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Yakov G. Berkovich, Zvonimir Janko

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Authors
Prof. Yakov G. Berkovich
18251 Afula
Israel
berkov@math.haifa.ac.il

Prof. Dr. Zvonimir Janko
Ruprecht-Karls-Universität Heidelberg
Mathematisches Institut
Im Neuenheimer Feld 288
69120 Heidelberg
Germany
janko@mathi.uni-heidelberg.de

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List of definitions and notations

Set theory

- $|M|$ is the cardinality of a set M (if G is a finite group, then $|G|$ is called its order).
- $x \in M$ ($x \notin M$) means that x is (is not) an element of a set M. $N \subseteq M$ ($N \nsubseteq M$) means that N is (is not) a subset of the set M; moreover, if $M \neq N \subseteq M$, we write $N \subset M$.
- $\emptyset$ is the empty set.
- N is called a nontrivial subset of M, if $N \neq \emptyset$ and $N \subset M$. If $N \subset M$, we say that N is a proper subset of M.
- $M \cap N$ is the intersection and $M \cup N$ is the union of sets M and N. If M, N are sets, then $N - M = \{x \in N \mid x \notin M\}$ is the difference of N and M.

Number theory and general algebra

- p is always a prime number.
- π is a set of primes; π' is the set of all primes not contained in π.
- m, n, k, r, s are, as a rule, natural numbers.
- If $\pi(m)$ is the set of prime divisors of m, then m is a π-number if $\pi(m) \subseteq \pi$.
- n_p is the p-part of n, and n_π is the π-part of n.
- $\mathrm{GCD}(m, n)$ is the greatest common divisor of m and n.
- $\mathrm{LCM}(m, n)$ is the least common multiple of m and n.
- $m \mid n$ should be read as: m divides n.
- $\mathrm{GF}(p^m)$ is the finite field containing p^m elements.
- F^* is the multiplicative group of a field F.
- $\mathcal{L}(G)$ is the lattice of subgroups of a group G.
- $\mathcal{L}_N(G)$ is the lattice of normal subgroups of a group G.
- If $n = p_1^{\alpha_1}, \dots, p_k^{\alpha_k}$ is the standard prime decomposition of n, then $\lambda(n) = \sum_{i=1}^{k} \alpha_i$.

Groups

We consider only finite groups which are denoted, with a pair exceptions, by upper case Latin letters.

- If G is a group, then $\pi(G) = \pi(|G|)$.
- G is a p-group if $|G|$ is a power of p; G is a π-group if $\pi(G) \subseteq \pi$.
- G is, as a rule, a finite p-group.
- $H \leq G$ means that H is a subgroup of G.

- $H < G$ means that $H \leq G$ and $H \neq G$ (in that case H is called a *proper* subgroup of G). $\{1\}$ denotes the group containing only one element.
- H is a nontrivial subgroup of G if $\{1\} < H < G$.
- H is a maximal subgroup of G if $G > \{1\}$, $H < G$ and it follows from $H \leq M < G$ that $H = M$.
- If H is a proper normal subgroup of G, then we write $H \lhd G$. Expressions "normal subgroup of G" and "G-invariant subgroup" are synonyms.
- A normal subgroup H of G is nontrivial provided $G > H > \{1\}$.
- H is a minimal normal subgroup of G if (a) H normal in G; (b) $H > \{1\}$; (c) $N \lhd G$ and $N < H$ implies $N = \{1\}$. Thus, the group $\{1\}$ has no minimal normal subgroup.
- A group G is *metabelian* if G/A is abelian for some abelian $A \lhd G$. Groups of class 2 are metabelian but the converse is not true.
- A p-group G is said to be *Dedekindian* if all its subgroups are normal.
- A group G is said to be *minimal nonabelian* if it is nonabelian but all its proper subgroups are abelian.
- $\mathcal{A}_1(G)$ is the set of all minimal nonabelian subgroups of a p-group G.
- A group H is said to be *capable* if there exists a group G such that $G/Z(G) \cong H$.
- A p-group G is said to be *metahamiltonian* if all its minimal nonabelian (so all nonabelian) subgroups are normal.
- $H \leq G$ is quasinormal if it is permutable with all subgroups of G.
- A p-group is said to be *modular* if all its subgroups are quasinormal.
- H is a maximal normal subgroup of G if $H < G$ and G/H is simple.
- The subgroup generated by all minimal normal subgroups of G is called the *socle* of G and denoted by $\mathrm{Sc}(G)$. We put, by definition, $\mathrm{Sc}(\{1\}) = \{1\}$.
- $\mathrm{N}_G(M) = \{x \in G \mid x^{-1}Mx = M\}$ is the normalizer of a subset M in G.
- $\mathrm{C}_G(x)$ is the centralizer of an element x in G : $\mathrm{C}_G(x) = \{z \in G \mid zx = xz\}$.
- (G) is the number of nonidentity cyclic subgroups of G.
- $\mathrm{C}_G(M) = \bigcap_{x \in M} \mathrm{C}_G(x)$ is the centralizer of a subset M in G.
- If $A \leq B$ and A, B are normal in G, then $\mathrm{C}_G(B/A) = H$, where $H/A = \mathrm{C}_{G/A}(B/A)$.
- $H < G$ is a TI-subgroup if $T \cap T^x = \{1\}$ for all $x \in G - \mathrm{N}_G(H)$.
- $A \text{ wr } B$ is the wreath product of the "passive" group A and the transitive permutation group B (in what follows we assume that B is a regular permutation group, as a rule, a p-group); B is called the active factor of the wreath product). Then the order of that group is $|A|^{|B|} \cdot |B|$.
- $\mathrm{Aut}(G)$ is the group of automorphisms of G (the automorphism group of G)
- $\mathrm{Inn}(G)$ is the group of all inner automorphisms of G.
- $\mathrm{Out}(G) = \mathrm{Aut}(G)/\mathrm{Inn}(G)$ is the outer automorphism group of G.
- $\mathcal{N}(G)$ is the norm of G, the intersection of normalizers of all subgroups of G.
- If $a, b \in G$, then $a^b = b^{-1}ab$ is a conjugate to a in G.
- $a \in G$ is real if it conjugate with a^{-1}.
- An element $x \in G$ inverts a subgroup $H \leq G$ if $h^x = h^{-1}$ for all $h \in H$.

- If $M \subseteq G$, then $\langle M \rangle = \langle x \mid x \in M \rangle$ is the subgroup of G generated by M.
- $M^x = x^{-1}Mx = \{y^x \mid y \in M\}$ for $x \in G$ and $M \subseteq G$.
- $[x, y] = x^{-1}y^{-1}xy = x^{-1}x^y$ is the *commutator* of elements x, y of G. If $M, N \subseteq G$ then $[M, N] = \langle [x, y] \mid x \in M, y \in N \rangle$ is a subgroup of G.
- $o(x)$ is the order of an element x of G.
- An element $x \in G$ is a π-element if $\pi(o(x)) \subseteq \pi$.
- G is a π-group, if $\pi(G) \subseteq \pi$. Obviously, G is a π-group if and only if all of its elements are π-elements.
- G' is the subgroup generated by all commutators $[x, y]$, $x, y \in G$ (i.e., $G' = [G, G]$), $G^{(2)} = [G', G'] = G'' = (G')'$, $G^{(3)} = [G'', G''] = (G'')'$ and so on. G' is called the *commutator* (or *derived*) subgroup of G.
- $\mathrm{Z}(G) = \bigcap_{x \in G} \mathrm{C}_G(x)$ is the center of G.
- $\mathrm{Z}_i(G)$ is the ith member of the upper central series of G; in particular, $\mathrm{Z}_0(G) = \{1\}$, $\mathrm{Z}_1(G) = \mathrm{Z}(G)$.
- $\mathrm{K}_i(G)$ is the ith member of the lower central series of G; in particular, $\mathrm{K}_2(G) = G'$. We have $\mathrm{K}_i(G) = [G, \dots, G]$ ($i \geq 1$ times). We set $\mathrm{K}_1(G) = G$.
- If G is nonabelian, then $\eta(G)/\mathrm{K}_3(G) = \mathrm{Z}(G/\mathrm{K}_3(G))$.
- $\mathcal{M}(G) = \langle x \in G \mid \mathrm{C}_G(x) = \mathrm{C}_G(x^p) \rangle$ is the Mann subgroup of a p-group G.
- $\mathrm{Syl}_p(G)$ is the set of p-Sylow subgroups of an arbitrary finite group G.
- S_n is the *symmetric* group of degree n.
- A_n is the *alternating* group of degree n.
- Σ_{p^n} is a Sylow p-subgroup of S_{p^n}.
- $\mathcal{H}_{2,p}$ is a nonabelian metacyclic p-group of order p^4 and exponent p^2.
- $\mathrm{GL}(n, F)$ is the set of all nonsingular $n \times n$ matrices with entries in a field F, the n-dimensional *general linear* group over F, $\mathrm{SL}(n, F) = \{A \in \mathrm{GL}(n, F) \mid \det(A) = 1 \in F\}$, the n-dimensional *special linear* group over F.
- If $H \leq G$, then $H_G = \bigcap_{x \in G} x^{-1}Hx$ is the *core* of the subgroup H in G and H^G, the intersection of all normal subgroups of G containing H, is the *normal closure* or *normal hull* of H in G. Obviously, H_G is normal in G.
- If G is a p-group, then $p^{b(x)} = |G : \mathrm{C}_G(x)|$; $b(x)$ is said to be the *breadth* of $x \in G$, where G is a p-group; $b(G) = \max\{b(x) \mid x \in G\}$ is the *breadth* of G.
- If $H \leq G$ and $|G : \mathrm{N}_G(H)| = p^{\mathrm{sb}(H)}$, when $\mathrm{sb}(H)$ is said to be the *subgroup breadth* of H. Next, $\mathrm{sb}(G) = \max\{\mathrm{sb}(H) \mid H \leq G\}$.
- $\Phi(G)$ is the Frattini subgroup of G (= the intersection of all maximal subgroups of G), $\Phi(\{1\}) = \{1\}$, $p^{\mathrm{d}(G)} = |G : \Phi(G)|$.
- $\Gamma_i = \{H < G \mid \Phi(G) \leq H, |G : H| = p^i\}$, $i = 1, \dots, \mathrm{d}(G)$, where $G > \{1\}$.
- If $H < G$, then $\Gamma_1(H)$ is the set of all maximal subgroups of H.
- $\exp(G)$ is the exponent of G (the least common multiple of the orders of elements of G). If G is a p-group, then $\exp(G) = \max\{o(x) \mid x \in G\}$.
- $k(G)$ is the number of conjugacy classes of G (= G-classes), the class number of G.
- K_x is the G-class containing an element x (sometimes we also write $ccl_G(x)$).

- C_m is the cyclic group of order m.
- $A \times B$ is the direct product of groups A and B.
- $G^m = G \times \cdots G$ (m times) is the direct product of m copies of a group G.
- $A * B$ is a central product of groups A and B, i.e., $A * B = AB$ with $[A, B] = \{1\}$. In particular, the direct product $A \times B$ is a central product of groups A and B.
- $E_{p^m} = C_p^m$ is the elementary abelian group of order p^m. G is an elementary abelian p-group if and only if it is a p-group $> \{1\}$ and G coincides with its socle. Next, $\{1\}$ is elementary abelian for each prime p, by definition.
- A group G is said to be *homocyclic* if it is a direct product of isomorphic cyclic subgroups (obviously, elementary abelian p-groups are homocyclic).
- $\mathrm{ES}(m, p)$ is an *extraspecial* group of order p^{1+2m} (a p-group G is said to be extraspecial if $G' = \Phi(G) = Z(G)$ is of order p). Note that for each positive integer m, there are exactly two nonisomorphic extraspecial groups of order p^{2m+1}.
- $S(p^3)$ is a nonabelian group of order p^3 and exponent $p > 2$.
- A *special* p-group is a nonabelian p-group G such that $G' = \Phi(G) = Z(G)$ is elementary abelian. Direct products of extraspecial p-groups are special.
- D_{2m} is the *dihedral* group of order $2m$, $m > 2$. Some authors consider E_{2^2} as the dihedral group D_4.
- Q_{2^m} is the *generalized quaternion* group of order $2^m \geq 2^3$.
- SD_{2^m} is the *semidihedral group* of order $2^m \geq 2^4$.
- M_{p^m} is a nonabelian p-group containing exactly p cyclic subgroups of index p (see Theorem 1.2).
- $\mathrm{cl}(G)$ is the *nilpotence class* of a p-group G.
- $\mathrm{dl}(G)$ is the *derived length* of a p-group G.
- $\mathrm{CL}(G)$ is the set of all G-classes.
- A p-group of *maximal class* is a nonabelian group G of order p^m with $\mathrm{cl}(G) = m-1$.
- A p-group is s-self-dual if every its subgroup is isomorphic to a quotient group.
- A p-group is q-self-dual if every its quotient group is isomorphic to a subgroup.
- $\mathrm{GL}(n, F)$ ($\mathrm{SL}(n, p)$) the general (special) linear group of degree n over the field F.
- $\Omega_m(G) = \langle x \in G \mid o(x) \leq p^m \rangle$, $\Omega_m^*(G) = \langle x \in G \mid o(x) = p^m \rangle$ and $\mho_m(G) = \langle x^{p^m} \mid x \in G \rangle$.
- A p-group G is said to be *regular*, if for any $x, y \in G$ there exists $z \in \langle x, y \rangle'$ such that $(xy)^p = x^p y^p z^p$.
- A p-group is *absolutely regular* if $|G/\mho_1(G)| < p^p$. Absolutely regular p-groups are regular.
- A p-group is *thin* if it is either absolutely regular or of maximal class.
- $G = A \cdot B$ is a *semidirect product* with *kernel* B and *complement* A.
- A group G is an extension of a normal subgroup N by a group H if $G/N \cong H$. A group G splits over N if $G = H \cdot N$ with $H \leq G$ and $H \cap N = \{1\}$ (in that case, G is a semidirect product of H and N with kernel N).

- $H^\# = H - \{e_H\}$, where e_H is the identity element of the group H. If $M \subseteq G$, then $M^\# = M - \{e_G\}$.
- An automorphism α of G is *regular* (= *fixed-point-free*) if it induces a regular permutation on $G^\#$ (a permutation is said to be *regular* if it has no fixed points).
- An *involution* is an element of order 2 in a group.
- A group G is said to *metacyclic* if it contains a normal cyclic subgroup C such that G/C is cyclic.
- A group G is said to be *minimal nonmetacyclic* if it is nonmetacyclic but all its proper subgroups are metacyclic.
- A subgroup A of a group G is said to be *soft*, if $C_G(A) = A$ and $|N_G(A) : A| = p$.
- A *section* of a group G is an epimorphic image of some subgroup of G.
- If $F = GF(p^n)$, then we usually write $GL(m, p^n)$, $SL(m, p^n)$, ... instead of $GL(m, F)$, $SL(m, F)$, ...
- $c_n(G)$ is the number of cyclic subgroups of order p^n in a p-group G.
- $c(G)$ is the number of non-identity cyclic subgroups of G.
- $s_n(G)$ is the number of subgroups of order p^n in a p-group G.
- $v_n(G)$ is the number of normal subgroups of order p^n in a p-group G.
- $e_n(G)$ is the number of subgroups of order p^n and exponent p in G.
- $M_p(m, n) = \langle a, b \mid o(a) = p^m,\ o(b) = p^n,\ a^b = a^{1+p^{m-1}}\rangle$ is the metacyclic minimal nonabelian group of order p^{m+n}.
- $M_p(m, n.1) = \langle a, b \mid o(a) = p^m,\ o(b) = p^n,\ [a, b] = c,\ [a, c] = [b, c] = c^p = 1\rangle$ is the nonmetacyclic minimal nonabelian group of order p^{m+n+1}.
- An $\mathcal{A}_n$-group is a p-group G all of whose subgroups of index p^n are abelian but G contains a nonabelian subgroup of index p^{n-1}. In particular, $\mathcal{A}_1$-group is a minimal nonabelian p-group for some p.
- $\alpha_n(G)$ is the number of $\mathcal{A}_n$-subgroups in a p-group G.
- $\mathcal{D}_n$-group is a 2-group all of whose subgroups of index p^n are Dedekindian, containing a non-Dedekindian subgroup of index p^{n-1} and which is not an $\mathcal{A}_n$-group.
- $\mathcal{MA}(G)$ is the set of minimal nonabelian subgroups of a p-group G.
- $\mathcal{MA}_k(G) = \{H \in \mathcal{MA}(G) \mid \Omega_k(H) = H\}$.
- $D_k(G) = \langle \mathcal{MA}_k(G)\rangle = \langle H \mid H \in \mathcal{MA}_k(G)\rangle$.
- $L_n = |\{x \in G \mid x^n = 1\}|$.
- If G is a metacyclic p-group and $w(G) = \max\{i \mid |\Omega_i(G)| = p^{2^i}\}$, then $R(G) = \Omega_{w(G)}(G)$. In that case, $G/R(G)$ is either cyclic or a 2-group of maximal class.

Characters and representations

- $\mathrm{Irr}(G)$ is the set of all *irreducible* characters of G over complex numbers.
- A character of degree 1 is said to be *linear*.
- $\mathrm{Lin}(G)$ is the set of all *linear* characters of G (obviously, $\mathrm{Lin}(G) \subseteq \mathrm{Irr}(G)$).

- $\mathrm{Irr}_1(G) = \mathrm{Irr}(G) - \mathrm{Lin}(G)$ is the set of all *nonlinear* irreducible characters of G; $\mathrm{n}(G) = |\mathrm{Irr}_1(G)|$.
- $\chi(1)$ is the *degree* of a character χ of G.
- χ_H is the *restriction* of a character χ of G to $H \leq G$.
- χ^G is the character of G induced from the character χ of some subgroup of G.
- $\bar{\chi}$ is a character of G defined as follows: $\bar{\chi}(x) = \overline{\chi(x)}$ (here $\bar{w}$ is the complex conjugate of a complex number w).
- $\mathrm{X}(G)$ is a character table of G.
- $\mathrm{Irr}(\chi)$ is the set of irreducible constituents of a character χ of G.
- 1_G is the principal character of G.
- $\mathrm{Irr}^{\#}(G) = \mathrm{Irr}(G) - \{1_G\}$.
- If χ is a character of G, then $\ker(\chi) = \{x \in G \mid \chi(x) = \chi(1)\}$ is the *kernel* of a character χ.
- $\mathrm{Z}(\chi) = \{x \in G \mid |\chi(x)| = \chi(1)\}$ is the *quasikernel* of χ.
- If N is normal in G, then $\mathrm{Irr}(G \mid N) = \{\chi \in \mathrm{Irr}(G) \mid N \nleq \ker(\chi)\}$.
- $\langle \chi, \tau \rangle = |G|^{-1} \sum_{x \in G} \chi(x)\tau(x^{-1})$ is the *inner product* of characters χ and τ of G.
- $\mathrm{I}_G(\phi) = \langle x \in G \mid \phi^x = \phi \rangle$ is the *inertia subgroup* of $\phi \in \mathrm{Irr}(H)$ in G, where $H \lhd G$.
- 1_G is the *principal character* of G ($1_G(x) = 1$ for all $x \in G$).
- $\mathrm{M}(G)$ is the *Schur multiplier* of G.
- $\mathrm{cd}(G) = \{\chi(1) \mid \chi \in \mathrm{Irr}(G)\}$.
- $\mathrm{mc}(G) = \mathrm{k}(G)/|G|$ is the *measure of commutativity* of G.
- $\mathrm{T}(G) = \sum_{\chi \in \mathrm{Irr}(G)} \chi(1)$.
- $\mathrm{f}(G) = \mathrm{T}(G)/|G|$.

Preface

This is the fifth volume of the series devoted to elementary parts of finite p-group theory. The material presented here has appeared in a book form for the first time.

Below we list some new characterizations of certain classes of p-groups and results presented in this volume:

(1) classification of non-Dedekindian groups in which the normal closures of nonnormal cyclic subgroups are nonabelian (abelian),
(2) (i) p-groups with all subgroups isomorphic to quotient groups (s-self-dual groups), (ii) minimal non-s-self p-groups,
(3) p-groups all of whose maximal subgroups, except one, are s-self-dual,
(4) nonabelian p-groups all of whose subgroups are q-self-dual,
(5) p-groups with all subgroups isomorphic to quotient groups (q-self-dual groups),
(6) minimal non-q-self-dual 2-groups,
(7) p-groups with absolutely regular normalizer of some subgroup,
(8) p-groups all of whose $\mathcal{A}_2$-subgroups are metacyclic,
(9) p-groups all of whose $\mathcal{A}_1$-subgroups are pairwise nonisomorphic,
(10) metacyclic groups of exponent p^e with a normal cyclic subgroup of order p^e,
(11) another proof of Baer's theorem about 2-groups with nonabelian norm,
(12) properties of metahamiltonian p-groups,
(13) p-groups all of whose nonnormal subgroups are elementary abelian,
(14) large elementary abelian subgroups of a p-group of maximal class,
(15) p-groups all of whose nonabelian two-generator subgroups are minimal nonabelian,
(16) p-groups all of whose three pairwise noncommuting elements generate a p-group of maximal class,
(17) § 190 p-groups containing a subgroup of maximal class and index p,
(18) groups G such that $H < G$ and $|H|^2 < |G|$ imply that $H \lhd G$,
(19) nonabelian p-groups in which any nonabelian subgroups contains its centralizer,
(21) groups in which the normal closure of any cyclic subgroup is abelian,
(22) nonabelian p-groups with an abelian subgroup of index p covered by minimal nonabelian subgroups,
(23) non-Dedekindian p-groups which are not generated by their nonnormal subgroups,
(24) certain groups cannot be normal subgroups of capable p-groups,
(25) nonabelian p-groups, $p > 2$, of exponent p^e all of whose cyclic subgroups of order p^e are normal,
(26) non-Dedekindian p-groups in which the normal closures of nonnormal cyclic subgroups have cyclic centers,

(27) groups all of whose minimal nonabelian (maximal abelian) subgroups are isolated, p-groups saturated by isolated subgroups,
(28) p-groups whose the proper Hughes subgroup have Frattini subgroup of order p,
(29) computation of orders of some 2-groups all of whose subgroups of given index are Dedekindian,
(30) an estimate of the number of minimal nonabelian subgroups in a group with a given quotient group G/A, where A is maximal abelian normal subgroup of G,
(31) the number of epimorphic images of maximal class and order in a 2-group,
(32) the number of cyclic subgroups of given order in a metacyclic p-group,
(33) the estimate of the number of minimal nonabelian subgroups in a p-group,
(34) p-groups G all of whose nonnormal subgroups contain G' in its normal closure, etc.

For further information, see Contents.

The problems from section "Research problems and themes," Parts V, are posed, if it is not stated otherwise, by the first author. That section contains more than 570 items some of which are solved by the second author. Many problems from the lists in previous three volumes were solved mainly by the second author and mathematicians from Shanxi Normal University.

There are, in the text, many hundreds exercises most of which are solved (these exercises, if it is not stated otherwise, written by the first author).

Sections and appendices written by the authors, are listed in the Author Index. Avinoam Mann wrote Appendix 65, Qinhai Zhang wrote § 205. In § 204 we have used papers of Robert van der Waall. Some sections were discussed with Isaacs and Mann. The problems were discussed with Mann and, of course, with the second author. M. Roitman (Haifa University) helped in whole project. We are indebted to all named mathematicians.

We added in the Bibliography of volume III those items that are cited in this volume and omitted from the so-obtained Bibliography those items that are not cited here.

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