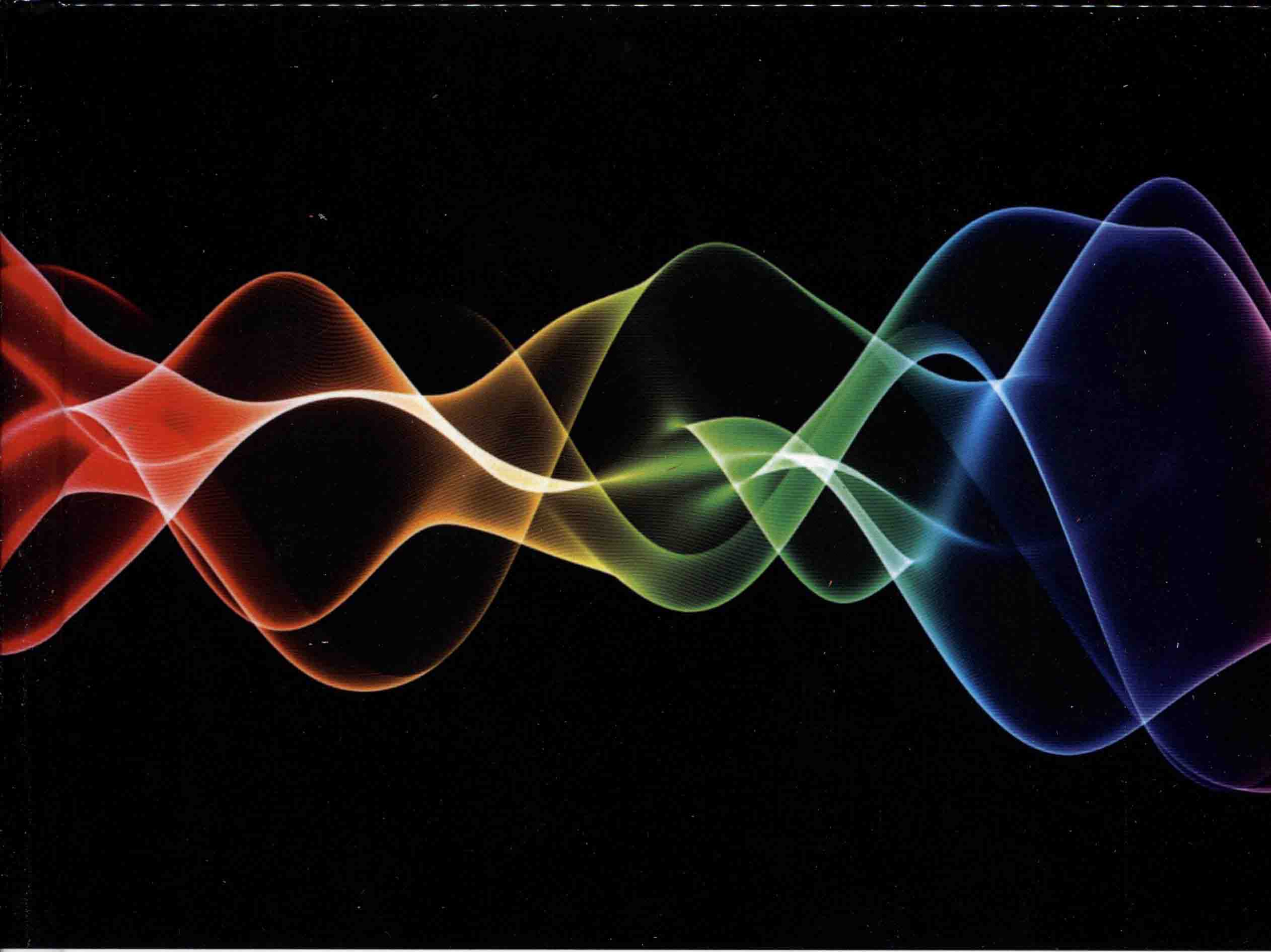

Fourier Transform: SIGNAL PROCESSING

Olga Moreira, Ph.D.

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About the Editor

Olga Moreira, Ph.D.

Olga Moreira obtained her Ph.D. in Astrophysics from the University of Liege (Belgium) in 2010, her BSc. in Physics and Applied Mathematics from the University of Porto (Portugal). Her post-graduate travels and international collaborations with the European Space Agency (ESA) and European Southern Observatory (ESO) led to great personal and professional growth as a scientist. Currently, she is working as an independent researcher, technical writer, and editor in the fields of Mathematics, Physics, Astronomy and Astrophysics.

List of Contributors

P. L. Butzer
Lehrstuhl A für Mathematik, RWTH Aachen University, 52056 Aachen, Germany

M. M. Dodson
Department of Mathematics, University of York, York YO1O 5DD, UK

P. J. S. G. Ferreira
IEETA/DETI, Universidade de Aveiro, 3810-193 Aveiro, Portugal

J. R. Higgins
I.H.P., 4 rue du Bary, 11250 Montclar, France

G. Schmeisser
Department of Mathematics, University of Erlangen-Nuremberg, 91058 Erlangen, Germany

R. L. Stens
Lehrstuhl A für Mathematik, RWTH Aachen University, 52056 Aachen, Germany

Francisco J. Mendoza-Torres
Facultad de Ciencias Físico-Matemáticas, Benemérita Universidad Autónoma de Puebla, Puebla, Mexico

Ma. Guadalupe Morales-Macías
Departamento de Matemáticas, Universidad Autónoma Metropolitana-Iztapalapa, México, D. F., Mexico

Salvador Sánchez-Perales
Instituto de Física y Matemáticas, Universidad Tecnológica de la Mixteca, Oaxaca, Mexico

Juan Alberto Escamilla-Reyna
Facultad de Ciencias Físico-Matemáticas, Benemérita Universidad Autónoma de Puebla, Puebla, Mexico

Jiufei Luo
School of Advanced Manufacture Engineering, Chongqing University of Posts and Telecommunications, Chongqing 400065, People's Republic of China

Shuaicheng Hou
School of Automation, Chongqing University, Chongqing 400044, People's Republic of China

Xinyi Li
Department of Mechanical Engineering, Chongqing University, Chongqing 400044, People's Republic of China.

Qi Ouyang
School of Automation, Chongqing University, Chongqing 400044, People's Republic of China

Yi Zhang
School of Advanced Manufacture Engineering, Chongqing University of Posts and Telecommunications, Chongqing 400065, People's Republic of China

Jesús Ramón Lerma-Aragón
Facultad de Ciencias, Universidad Autónoma de Baja California, México

Josué Álvarez-**Borrego**
CICESE, División de Física Aplicada, Departamento de Óptica, México

Tao Guan
College of Electronic Science and Engineering, National University of Defense Technology, Kaifu, Changsha 410073, Hunan, China

Dongxiang Zhou
College of Electronic Science and Engineering, National University of Defense Technology, Kaifu, Changsha 410073, Hunan, China

Chao Xu
College of Electronic Science and Engineering, National University of Defense Technology, Kaifu, Changsha 410073, Hunan, China

Yunhui Liu
Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, Hong Kong, China.

Lebing Pan
Key Laboratory of Wireless Sensor Networks, Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences, 865 Changning Road, Shanghai 200050, China
University of Chinese Academy of Sciences, No.19A Yuquan Road, Beijing 100049, China

Shiliang Xiao
Key Laboratory of Wireless Sensor Networks, Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences, 865 Changning Road, Shanghai 200050, China
University of Chinese Academy of Sciences, No.19A Yuquan Road, Beijing 100049, China

Xiaobing Yuan
Key Laboratory of Wireless Sensor Networks, Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences, 865 Changning Road, Shanghai 200050, China

Baoqing Li
Key Laboratory of Wireless Sensor Networks, Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences, 865 Changning Road, Shanghai 200050, China

Siwar Rekik
Université de Bretagne Occidentale, Brest, France
Canadian University of Dubai, Dubai, UAE

Driss Guerchi
Canadian University of Dubai, Dubai, UAE

Sid-Ahmed Selouani
University of Moncton, Shippagan, NB, Canada

Habib Hamam
University of Moncton, Moncton, NB, Canada.

Vinod Chandran
School of Electrical Engineering and Computer Science Queensland University of Technology

Qiong Wu, Qilian Liang
Department of Electrical Engineering University of Texas at Arlington

Xiumei Li
School of Information Science and Engineering, Hangzhou Normal University, China

Guoan Bi
School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore

Shenghong Li
School of Electronic Information and Electrical Engineering, Shanghai Jiao Tong University, China

Jiexiao Yu
School of Electronic Information Engineering, Tianjin University, Tianjin, China

Liang Zhang
School of Electronic Information Engineering, Tianjin University, Tianjin, China

Kaihua Liu
School of Electronic Information Engineering, Tianjin University, Tianjin, China

Deliang Liu
School of Electronic Information Engineering, Tianjin University, Tianjin, China

Steffen Weimann
Institute of Applied Physics, Abbe School of Photonics, Friedrich-Schiller-Universit t Jena, Max-Wien Platz 1, 07743 Jena, Germany

Armando Perez-Leija
Institute of Applied Physics, Abbe School of Photonics, Friedrich-Schiller-Universit t Jena, Max-Wien Platz 1, 07743 Jena, Germany

Maxime Lebugle
Institute of Applied Physics, Abbe School of Photonics, Friedrich-Schiller-Universit t Jena, Max-Wien Platz 1, 07743 Jena, Germany

Robert Keil
Institut fu¨r Experimentalphysik, Universita¨t Innsbruck, Technikerstrae 25, 6020 Innsbruck, Austria

Malte Tichy
Department of Physics and Astronomy, University of Aarhus, 8000 Aarhus, Denmark

Markus Grfe
Institute of Applied Physics, Abbe School of Photonics, Friedrich-Schiller-Universit t Jena, Max-Wien Platz 1, 07743 Jena, Germany

Rene´ Heilmann
Institute of Applied Physics, Abbe School of Photonics, Friedrich-Schiller-Universit t Jena, Max-Wien Platz 1, 07743 Jena, Germany

Stefan Nolte
Institute of Applied Physics, Abbe School of Photonics, Friedrich-Schiller-Universit t Jena, Max-Wien Platz 1, 07743 Jena, Germany

Hector Moya-Cessa
INAOE, Coordinacion de Optica, Luis Enrique Erro No. 1, Tonantzintla, Puebla 72840, Mexico

Gregor Weihs
Institut fu¨r Experimentalphysik, Universita¨t Innsbruck, Technikerstrae 25, 6020 Innsbruck, Austria

Demetrios N. Christodoulides
CREOL, The College of Optics & Photonics, University of Central Florida, Orlando, Florida 32816, USA

Alexander Szameit
Institute of Applied Physics, Abbe School of Photonics, Friedrich-Schiller-Universit t Jena, Max-Wien Platz 1, 07743 Jena, Germany

Katherine M. M. Tant
Department of Mathematics and Statistics

Anthony J. Mulholland
Department of Mathematics and Statistics

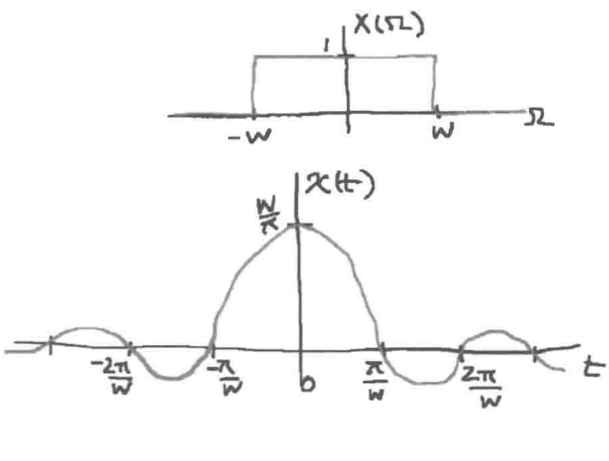
Matthias Langer
Department of Mathematics and Statistics

INTRODUCTION

In mathematics, Fourier analysis (English pronunciation) is the study of the way general functions may be represented or approximated by sums of simpler trigonometric functions. Fourier analysis grew from the study of Fourier series, and is named after Joseph Fourier, who showed that representing a function as a sum of trigonometric functions greatly simplifies the study of heat transfer.

Today, the subject of Fourier analysis encompasses a vast spectrum of mathematics. In the sciences and engineering, the process of decomposing a function into oscillatory components is often called Fourier analysis, while the operation of rebuilding the function from these pieces is known as Fourier synthesis. For example, determining what component frequencies are present in a musical note would involve computing the Fourier transform of a sampled musical note. One could then re-synthesize the same sound by including the frequency components as revealed in the Fourier analysis. In mathematics, the term Fourier analysis often refers to the study of both operations.

The decomposition process itself is called a Fourier transformation. Its output, the Fourier transform, is often given a more specific name, which depends on the domain and other properties of the function being transformed. Moreover, the original concept of Fourier analysis has been extended over time to apply to more and more abstract and general situations, and the general field is often known as harmonic analysis. Each transform used for analysis (see list of Fourier-related transforms) has a corresponding inverse transform that can be used for synthesis.



Applications

Fourier analysis has many scientific applications – in physics, partial differential equations, number theory, combinatorics, signal processing, imaging, probability theory, statistics, forensics, option pricing, cryptography, numerical analysis, acoustics, oceanography, sonar, optics, diffraction, geometry, protein structure analysis, and other areas.

This wide applicability stems from many useful properties of the transforms:

- The transforms are linear operators and, with proper normalization, are unitary as well (a property known as Parseval's theorem or, more generally, as the Plancherel theorem, and most generally via Pontryagin duality).
- The transforms are usually invertible.

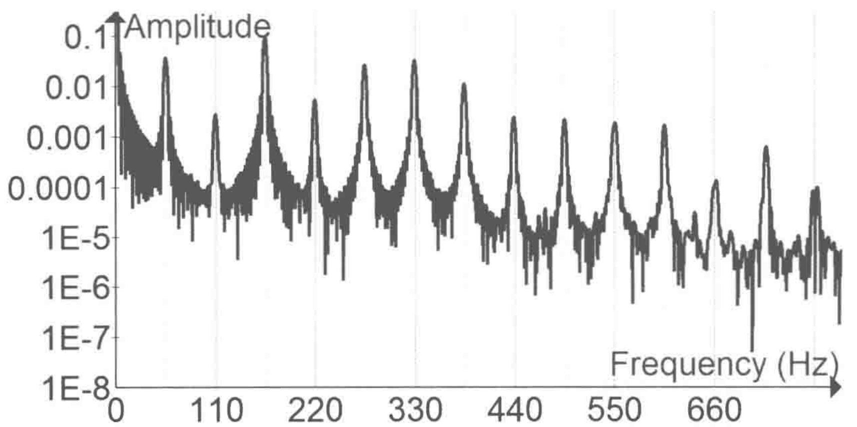
- The exponential functions are eigenfunctions of differentiation, which means that this representation transforms linear differential equations with constant coefficients into ordinary algebraic ones (Evans). Therefore, the behavior of a linear time-invariant system can be analyzed at each frequency independently.

- By the convolution theorem, Fourier transforms turn the complicated convolution operation into simple multiplication, which means that they provide an efficient way to compute convolution-based operations such as polynomial multiplication and multiplying large numbers (Knuth).

- The discrete version of the Fourier transform (see below) can be evaluated quickly on computers using Fast Fourier Transform (FFT) algorithms.

In forensics, laboratory infrared spectrophotometers use Fourier transform analysis for measuring the wavelengths of light at which a material will absorb in the infrared spectrum. The FT method is used to decode the measured signals and record the wavelength data. And by using a computer, these Fourier calculations are rapidly carried out, so that in a matter of seconds, a computer-operated FT-IR instrument can produce an infrared absorption pattern comparable to that of a prism instrument.

Fourier transformation is also useful as a compact representation of a signal. For example, JPEG compression uses a variant of the Fourier transformation (discrete cosine transform) of small square pieces of a digital image. The Fourier components of each square are rounded to lower arithmetic precision, and weak components are eliminated entirely, so that the remaining components can be stored very compactly. In image reconstruction, each image square is reassembled from the preserved approximate Fourier-transformed components, which are then inverse-transformed to produce an approximation of the original image.



Applications in signal processing

When processing signals, such as audio, radio waves, light waves, seismic waves, and even images, Fourier analysis can isolate narrowband components of a compound waveform, concentrating them for easier detection or removal. A large family of signal processing techniques consist of Fourier-transforming a signal, manipulating the Fourier-transformed data in a simple way, and reversing the transformation.[2]

Some examples include:

- Equalization of audio recordings with a series of band pass filters;

- Digital radio reception without a super heterodyne circuit, as in a modern cell phone or radio scanner;

- Image processing to remove periodic or anisotropic artifacts such as jaggies from interlaced video, stripe artifacts from strip aerial photography, or wave patterns from radio frequency interference in a digital camera;

- Cross correlation of similar images for co-alignment;

- X-ray crystallography to reconstruct a crystal structure from its diffraction pattern;

- Fourier transform ion cyclotron resonance mass spectrometry to determine the mass of ions from the frequency of cyclotron motion in a magnetic field;

- Many other forms of spectroscopy, including infrared and nuclear magnetic resonance spectroscopies;

- Generation of sound spectrograms used to analyze sounds;

- Passive sonar used to classify targets based on machinery noise.

Variants of Fourier analysis

Fourier transform

Most often, the unqualified term Fourier transform refers to the transform of functions of a continuous real argument, and it produces a continuous function of frequency, known as a frequency distribution. One function is transformed into another, and the operation is reversible. When the domain of the input (initial) function is time (t), and the domain of the output (final) function is ordinary frequency, the transform of function s(t) at frequency f is given by the complex number:

$$S(f) = \int_{-\infty}^{\infty} s(t) \cdot e^{-i2\pi ft} dt.$$

Evaluating this quantity for all values of f produces the frequency-domain function. Then s(t) can be represented as a recombination of complex exponentials of all possible frequencies:

$$s(t) = \int_{-\infty}^{\infty} S(f) \cdot e^{i2\pi ft} df,$$

which is the inverse transform formula. The complex number, S(f), conveys both amplitude and phase of frequency f.

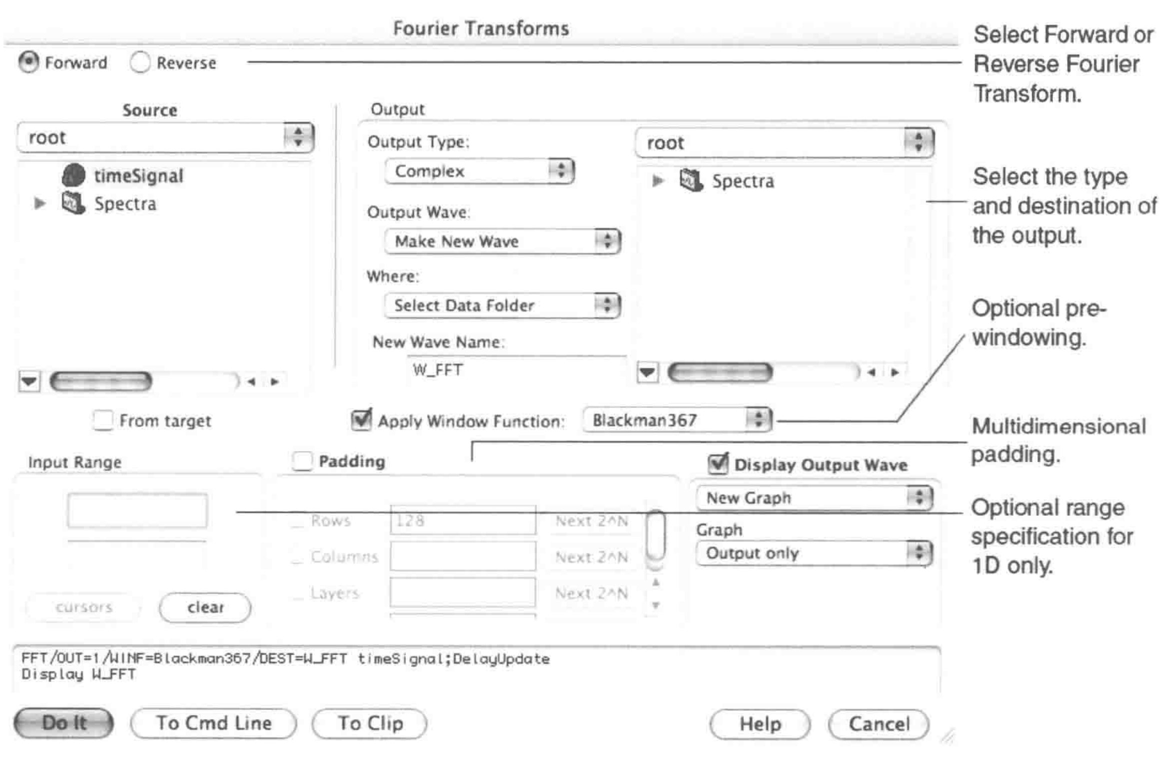
See Fourier transform for much more information, including:

- conventions for amplitude normalization and frequency scaling/units
- transform properties
- tabulated transforms of specific functions
- an extension/generalization for functions of multiple dimensions, such as images.

The Fourier Transform´s ability to represent time-domain data in the frequency domain and vice-versa has many applications. One of the most frequent applications is analyzing the spectral (frequency) energy contained in data that has been sampled at evenly-spaced time intervals. Other applications include fast computation of convolution (linear systems responses, digital filtering, correlation (time-delay estimation, similarity measurements) and time-frequency analysis.

The fast version of this transform, the Fast Fourier Transform (or FFT) was first developed by Cooley and Tukey[2] and later refined for even greater speed and for use with different data lengths through the "mixed-radix" algorithm. Igor computes the FFT using a fast multidimensional prime factor decomposition Cooley-Tukey algorithm.

While the Fourier Transform is mathematically complicated, Igor´s Fourier Transforms dialog makes it easy to use:



Spectral Windowing

The FFT computation presumes that the input data repeats over and over. This is important when the initial and final values of your data are not the same: the discontinuity causes aberrations in the spectrum computed by the FFT. "Windowing" smooths the ends of the data to eliminate these aberrations.

Power Spectra

"Power Spectra" answer the question "which frequencies contain the signal´s power?" The answer is in the form of a distribution of power values as a function of frequency, where "power" is considered to be the average of the signal^2. In the frequency domain, this is the square of FFT´s magnitude.

Power spectra can be computed for the entire signal at once (a "periodogram") or periodograms of segments of the time signal can be averaged together to form the "power spectral density".

Hilbert Transform

The Hilbert Transform computes a time-domain signal that is 90 degrees "out of phase" with the input signal. One-dimensional applications include computing the envelope of a modulated signal and the measurement of the decay rate of an exponentially decaying sinusoid often encountered in underdamped linear and non-linear systems.

Time Frequency Analysis

When you compute the Fourier spectrum (or Power Spectra) of a signal you dispose of all the phase information contained in the Fourier transform. You can find out which frequencies a signal contains but you do not know when these frequencies appear in the signal. For example, consider the signal:

$$f(t) = \begin{cases} \sin(2\pi f_1 t) & 0 \le t < t_1 \\ \sin(2\pi f_2 t) & t_1 \le t < t_2 \end{cases}$$

Fourier series

The Fourier transform of a periodic function, $s_p(t)$, with period P, becomes a Dirac comb function, modulated by a sequence of complex coefficients:

$$S[k] = \frac{1}{P}\int_P s_P(t)\cdot e^{-i2\pi \frac{k}{P}t}\,dt$$ for all integer values of k,

and where $\int_P$ is the integral over any interval of length P.

The inverse transform, known as Fourier series, is a representation of $s_p(t)$ in terms of a summation of a potentially infinite number of harmonically related sinusoids or complex exponential functions, each with an amplitude and phase specified by one of the coefficients:

$$s_P(t) = \sum_{k=-\infty}^{\infty} S[k]\cdot e^{i2\pi\frac{k}{P}t} \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \sum_{k=-\infty}^{+\infty} S[k]\,\delta\left(f-\frac{k}{P}\right).$$

When $s_p(t)$, is expressed as a periodic summation of another function, s(t):

$$s_P(t) \overset{\text{def}}{=} \sum_{k=-\infty}^{\infty} s(t-kP),$$

the coefficients are proportional to samples of $S(f)$ at discrete intervals of 1/P:

$$S[k] = \frac{1}{P}\cdot S\left(\frac{k}{P}\right)$$

A sufficient condition for recovering s(t) (and therefore $S(f)$) from just these samples is that the non-zero portion of s(t) be confined to a known interval of duration P, which is the frequency domain dual of the Nyquist–Shannon sampling theorem.

Convolution and Correlation

You can use convolution to compute the response of a linear system to an input signal. The linear system is defined by its impulse response. The convolution of the input signal and the impulse response is the output signal response. Digital filtering is accomplished by defining a linear system´s impulse response that when convolved with the signal accomplishes the desired result (low-pass or high-pass filter).

The correlation algorithm is very similar mathematically to convolution, but is used for different purposes. It is most frequently used to identify the time delay at which two signals "line up", or are "most similar".

Smoothing

Smoothing removes short-term variations, or "noise" to reveal the important underlying form of the data.

The simplest form of smoothing is the "moving average" which simply replaces each data value with the average of neighboring values. (Other terms for this kind of smoothing are "sliding average", "box smoothing", or "boxcar smoothing".)

Igor´s Smooth operation performs box smoothing, "binomial" (Gaussian) smoothing, and Savitzky-Golay (polynomial) smoothing. The different smoothing algorithms compute weighted averages that multiply neighboring values by differing weights or "coefficients" to compute the smoothed value.

Digital Filters

Digital filters are a natural tool when data is already digitized. Reasons for applying digital filtering to data include:

- Elimination of unwanted signal components ("noise")
- Enhancing of desired signal components
- Detecting the presence of certain signals
- Simulation of linear systems (compute the output signal given the input signal and the system´s "transfer function")
- Digital filters generally come in two flavors: Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters.

Igor implements FIR digital filtering primarily through time-domain convolution using the Smooth or Smooth Custom commands. (In spite of its name, Smooth Custom convolves data with user-supplied filter coefficients to implement any kind of FIR filter, low-pass, high-pass, band-pass, etc.)

Design of the FIR filter coefficients used with Smooth Custom is most easily accomplished using the Igor Filter Design Laboratory (a separate product which also requires Igor Pro).