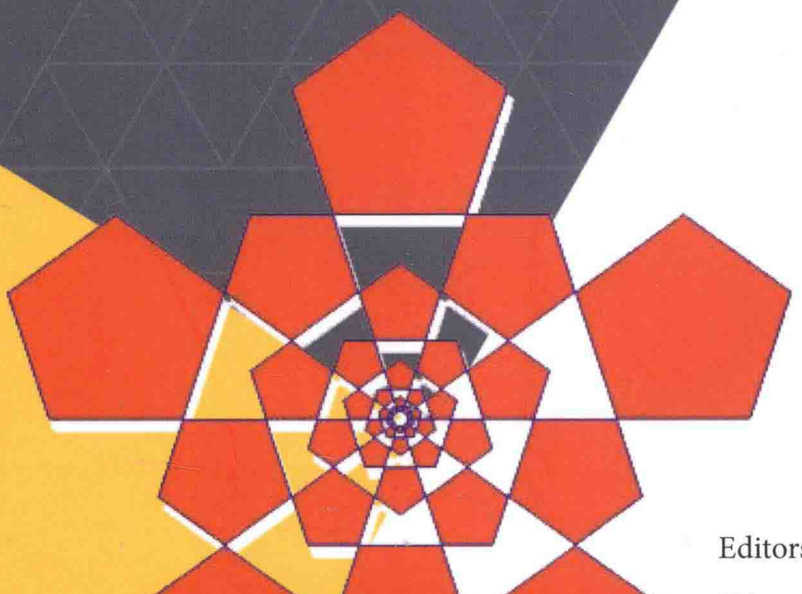
Dynamical Systems, Number Theory and Applications

A Festschrift in Honor of Armin Leutbecher's 80th Birthday



Editors

Thomas Hagen
Florian Rupp
Jürgen Scheurle

World Scientific

Dynamical Systems, Number Theory and Applications

A Festschrift in Honor of Armin Leutbecher's 80th Birthday

Editors

Thomas Hagen (University of Memphis, USA)

Florian Rupp (German University of Technology, Oman)

Jürgen Scheurle (Technische Universität München, Germany)

World Scientific

NEW JERSEY · LONDON · SINGAPORE · BEIJING · SHANGHAI · HONG KONG · TAIPEI · CHENNAI · TOKYO

Published by

World Scientific Publishing Co. Pte. Ltd.
5 Toh Tuck Link, Singapore 596224
USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data
Names: Leutbecher, Armin. | Hagen, Thomas, 1972– editor. | Rupp, Florian, 1978– editor. |
Scheurle, Jürgen, 1951– editor.
Title: Dynamical systems, number theory and applications : a festschrift in honor of
Armin Leutbecher's 80th birthday / edited by Thomas Hagen (University of Memphis, USA),
Florian Rupp (German University of Technology, Oman), Jürgen Scheurle (Technische Universität
München, Germany).
Description: New Jersey : World Scientific, 2016. | Includes bibliographical references.
Identifiers: LCCN 2015041750 | ISBN 9789814699860 (hardcover : alk. paper)
Subjects: LCSH: Number theory. | Dynamics. | Topological algebras.
Classification: LCC QA241 .D965 2016 | DDC 512.7--dc23
LC record available at http://lccn.loc.gov/2015041750

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

Printed by FuIsland Offset Printing (S) Pte Ltd Singapore

Dynamical Systems, Number Theory and Applications

A Festschrift in Honor of Armin Leutbecher's 80th Birthday

Preface

Deep and subtle links connect Dynamical Systems and Number Theory. The past few years have witnessed the solution of several problems on the frontiers of mathematical research, obtained by combining methods and insights from these two independent fields.

Having evolved from elementary Number Theory and first considerations about prime numbers, modern Number Theory aims to develop unifying results, like the structure of rational points on varieties. Methods from Complex Analysis, Group Theory and Algebraic Geometry, above all, shed new light on number-theoretic structures and lead to the improvement of classical methods beyond the seminal works of the 18th, 19th and 20th century. As we have seen in the proof of Fermat's Last Theorem, achievements are based on connections between different mathematical fields and on the transfer of methods and results from one field to another.

As an example in Number Theory, the classical and still interesting case is the upper complex half plane, which, on the one hand, can be viewed as a set of points or the domain of definition for interesting geometric curves and corresponding generating or zeta functions, or, on the other hand, as a group of Möbius transformations (modulo some stabilizer). Hence, numbers and groups immediately get connected, and interpretations of, say, the zeta function for groups become a subject of research. Moreover, generalizations of the thus derived group structure are natural with properties of zeta functions on these more general domains of definition arising as exciting and non-trivial research objects among many other topics of interest. Thus, groups and their representation, complex functions, connections between different algebraic structures as well as their applications in Crystallography, Quantum Physics, Coding Theory and other disciplines are areas of number-theoretic focus. In this way, for instance, recent applications link lattices and the zeta function to Physics, cf. [1].

The mathematical theory of Dynamical Systems evolved as a geometric theory of phase space where a discrete or continuous time-parameterized propagation of states is viewed as the action of a (semi-)group. Such groups are, for instance, generated by the solutions of iterated function systems or initial (-boundary) value problems associated to ordinary or partial differential equations. Above all, questions of stability, long-term behavior, singularities and blow-up, and attraction by stationary solutions are of great interest as they may reflect real-world phenomena visible in

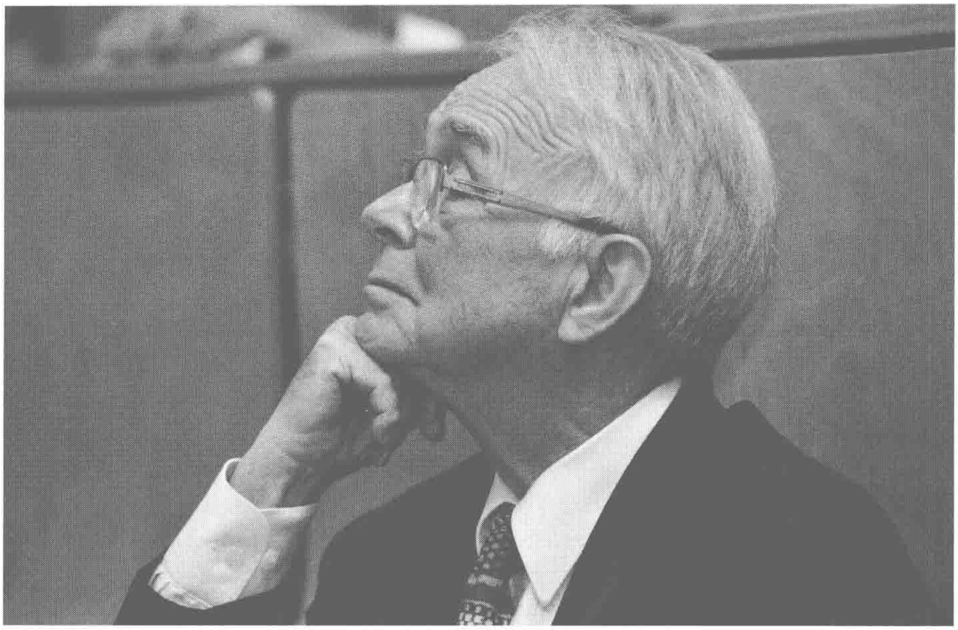


Fig. 1. Armin Leutbecher at the colloquium on the occasion of his 80th birthday, held at TU München on 17 October 2014.

physical experiments. At the core of classical Dynamical Systems Theory lies its geometric character and its objective to study the (long-)time behavior of all (or at least sufficiently large) subsets in phase space. Here, curves and their properties play an essential role. Methods of Algebraic Geometry are widely used in this context. Moreover, it is clear that such considerations must be supported by reductions of a problem's complexity, often connected to underlying symmetries of the equation of motion (cf. Noether's Theorem) where groups and their stabilizers are central. Modern computational methods and novel techniques from the theory of Evolution Equations, Differential Geometry and Algebraic Topology add to the expanding field of methodology embraced and often appropriated by Dynamical Systems Theory. This includes the development of efficient algorithms that preserve the underlying geometric structures of the dynamics, and the consideration of physical or geometric characteristics that are present in evolution models of real-world processes. It is not surprising that one can find many important and innovative applications of Dynamical Systems in Engineering and both the Natural and Social Sciences.

Although working in seemingly completely different fields, researchers in Number Theory, Dynamical Systems, Evolution Equations and several other, closely related disciplines share common interests concerning algebraic, geometric and analytic concepts, among them groups and semi-groups, spectral and operator-theoretic methods, harmonic functions, trajectories/orbits (in space) or curves (in the complex plane) and their properties. It is worth noting that such common interests are not restricted to those just mentioned. Hence, we would like to refer, for instance, to [2], [3] and [6] for additional examples.

This volume, dedicated to Professor Dr. Armin Leutbecher on the occasion of his 80th birthday, intends to broaden the mutual understanding of researchers from the fields of Number Theory and Dynamical Systems and some closely connected disciplines in Pure and Applied Mathematics. Therefore, a carefully chosen selection of genuine research papers as well as survey articles from these fields is presented here in a way experts and non-experts in the respective disciplines will benefit from. All research contributions were diligently refereed. Of course, only selected highlights could be chosen, mostly based on Armin Leutbecher's personal preferences and scientific activities during a long and still ongoing career as a scholar and academic teacher of exquisite taste and wisdom. Let us just mention Complex Analysis as the mathematical subject where Armin Leutbecher's seminal lectures [4] and [5] are foundational for many high-impact works, covering topics in Number Theory and Dynamical Systems.

Reflecting the wide range of Armin Leutbecher's interests, the articles presented cover a broad spectrum of topics in both the fields of Number Theory and Dynamical Systems and thus serve as an excellent overview of methods relevant in those two fields of study. To be more specific, these articles cover topics dealing with groups and symmetries, discrete and continuous dynamical systems, problems in Coding Theory, questions in Differential Geometry, evolution equations from Material Science and Fluid Dynamics, computational aspects of Dynamical Systems, connections to Number Theory, and some historical developments. The spectrum of topics in Number Theory, Dynamical Systems and adjacent areas of mathematics is broad by design. The application-oriented nature of the works included may serve as a blueprint for further research on related questions and stimulate the exchange of insights and ideas between these central disciplines of modern mathematics.

Apart from a biographical note on Armin Leutbecher and a historical article on mechanical integration devices, all other contributions are ordered alphabetically by the surname of the first author. As editors we would like to thank the contributors, the reviewers and the publishing company for their excellent work, critiques and comments, and suggestions and editorial assistance, respectively.

Memphis and Munich, August 2015 — *Th. Hagen, F. Rupp & J. Scheurle*

References

[1] A. Connes & M. Marcolli (2004): From Physics to Number Theory via Noncommutative Geometry. Part I: Quantum Statistical Mechanics of Q-lattices, arXiv:math/0404128.

[2] A. Knauf (1998): Number Theory, Dynamical Systems and Statistical Mechanics, lecture notes, Max-Planck Institut für Mathematik in den Naturwissenschaften, Leipzig.

[3] S. Kolyada, Y. Manin, M. Möller, P. Moree, & T. Ward (Eds., 2010): Dynamical Numbers: Interplay Between Dynamical Systems and Number Theory, Contemporary Mathematics, AMS.
[4] A. Leutbecher (1990): Vorlesungen zur Funktionentheorie I. Mathematisches Institut der Technischen Universität München.
[5] A. Leutbecher (1991): Vorlesungen zur Funktionentheorie II. Mathematisches Institut der Technischen Universität München.
[6] J.H. Silverman (2007): The Arithmetic of Dynamical Systems, Graduate Texts in Mathematics, 241, Springer.

Biographical Note on Armin Leutbecher

S. Walcher
Lehrstuhl A für Mathematik, RWTH Aachen University, D-52056 Aachen
E-mail: walcher@rwth-aachen.de

Armin Leutbecher was born on August 11, 1934 in Nancheng (China). He studied Mathematics and Physics from 1954 to 1961 in Göttingen, Tübingen and finally Münster, finishing the "Staatsexamen für das Höhere Lehramt" in 1961. In 1963 he received his doctoral degree at the University of Münster, with Hans Petersson as principal advisor. He proceeded there with postdoctoral work, culminating in his "Habilitation" in 1968. In 1970 he accepted an offer for a position at TU München, where he remained and was promoted to Professor in 1975. Even after his retirement in 1999 he continued to be active in research and teaching, and served as president of the Hurwitz-Gesellschaft, an alumni organization which he co-founded in 1996.

Notable in Armin Leutbecher's research are his broad range of mathematical knowledge and activity, and the excellent quality of the articles which satisfy his own high standards for publication.

The range of his work reaches from automorphic forms to algebra and algebraic number theory. Several of his manuscripts (including some work on Dynamical Systems) were circulated only privately or in newsletters, although those who read them agreed that they deserved publication in a journal. His research combined deep mathematical insight with elegance and rigor, and has continuing impact.

At TU München, Armin Leutbecher was renowned for giving excellent lectures which were characterized by originality, rigor and high standards, and succeeded in conveying the beauty of mathematics to many generations of students. In particular, his lectures on Linear Algebra and Analysis for first- and second-year students had a lasting influence. He led a number of students to mathematical research and advised seven doctoral students at TU München.

The following selection of his publications gives an impression of Armin Leutbecher's work:

- A. Leutbecher (1967): *Über die Heckeschen Gruppen* $G(\lambda)$. Abh. Math. Semin. Univ. Hamb. 31, 199-205.
- A. Leutbecher (1978): *Euklidischer Algorithmus und die Gruppe* GL_2.

Math. Ann. 231, 269-285.

- A. Leutbecher & J. Martinet (1982): *Lenstra's constant and Euclidean number fields.* Astérisque **94**: 87-131.
- A. Leutbecher & G. Niklasch (1989): *On cliques of exceptional units and Lenstra's construction of Euclidean fields.* In: Lecture Notes in Math. 1380, Springer, Heidelberg, 150-178.
- A. Leutbecher (1996): Zahlentheorie – Eine Einführung in die Algebra, Springer, Heidelberg.

Contents

Preface v

Biographical Note on Armin Leutbecher ix
S. Walcher

1. Das Jahr 1934 ... 1
J. Fischer

2. Explicit Expressions for Equivariant Minimal Lagrangian Surfaces 19
J.F. Dorfmeister & H. Ma

3. Rational Parameter Rays of the Multibrot Sets 49
D. Eberlein, S. Mukherjee & D. Schleicher

4. The Matovich-Pearson Equations Revisited 85
T. Hagen

5. Diffeomorphisms with Stable Manifolds as Basin Boundaries 99
S. Hayes & Ch. Wolf

6. A New Type of Functional Equations of Euler Products 113
B. Heim

7. The Hexagonal Lattice and the Epstein Zeta Function 127
A. Henn

8. On Putative q-Analogues of the Fano Plane 141
Th. Honold & M. Kiermaier

9. Integral Orthogonal Groups 177
A. Krieg

10. The Role of Fourier Analysis in X-Ray Crystallography 197
F. Rupp & J. Scheurle

11. An Elementary Proof for Joint Continuity of Semiflows 211
S. Schmitz

12. A Convergent String Method 221
H. Schwetlick & J. Zimmer

13. Variational Symmetries and Pluri-Lagrangian Systems 255
Y.B. Suris

Chapter 1

Das Jahr 1934 ...

Joachim Fischer[a]
Technische Universität München, Fakultät für Mathematik,
D-85747 Garching, Germany
E-mail: jea.fischer@t-online.de

Contents

1. Einleitung 1
2. Vor 1934 2
3. 1934 6
4. Epilog 15
References 16

1. Einleitung

Im Geburtsjahr 1934 von Armin Leutbecher erblickten unter anderem auch die Welt: Giorgio Armani, Brigitte Bardot, Alfred Biolek, Sophia Loren, Shirley MacLaine und Sidney Pollack; oder, wenn wir das Gebiet der Prominenten und Stars einmal etwas hinter uns lassen und uns dafür den wirklich und langfristig bedeutenden Personen zuwenden: der Logiker Paul Cohen, der Schriftsteller Harlan Ellison, der Kosmonaut Juri Alexejewitsch Gagarin, die Verhaltensforscherin Jane Goodall, der Astronom Carl Sagan, oder die Mathematiker bzw. Informatiker Tony Hoare, Robin Milner, Gilbert Strang und Niklaus Wirth ... um nur einige aufzuzählen. Man sieht also insbesondere schon: Der Leutbechersche Jahrgang 1934 war ein *guter* Jahrgang!

Das gilt nicht nur für Personen, sondern auch für manche Teilgebiete menschlicher Aktivitäten. Es wird also niemanden verwundern – und den Jubilar zu allerletzt –, wenn ich mich auf dem Terrain der "Mechanischen Integration", dem ich seit fast 30 Jahren meine (knappen) Mußestunden widme, ebenfalls im Jahr 1934 umsehe und finde, daß sich dort einige höchst bedeutsame Entwicklungen oder Teilschritte dazu festmachen lassen ... Ich hatte ja die Ehre und das Vergnügen, zur Inaugurations-Feier der von Armin Leutbecher ins Leben gerufenen Hurwitz-Gesellschaft am 20. März 1998 den Festvortrag über "Instrumente zum Integrieren,

[a]Diplomarbeit August 1973 an der TU München: "Dirichletreihen mit Funktionalgleichung"; Aufgabensteller: Prof. Dr. Armin Leutbecher.

oder: Mathematik und Feinmechanik. Ein Streifzug durch knapp zwei Jahrhunderte (verbunden mit der Vorführung ausgewählter historischer Geräte)" halten zu dürfen:

Fig. 1. V.l.n.r.: Reinsch, Leutbecher, Scheurle, Schleicher, Vachenauer, Fischer, Hartl, Kredler.

Und die wenigen, die sich auf diesem Gebiet sonst noch tummeln, werden mit mir die Leidenschaft teilen, daß die Kombination von (teilweise sehr guter) Mathematik und Feinmechanik – meist sogar Feinstmechanik – so reizvoll ist, daß es schwerfällt, sich davon zu lösen.– Bevor wir einen Blick in das Jahr 1934 werfen, schließen wir kurz einen Bogen zu heute mit dem Hinweis, daß sich 2014 auch die Erfindung des exakt messenden, rein mechanischen Planimeters zum 200sten Male gejährt hat ... Also: Jubiläen allenthalben!

2. Vor 1934

Was war der Stand der Mechanischen Integration im Jahr 1934? Erstaunlicherweise gibt es allein schon viele runde Jubiläen, die man 1934 hätte begehen können. So hatte 120 Jahre zuvor, 1814, Johann Martin Hermann (1785-1841) das erste theoretisch exakt messende Planimeter erfunden und damit eigentlich das Gebiet der mechanischen Integration eröffnet. Leider veröffentlichte er wohl aufgrund einer einsetzenden Krankheit nichts dazu, und die, die von seiner Erfindung, von der Möglichkeit exakten mechanischen Integrierens und sogar von dem erfolgreichen Test eines Prototyps wußten, trugen es nicht weiter; die Erfindung wurde rasch vergessen und der Prototyp 1848 verschrottet. Erhalten blieb neben fragmen-

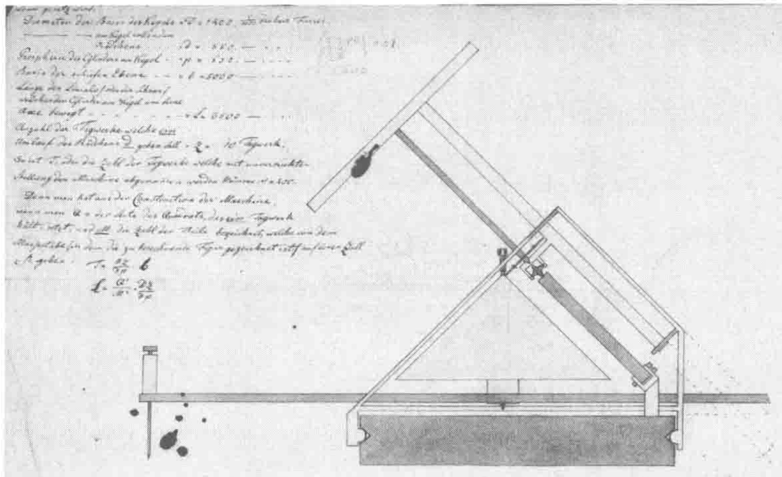


Fig. 2. Seitenriß des Hermannschen Planimeters (einzig erhaltene Zeichnung des 1848 verschrotteten Prototyp-Instruments); Nachlaß Bauernfeind, Deutsches Museum München, Archiv NL 49/14.

tarischen Dokumenten nur ein kolorierter Seitenriß mit Maßangaben, der immerhin eine Rekonstruktion erlaubt (mehr dazu in Fischer 2014):

40 Jahre nach Hermann, also 1854 und damit 80 Jahre vor dem Jahr 1934, machte Jakob Amsler (1823-1912) mit seiner Erfindung des Polarplanimeters Furore. Dieser einfachste Mechanismus, der nur aus einem zweiarmigen Gelenk und einer passend montierten Meßrolle bestand, trug wesentlich dazu bei, daß die mechanische Integration aufgrund der Einfachheit und dadurch der preiswerten Herstellung des Instruments Einzug hielt in die Büros und Kontore aller, die mit Flächenberechnung zu tun hatten: Katasterbüros, Vermesser, aber bald auch Bauingenieure, Maschinenbauer, Elektrotechniker.

Über kurz oder lang gab es fast kein Gebiet im technisch-naturwissenschaftlich-mathematischen Bereich, in dem nicht planimetriert wurde. Aber Amsler blieb nicht dort stehen; er skizzierte schon in seiner ersten Veröffentlichung zum Polarplanimeter weitere Anwendungen für andere Integrale, die nicht in erster Linie Flächeninhalte darstellten: Integratoren, später meist zutreffender Momentenplanimeter oder Potenzplanimeter genannt, zur Integration nicht nur über cartesisches $f(x)$, sondern zugleich – jedoch ohne zeichnerisch-rechnerischen Zwischenschritt – über $f(x)^2$, $f(x)^3$ und/oder $f(x)^4$; oder Harmonische Analysatoren zur Bestimmung der Fourier-Koeffizienten (Amsler 1856).

1884, 50 Jahre vor 1934, legte Amsler nochmals nach und stellte einerseits die Übertragung der Flächenmessung in der Ebene auf die Kugel mittels des wiederum äußerst einfachen Sphärischen Polarplanimeters vor, und gab andererseits eigene Ideen zu den sogenannten Präzisionsplanimetern an die Öffentlichkeit (Amsler 1884), nachdem Gottlieb Coradi in Zürich ein paar Jahre davor mit seinen – in

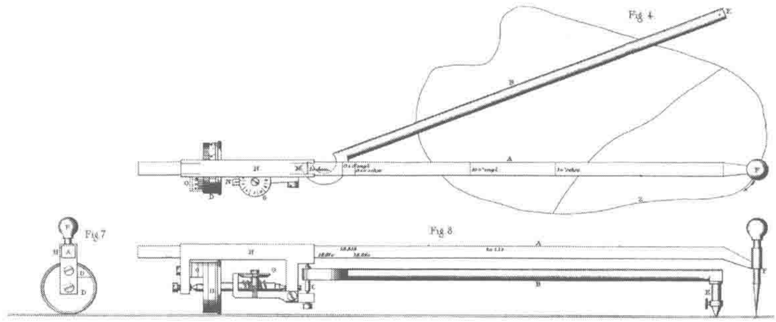


Fig. 3. Polarplanimeter von Jakob Amsler; Figuren 4, 7 und 8 aus Amsler 1856.

Zusammenarbeit mit Friedrich Hohmann entstandenen – entsprechenden Instrumenten debütiert hatte (Fischer 1995; Fischer 2002).

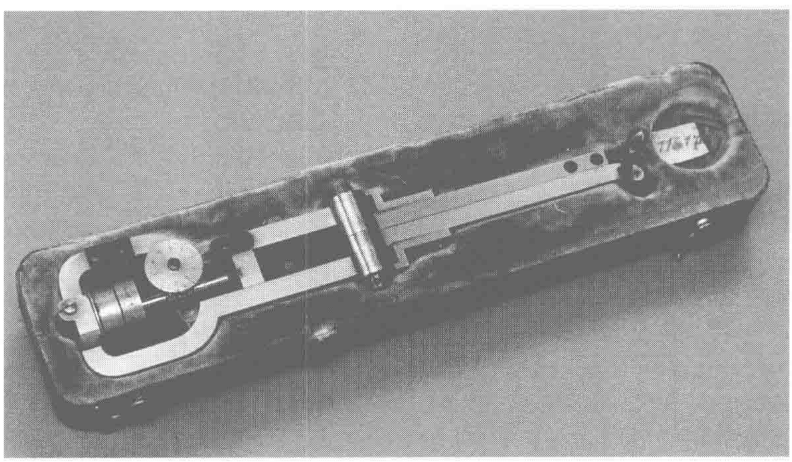


Fig. 4. Sphärisches Polarplanimeter im Etui (einziges derzeit bekanntes Exemplar; ein zur Beschwerung dienendes Polgewicht ist wohl leider abhanden gekommen), um 1883/84; Musée des arts et métiers, Paris, Inv.-Nr. 11617.

Das sphärische Polarplanimeter entsteht ganz einfach aus dem ebenen Polarplanimeter, indem die Arme des Planimetergelenks so nach unten abgeknickt werden, daß die Bewegung auf einer Kugeloberfläche möglich wird; die Theorie bleibt dabei wesentlich dieselbe.

1894, 40 Jahre vor 1934, sah dann die ersten kommerziell hergestellten Harmonischen Analysatoren; sie wurden aber nicht nach den ursprünglichen Amslerschen Ideen, sondern nach denen von Olaus Magnus Friedrich Erdmann Henrici (1840-1918) bei Coradi gebaut: