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58TH EDITION

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A Ready-Reference Book of Chemical and Physical Data

HANDBOOK OF CHEMISTRY AND PHYSICS
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In collaboration with a large number of professional chemists and physicists whose assistance is acknowledged in the list of general collaborators and in connection with the particular tables or sections involved.

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PREFACE TO THE 58th EDITION

CRC Press, Inc. is dedicated to establishment and development of data sources that provide superior value to its customers. It has long been committed to enhancing the availability of reference data. When the *CRC Handbook of Chemistry and Physics* was first published in 1913 it filled a void by supplying data especially useful to, but certainly not restricted to, undergraduate students.

Growth of scientific research, increase in college enrollments, emphasis on graduate schools, need for new data for the physical sciences, and the excellent acceptance of the *CRC Handbook of Chemistry and Physics* resulted in the *Handbook* being revised and expanded annually except for certain years during World Wars I and II. With publication of this current edition, the *CRC Handbook of Chemistry and Physics* will have been published continuously for 65 years and will have been revised 57 times.

Even though some material has been deleted from the *Handbook* from edition to edition, the quantity of new and revised material added over the years has caused the book to be greatly increased in physical size. This present edition is more than 50 times larger than the first edition. As a result of the great increase in the size of the book, we have been advised to divide the *Handbook* into more than one volume. In response to these valued suggestions we instituted a program, part of which embraced publication of single or multivolume books containing extensive quantities of data on individual subjects or areas. For example, two areas among many covered in this program are 1. absorption spectra and 2. materials.

Implementation of this program with publication of specialized and extensive books permits us to continue to publish the *CRC Handbook of Chemistry and Physics* as a single volume reference book. As in the past, the *Handbook* contains a reasonable amount of a variety of physical and mathematical data not presently found in any other single volume reference book. Even so, it cannot contain as wide a variety of information as we would wish. The type of data placed into the *CRC Handbook of Chemistry and Physics* is the result of input from many sources including editors, advisors, current and previous contributors, and, very importantly, users of the *Handbook*. As always, we continue to value suggestions from users of our books.

Included among the improvements of the book are new and better data for the cryogenic properties of gases with the data being presented in SI units.The table of the Electron Work Function of the Elements has been updated and revised. A table of the Natural Width in eV of Indicated L X-ray Lines has been added. In the section of the book dealing with definitions, some definitions have been added and some have been revised. Also included among some of the improvements has been the resetting of certain tables and pages so they can be more easily read and utilized. Likewise, in keeping with custom we have corrected all typographical errors that have been brought to our attention.

Robert C. Weast
April 11, 1977

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The Publishers and Editors will be grateful to readers of this Handbook who will call their attention to errors that may be discovered. Suggestions for improvement are also welcome.

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USE OF LOGARITHMS

LAWS OF EXPONENTS

For a any real number and m a positive integer, the exponential a^m is defined as

$$\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{m \text{ terms}}$$

Using this definition, it is easy to show that the following three *laws of exponents* hold:

I. $a^m \cdot a^n = a^{m+n}$

II. $$\frac{a^m}{a^n} = \begin{cases} a^{m-n} & \text{if } m > n \\ 1 & \text{if } m = n \\ \dfrac{1}{a^{n-m}} & \text{if } m < n \end{cases}$$

III. $(a^m)^n = a^{mn}$

The n-th root function is defined as the inverse of the n-th power function; that is, if

$$b^n = a, \quad \text{then} \quad b = \sqrt[n]{a}.$$

If n is odd, there will be a unique real number satisfying the above definition for $\sqrt[n]{a}$, for any real value of a. If n is even, for positive values of a there will be two real values for $\sqrt[n]{a}$, one positive and one negative. By convention, the symbol $\sqrt[n]{a}$ is understood to mean the positive value in this case. If n is even and a is negative, there are no real values for $\sqrt[n]{a}$.

If we now attempt to extend the definition of the exponential a^t to all rational values of the exponent t, in such a way that the three laws of exponents continue to hold, it is easily shown that the required definitions are:

$$a^0 = 1$$

$$a^{p/q} = \sqrt[q]{a^p}$$

$$a^{-t} = \frac{1}{a^t}$$

In order to avoid difficulties with imaginary numbers and division by zero, a must now be restricted to be positive.

With this extended definition, it is possible to restate the second law of exponents in a simpler form:

II′. $\dfrac{a^m}{a^n} = a^{m-n}$

It is shown in advanced calculus that this definition may be further extended so that the exponent may be any real number, and the laws of exponents continue to hold. When the quantity a^x thus defined is viewed as a function of the exponent x, with the base a held constant, it is a continuous function. Also, if $a > 1$, the exponential function is monotone increasing, and if $0 < a < 1$, it is monotone decreasing.

Any monotone function has a single-valued inverse function, which is also monotone. Furthermore, if the original function is continuous, so is the inverse. Therefore, the inverse function to the exponential function a^x exists for all positive values of a, except $a = 1$. This function is given the name *logarithm to the base a*, abbreviated $\log_a$. That is, if

$$x = a^y, \quad \text{then} \quad y = \log_a x.$$

This function is defined and continuous for all positive values of x. It is monotone increasing if $a > 1$, and monotone decreasing if $a < 1$.

If the laws of exponents are rewritten in terms of logarithms, they become the *laws of logarithms*:

I. $\log_a (xy) = \log_a x + \log_a y$

II. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

III. $\log_a (x^n) = n \log_a x$

Logarithms derive their main usefulness in computation from the above laws, since they allow multiplication, division, and exponentiation to be replaced by the simpler operations of addition, subtraction, and multiplication, respectively. See the examples which follow.

Further recourse to the definition of logarithm leads to the following formula for change of base

$$\log_a x = \log_b x / \log_b a = (\log_b x) \cdot (\log_a b)$$

Two numbers are commonly used as bases for logarithms. Logarithms to the base 10 are most convenient for use in computation. These logarithm are called common or Briggsian logarithms.

The other usual base for logarithms is an irrational number denoted by e, whose value is approximately 2.71828 .... These logarithms are called natural, Naperian, or hyperbolic logarithms, and occur in many formulas of higher mathematics. The abbreviation ln is frequently used for the natural logarithm function.

Other bases for logarithms, such as 2 and 3, occur in certain applications. These applications are so infrequent that separate tables for these bases are usually not given. Instead, the formulas for change of base are applied to common or natural logarithms.

If the formulas for change of base are applied to the two usual bases, the following formulas result:

$$\log_{10} x = \log_e x / \log_e 10 = (\log_{10} e)(\log_e x) = M \log_e x$$
$$= 0.43429\ 44819 \log_e x$$

$$\log_e x = \log_{10} x / \log_{10} e = (\log_e 10)(\log_{10} x) = \frac{1}{M} \log_{10} x$$
$$= 2.30258\ 50930 \log_{10} x$$

The following remarks apply to common logarithms.

Since most numbers are irrational powers of ten, a common logarithm, in general, consists of an integer, which is called the characteristic, and an endless decimal, the mantissa.

It is to be observed that the common logarithms of all numbers expressed by the same figures in the same order with the decimal point in different positions have different characteristics but the same mantissa. To illustrate:—if the decimal point stands after the first figure of a number, counting from the left, the characteristic is 0; if after two figures, it is 1; if after three figures, it is 2; and so forth. If the decimal point stands before the first significant figure the characteristic is -1, usually written $\bar{1}$; if there is one zero between the decimal point and the first significant figure it is $\bar{2}$, and so on. For example: log 256 = 2.40824, log 2.56 = 0.40824, log 0.256 = $\bar{1}.40824$, log 0.00256 = $\bar{3}.40824$. The two latter are often written log 0.256 = 9.40824 − 10, log 0.00256 = 7.40824 − 10.

Notice that, although the common logarithm of a number less than one is a negative number, it is customarily written as a negative characteristic and a *positive* mantissa, since the mantissas are usually given in tables as positive numbers. This is the reason that the negative sign is written above the characteristic, since it does not apply to the mantissa. Thus log 0.00256 = $\bar{3}.40824$ = 7.40824 − 10 = −2.59176.

A method of determining characteristics of logarithms is to write the number with one figure to the left of the decimal point multiplied by the appropriate power of 10. The characteristic is then the exponent used. For example:

$$256\ 000\ 000 = 2.56 \times 10^8 \qquad \log = 8.40824$$

$$0.000\ 000\ 256 = 2.56 \times 10^{-7} \qquad \log = \bar{7}.40824 \quad \text{or} \quad 3.40824 - 10$$

Inasmuch as the characteristic may be determined by inspection, the mantissas only are given in tables of common logarithms.

USE OF LOGARITHM TABLES

To find the common logarithm of a number:

(*Note:* This description and examples refer specifically to the table entitled "Five-Place Logarithms." For the other tables, there will be minor differences from this description. Most of these differences are obvious. Notes with the individual tables explain any differences which are not immediately obvious.)

For a number of four figures, take out the tabular mantissa on a line with the first three figures of the number and under its fourth figure. The characteristic is determined as previously explained.

For a number of less than four figures, supply zeros to make a four figure number and take the value of the mantissa from the tables as before. For example: log 2 = log 2.000 = 0.30103.

(Notice that in some of the tables not all of the digits of the logarithm are given for every value. For example, in the table of five-place common logarithms, the first two digits of each mantissa are given only once for each line. The remaining three digits of each mantissa are given in the correct place in the table. When the leading two digits are not given on a line, they should be taken from the last line above it on which they do appear. When a mantissa is marked with an asterisk, it indicates that the value for the leading digits is to be taken from the *next* line instead of the present line. Similar remarks apply to the other tables in which this method of presenting the values are used.)

For a number of more than four figures, interpolation must be used. There are several precautions that must be observed when interpolating:

1. Linear interpolation, as described below, may only be used to add one extra digit to the argument (i.e., in the present case, for a four-digit argument).
2. Even though the mantissas given in the table are accurate to five decimal places, interpolated values are accurate only to the same number of places as in the argument, i.e. four places.

3. Because of the rapidly changing values in this region of the table, interpolation is not accurate if the first two digits of the argument are 11 or 12. For this reason, the table has been extended at the end so that such values may be read directly from the table with five-digit arguments, without interpolation. (The four-place table is not so extended. If interpolation is required in this section of the four-place table, the value should be read instead from the five-place table without interpolation.)

If the above precautions cannot be observed, then higher order interpolation should be used.

Where applicable, linear interpolation is carried out as follows:

Take the tabular value of the mantissa for the first four figures; find the difference between the mantissa and the next greater tabular mantissa and multiply the difference so found by the remaining figures of the number as a decimal and add the product to the mantissa of the first four figures. For example, to find log 46.762:

$$\log 46.76 = 1.66987$$

Tabular difference between this mantissa and that for 4677 is .00010

$$\begin{aligned} \therefore \log 46.762 &= 1.66987 + .2 \times .00010 \\ &= 1.66987 + .00002 \\ &= 1.66989 \end{aligned}$$

In the four-place logarithm table, a column of proportional parts is given at the end of each line. The number in the column under the fourth digit of the argument is the amount that must be added to any mantissa in that line to interpolate for the fourth digit. This number is to be added to the last place of the mantissa. These numbers are averages for the entire line, so may be off by 1 in the last place.

For example, to find log 33.74

$$\begin{aligned} \log 33.7 &= 1.5276 \\ \text{proportional part for } 4 &= \underline{5} \\ \therefore \log 33.74 &= 1.5281 \end{aligned}$$

To find the number corresponding to a given logarithm:

(*Note:* This number is called the antilogarithm, and is denoted by $\log^{-1}$. Since the logarithm function is the inverse of the exponential function, $\log_a^{-1} x = a^x$. Therefore, any procedure or table which calculates antilogarithms may also be used to calculate exponentials, and vice-versa. In particular, tables of e^x may be used to compute antilogarithms to the base e.)

The procedure given below refers to the five-place logarithm table. As before, any significant deviation for other tables will be noted.

If the mantissa is found exactly in the table, join the figure at the top which is directly above the given mantissa to the three figures on the line at the left and place the decimal point according to the characteristic of the logarithm. For example,

$$\log^{-1} 3.39967 = \text{antilog } 3.39967 = 2510.$$

If the mantissa is not found exactly in the table it is necessary to interpolate. For example, to find antilog 3.40028, we find in the table

$$\begin{aligned} \text{antilog } 3.40019 &= 2513. \\ \text{antilog } \underline{3.40037} &= 2514. \\ \text{tabular difference} \quad 18 & \end{aligned}$$

The required difference is 9, so we must add $\frac{9}{18} = .5$.

$$\therefore \text{ antilog } 3.40028 = 2513.5$$

The same precautions must be observed for interpolation in finding antilogarithms as in finding logarithms.

Tables of natural logarithms are used in the same way as tables of common logarithms, except that they contain both the characteristics and the mantissas of the logarithms.

Examples of the use of logarithms in computation follow. Almost all computation with logarithms is done with common logarithms, since the computation of the characteristic is simpler, and since only the significant digits of the argument need be given in the table, without regard for the decimal point location. These examples all use the table of five-place common logarithms.

1. $52600 \times 0.00381 \times 2.74 = 549.11$

log 52600 = 4.72099
log 0.00381 = $\bar{3}.58092$*
log 2.74 = 0.43775
adding = 2.73966
antilog = 549.11

*For numbers less than one, the characteristic is negative whereas the mantissa is positive.

The sum is the logarithm of the product, the mantissa of which is 73966. On looking up this mantissa in the logarithm tables we see that it corresponds to the digits 54911. The characteristic is 2, hence there are three figures before the decimal point. The number corresponding to the logarithm, called the antilogarithm, is 549.11.

2. $0.00123 \div 52.7 = 0.000\ 023\ 34$

log 0.00123 = $\bar{3}.08991$
log 52.7 = 1.72181
subtracting = $\bar{5}.36810$
antilog = 0.000 023 34

An alternative method:

log 0.00123 = 7.08991 − 10
log 52.7 = 1.72181
5.36810 − 10

The characteristic 5(5. − 10) shows four zeros after the decimal point before the first significant figure.

3. $\dfrac{273 \times 780}{292 \times 760} \times 15 \times 0.09 = 1.2954$

log 273 = 2.43616
log 780 = 2.89209
log 15 = 1.17609
log 0.09 = $\bar{2}.95424$
log numerator = 5.45858

log 292 = 2.46538
log 760 = 2.88081
log denominator = 5.34619

log numerator = 5.45858
log denominator = 5.34619
subtracting = 0.11237
antilog = 1.2954

As division may be accomplished by multiplying by the reciprocal of a number, the above may be considerably simplified. The logarithm of the reciprocal of a number, called the cologarithm, is readily obtained from the table by subtracting the logarithm of the number