The problem of gravitational-electromagnetic coupling (which is an important subject of study in this volume) has been widely investigated from the perspective of the interaction of various types of particles on backgrounds with various symmetries, the approach being done in both linear and nonlinear theories. The obtained results, even in some inchoate phases, were proved to be of a real interest in clarification of some important aspects in certain physical branches, such as cosmology and astrophysics. The great number of applications based on the results obtained in this field by the world scientists (results emphasized by awarding of the Nobel Prize in 1993

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to the renormalization theory of quantum electrodynamics; here, as one knows, these divergences are eliminated by description of the photon by means of a massive vector field, subsequently taking the limit when the photon mass goes to zero.

to physicists Russel A. Hulse and Joseph H. Taylor from Princeton University, New Jersey, USA, who evidenced for the first time in 1974, even if indirectly, the highly controversial – at least regarding their concrete physical existence – gravitational waves) stands for an important reason which fully justified the initiation of several researches on properties of the interaction processes involving the two fundamental physical fields (the electromagnetic and gravitational fields). From this perspective, since the approached subjects have both theoretical and practical finalization, the present volume is intended to fill up some “gaps” in scientific research. So, among the applications of the exposed results, we mention the possibility of using a scattered field in order to be able to make distinction between various types of cosmic objects (for instance, between the black holes and compact cosmic objects with large and very large spatial extension), when the observation process is performed at big distances from the source of the scattering gravitational field with axial symmetry, as well as the possibility of explaining the phenomenon of intensification of the radiation – both in visible and invisible spectra – emitted by the arms of spiral galaxies with an intense own magnetic field, on account of the photon-graviton (and vice-versa) conversion process, within the framework of various configurations of electromagnetic background fields. One of the most discouraging feature of the theories on high-spin fields, even if these fields are considered to be free, rests in the very complicated formalism required by the description of such fields. This formalism becomes even more complicated if, for example, we intend to couple the high-spin fermions with gravitation, in the frame of a local theory with interaction. This is one of the main reasons why until recently all the issues related to the use of high-spin fields has been least addressed regarding concrete calculations. Nevertheless, we have to mention that during the last three decades the electronic calculus technique was intensively developed, in strong connection with emergence of several very performant software packages, capable to process information not only numerically, but especially analytically; we recall, in this respect, the software products such as Reduce, Maple, Mathematica, etc., as being the best known and currently used. Nowadays,

we are in possession of very powerful computer equipments, not only at working offices, but also at our home private facilities, and this situation has revolutionized the research work. This way, the study of several complex processes of scattering, transformation and cosmological generation of particles became possible. These processes are very important in understanding of certain physical processes, not only at atomic and sub-atomic scale, but also at cosmic scale. A somewhat exotic phenomenon in this respect is furnished by the process of transformation/conversion of the photons into gravitons and vice-versa (or, classically, transformation/conversion of the electromagnetic into gravitational waves and vice-versa); at first sight this process seems less plausible, but it is perfectly possible from the quantum point of view, having important applications in astrophysics. In this connection it is important to mention that the existence of the gravitational waves is not anymore a controversial problem since, in 1974, Russel A. Hulse and Joseph H. Taylor from Princeton University put into evidence, in an indirect way, the existence of the gravitational waves and more recently, in September 2015, the two detectors of LIGO simultaneously registered a gravitational wave signal – the first direct detection ever achieved<sup>3</sup>. Due to the smallness of the gravitational coupling constant, the study of the photon-graviton conversion processes leads to very small scattering cross-sections. This was, in fact, the main reason why physicists oriented their attention mostly to the cosmic space, instead of laboratory experiments. In astrophysics there certainly exist the possibility that, under some special circumstances, the graviton generation rate in quantum processes, be as big as that corresponding to classical ones. However, recent results concerning this process show that the scattering cross-sections can receive detectable (measurable) values, even in laboratory experiments.

Before beginning a short presentation of the book, it is worth mentioning an important fact regarding the massless spinorial field and its corresponding particles/quanta. This remark concerns the

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<sup>3</sup>The observation was reported in B.P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), *Phys. Rev. Lett.* 116, 061102 (2016), <http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.116.061102>.

fact that, so far, neutrinos have been considered as quanta of the massless spinorial field. Nevertheless, several recently performed experiments (Super-Kamiokande I – 1996 Apr., Super-Kamiokande II – 2002 Oct., Super-Kamiokande III – 2006 Jul., Super-Kamiokande IV – 2008 Sep.; Sudbury Neutrino Observatory (SNO) – 1999 May–2006 Nov.) contradict this theory, showing that the neutrino rest-mass is in fact non-zero. This experimental result has been theoretically suspected much time ago (as a proof, in the Appendix H are given the upper mass limits of the electron-neutrino and muon-neutrino), supposition turning into a certitude.

The experimental discovery of *neutrino oscillation*, and thus *neutrino mass*, by the *Super-Kamiokande Observatory* and the *Sudbury Neutrino Observatory* was rewarded with the 2015 Nobel Prize in Physics that was shared by Japanese physicist Takaaki Kajita (b. 1959) from Institute for Cosmic Ray Research, University of Tokyo, and Canadian physicist Arthur B. McDonald (b. 1943) from Princeton University and Queen's University, for their early pioneering observations of these oscillations.

Neutrino oscillation is a quantum phenomenon by which a neutrino created with a specific lepton flavour (electron, muon or tauon) can subsequently change its flavour, turning – for example – from an electron-neutrino into a muon-neutrino. The basic physics behind neutrino oscillation can be found in any system of coupled harmonic oscillators. The most simple example is a system of two mechanical pendulums connected by a weak spring (having a small spring constant).

The probability of determination of a particular flavour for a certain neutrino varies periodically together with its spatial displacement. The neutrino oscillation was first predicted by the Italian nuclear physicist Bruno Pontecorvo (1913–1993) in 1957, since then being observed in various contexts, such as: *solar neutrino oscillation* (the Ray Davis's Homestake Experiment in the late 1960s; the Sudbury Neutrino Observatory provided clear evidence of neutrino flavor change in 2001. Notably, the existence of neutrino oscillation resolved the long-standing solar neutrino problem – Arthur B. McDonald), *atmospheric neutrino oscillation* (the large detectors such

as IMB, MACRO, and Kamiokande II have observed a deficit in the ratio of the flux of muon to electron flavor atmospheric neutrinos; the Super-Kamiokande experiment provided a very precise measurement of neutrino oscillation in an energy range of hundreds of MeV to a few TeV), *reactor neutrino oscillation* (the Daya Bay experiment, in 2012), *beam neutrino oscillation* (the MINOS, K2K, Super-K, LSND, MiniBooNE, Gran Sasso, T2K, Super-Kamiokande, NOvA experiments).

Neutrino oscillation is of great theoretical and experimental interest, as the precise properties of the process can shed light on several properties of the neutrino. In particular, it implies that the *neutrino has a non-zero rest mass*, which requires a modification to the Standard Model of particle physics. However, the question of how neutrino masses arise has not been answered conclusively. There exist several attempts to explain how neutrinos acquire their mass (such as the Majorana mass term, the seesaw mechanism, the leptogenesis, etc.), but none of them has prevailed so far.

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The book is organized in two parts: the first has a monographic character (based on reference books on the subject and consisting in basic elements necessary to understand the investigations to follow), and the second has a pronounced applicative feature (being composed mostly by authors' original research). The volume also contains eight appendices with relations and formulas necessary to understand the main text as well as certain auxiliary elements/information included for a better understanding of several notions introduced during the exposure, together with a paragraph with some useful formulas met in quantum field theory.

The first chapter is called *Classical and quantum free fields* and contains a short description of classical and quantum theory of free fields with the spin 0, 1/2, 1, 3/2, and 2. Although this subject was exhaustively approached by many famous scientists in several excellent books, the authors decided to include these fundamental



notions/results for two reasons: first, all further applications presented in the book (and even all further theoretical considerations designated to prepare an appropriate understanding of the interaction processes studied in the second part of the book) directly appeal a lot of elements belonging to quantum field theory, and second, the authors wanted to ease – and even eliminate – the reader's effort of searching elsewhere for fundamentals of quantum field theory.

Being a fundamental and well-adjusted subject, the authors preferred to use the manner found in masters' investigations. Thus, in this chapter are briefly exposed the main characteristics of the fields having spins between 0 and 2, following as closely as possible the way of presentation used by these consecrated authors. This has been done extensively, keeping even the metric signature and the notation conventions used in their original works. As such masters the authors chose N.N. Bogoliubov and D.V. Shirkov (Ref. [41]) for quantization of the scalar, spinorial and electromagnetic fields, and D. Lurie (Ref. [252]) for quantization of the Rarita–Schwinger field. Regarding the free gravitational field, the authors preferred to expose a new variant, somewhat more natural (see subchapter I.3.7.4) as compared to procedures exposed in references 181, 182, 219, and 275.

The second chapter of the volume, named *The gravitational transmutations hypothesis* can be considered as an introduction to the second part of the book. It contains a survey on the main aspects of the quantum study of particles' interaction in gravitational (and electromagnetic) fields with applications in astrophysics, as they appear in scientific literature: particle diffusion in external gravitational field, transformation of the photon into graviton in an external electromagnetic field, the gravitational/electromagnetic radiation of deceleration of particles in electromagnetic/gravitational external fields (*bremsstrahlung*), the gravitational/electromagnetic annihilation/creation of elementary particles, the gravitational Compton effect, the diffusion of elementary particles through virtual gravitons, and the gravitational radiation of nuclei.

The third chapter is titled *Tomonaga–Schwinger representation of dynamics of a quantum physical system. Matrix elements of the*