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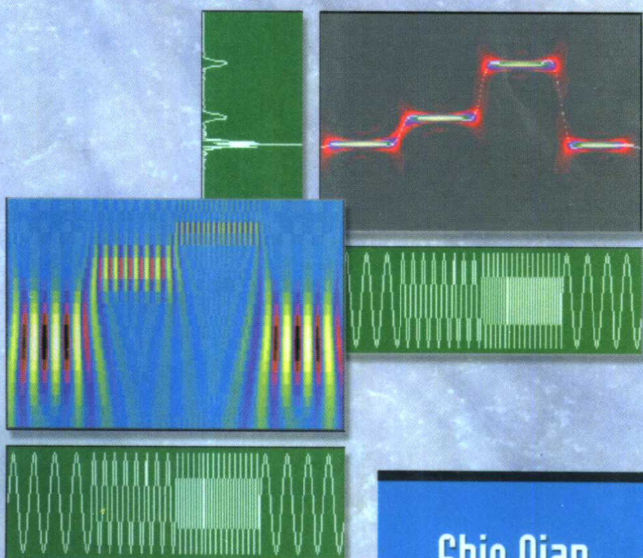
（英文版）

Introduction to

Time-Frequency and Wavelet Transforms



Shie Qian

（美）钱世锷 著

机械工业出版社
China Machine Press

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Preface

For a long time, I wondered if the recently popularized time-frequency and wavelet transforms were merely academic exercises. Do applied engineers and scientists really need signal processing tools other than the FFT? After 10 years of working with engineers and scientists from a wide variety of disciplines, I have finally come to the conclusion that, so far, neither the time-frequency nor wavelet transform appear to have had the revolutionary impact upon physics and pure mathematics that the Fourier transform has had. Nevertheless, they can be used to solve many real-world problems that the classical Fourier transform cannot.

As James Kaiser once said, "*The most widely used signal processing tool is the FFT; the most widely misused signal processing tool is also the FFT.*" Fourier transform-based techniques are effective as long as the frequency contents of the signal do not change with time. However, when the frequency contents of the data samples evolve over an observation period, time-frequency or wavelet transforms should be considered. Specifically, the time-frequency transform is suited for signals with slow frequency changes (narrow instantaneous bandwidth), such as sounds heard during an engine run-up or run-down, whereas the wavelet transform is suited for signals with rapid changes (wide instantaneous frequency bandwidth), such as sounds associated with engine knocking. The success of applications of the time-frequency and wavelet transforms largely hinges on understanding their fundamentals. It is the goal of this book to provide a brief introduction to time-frequency and wavelet transforms for those engineers and scientists who want to use these techniques in their applications, and for students who are new to these topics.

Keeping this goal in mind, I have included the two related subjects, time-frequency and wavelet transforms, under a single cover so that readers can grasp the necessary information and come up to speed in a short time. Professors can cover these topics in a single semester. The coexistence of the time-frequency and wavelet approaches in one book, I believe, will help comparative understanding and make complementary use easier.

This book can be viewed in two parts. While Chapters Two through Six focus on linear transforms, mainly the Gabor expansion and the wavelet transform, Chapters Seven through Nine are dedicated to bilinear time-frequency representations. Chapter Ten can be thought of as a combination of time-frequency and time-scale (that is, wavelets) decomposition. The presentation of the wavelet transform in this book is aimed at readers who need to know only the basics and perhaps apply these new techniques to solve problems with existing commercial software. It may not be sufficient for academic researchers interested in creating their own set of basic functions by techniques other than the elementary filter banks introduced here.

All chapters start with the discussion of basic concepts and motivation, then provide theoretical analysis and, finally, numerical implementation. Most algorithms introduced in this book are a part of the software package, Signal Processing Toolset, a National Instruments product. Visit `www.ni.com` for more information about this software.

This book is neither a research monograph nor an encyclopedia, and the materials presented here are believed to be the most basic fundamentals of time-frequency and wavelet analysis. Many theoretically excellent results, which are not practical for digital implementation, have been omitted. The contents of this book should provide a strong foundation for the time-frequency and wavelet analysis neophyte, as well as a good review tutorial for the more experienced signal-processing reader.

I wrote this book to appeal to the reader's intuition rather than to rely on abstract mathematical equations and wanted the material to be easily understood by a reader with an engineering or science undergraduate education. To achieve this, mathematical rigor and lengthy derivation have been sacrificed in many places. Hopefully, this style will not unduly offend purists.

On the other hand, "*Formulas were not invented simply as weapons of intimidation*" [22]. In many cases, mathematical language, I feel, is much more effective than plain English. Words are sometimes clumsy and ambiguous. For me, it is always a joy to refresh my knowledge of what I learned in school but have not used since.

Some of the material presented in this book is the result of collaborative work which so greatly profited from the contributions of friends and colleagues that I must mention them. It was my graduate advisor, Professor Joel M. Morris, who led me into such a fascinating field. Motivated by the suppression of cross-term interference, in the early 90s the idea of the decomposition of the Wigner-Ville distribution emerged, which led to a series of interesting results. With Shidong Li and Kai Chen, the relationship of the most similar dual and the pseudo inverse was discovered. Based on Wexler and Raz's periodic discrete Gabor expansion [225], Dapang Chen and I obtained its infinite counterpart which resulted in an interesting time-dependent spectrum, the time-frequency distribution series, also known as the Gabor spectrogram. To improve the time-frequency resolution, we also proposed the adaptive Gabor expansion which turned out to be the same scheme as that employed by the matching pursuit method independently developed by Stéphane Mallat and Zhifeng Zhang during the same time period [172]. With Qinye Yin, such an adaptive decomposition scheme was generalized into the Gaussian chirplet cases. The fast-refinement algorithm initially appeared when Qinye Yin visited Austin, Texas. As a result of his insightful observation, the computation of the adaptive Gaussian chirplet approximation has been significantly improved. All these years later, I clearly remember a discussion at Xiang-Geng Xia's office in Malibu, California, in front of the magnificent beach there. The subject was the Gabor expansion-based time-varying filter. As a result of that discussion, a few days later Xiang-Geng called me and said, "With a tight frame, the iteration of the time-varying filter converges!" That memory is indelible.

I would also like to thank Professors Xiang-Geng Xia and Richard G. Baraniuk for their contributions in Chapter 5 and Section 8.3, respectively.

In a larger sense, this book is the result of the enthusiasm and support from numerous customers, colleagues, and friends. I want to take this opportunity to express my sincere thanks to all of them. Particularly, I would like to thank Dr. James Truchard and Jeff Kodosky, the founders of National Instruments Corporation. It is their great enthusiasm and continuous support that keep such a "non-profitable" project evolving and making all those interesting applications take place.

This book has been an on-off project for almost three years and I extend my thanks to Bernard Goodwin at Prentice Hall for his endless patience and generous assistance. There were so many errors in the original draft that I dare not look at it again. Mahesh Chugani carefully read the entire manuscript. His numerous comments and suggestions improved the book significantly.

My deepest thanks are reserved for my mother, Yuzhen Wu, and my family: my wife, Jun, and daughter, Nancy. I am very grateful for their understanding, support, and patience during this formidable project.

錢世鍔
Shie Qian
Austin, Texas

Contents

CHAPTER 1

Introduction

*D*ue to physical limitations, usually we are only able to study a system through signals associated with the system rather than physically opening up the system. For example, physicists and chemists use the spectrum generated by the prism, without breaking up molecules, to distinguish different types of matter.[1] Astronomers apply spectra as well as the Doppler effect, discovered by an Austrian physicist *Christian Johann Doppler* in 1842, to determine distances to planets that a human being may never be able to reach. Doctors utilize the electrocardiograph (ECG), without opening up the body, to trace the electrical activity of the heart and diagnose whether or not a patient suffers from heart problems. Indeed, signal processing has played a fundamental role in the history of civilization. Prior to World War II, however, signal processing was primarily a part of physics. Signals that scientists and engineers dealt with were mainly analog by nature. It was the *sampling theorem*, proved by the mathematician *J. Whittaker* in 1935 [229] and applied to communication by *Claude Shannon* in 1949 [46], that led to a new era of signal processing. Modern signal processing can be thought of as the combination of physics as well as statistics. Because of the discovery of the sampling theorem and the advance of the digital computer over the last couple of decades, we are now able to employ elegant mathematical approaches, such as the virtual prism — Fourier transform, to process all different kinds of signals that our ancestors never would have been able to imagine. Applications of modern signal processing range from the control of the *Mars Pathfinder* more than twenty million miles away to the discovery of abnormal cells inside the body.

1. Spectrum analysis was jointly discovered by the German chemist *Robert Wilhelm Bunsen* (1811 - 1899) and the German physicist *Gustar Robert Kirchhoff* (1824 - 1887). Contrary to popular belief, Bunsen had little to do with the invention of the Bunsen burner, a gas burner used in scientific laboratories. Although *Bunsen* popularized the device, credit for its design should go to the British chemist and physicist *Michael Faraday* (1791 - 1867).

A fundamental mathematical tool employed in signal processing is a *transform*. When we are asked to multiply the Roman numerals LXIV and XXXII, only a few of us will be able to give the correct answer right away. However, if the Roman numerals are first translated into Arabic numerals, 64 and 32, then all of us can get 2048 immediately. The process of converting the unfamiliar Roman numerals into common Arabic numerals is a typical example of transforms [22]. By properly applying transforms, we can simplify calculations or make certain attributes of the signal explicit.

One of the most popular transforms known to scientists and engineers is the Fourier transform that converts a signal from the time domain to the frequency domain. Two hundred years ago, during the study of heat propagation and diffusion, *Jean Baptiste Joseph Fourier* found a series of harmonically related sinusoids to be useful in representing the temperature distribution through a body. The method of computing the weight of each sinusoidal function is now known as the *Fourier transform*. The Fourier transform can not only benefit the study of heat distribution, but is also extremely useful for many other mathematical operations, such as solutions to differential equations. The application of the Fourier transform with which scientists and engineers are most familiar may be convolution theory. By applying the Fourier transform, one can convert time-consuming convolutions into more efficient multiplications.

In fact, the Fourier transform is not simply a mathematical trick to make calculations easier; it also acts as a mathematical *prism* to beak down a signal into a group of waveforms (different frequencies), as a prism breaks up light into a color spectrum. With the help of the Fourier transform, we can interpret radiation from distant galaxies, diagnose a developing fetus, and make inexpensive cellular phone calls. With the establishment of quantum mechanics, the significance of Fourier's discovery becomes even more obvious. By using the Fourier transform, for instance, we can quantitatively describe a fundamental and inescapable property of the world – the *Heisenberg uncertainty principle*. That is, in certain pairs of quantities, such as the position and velocity of a particle, cannot both be predicted with complete accuracy.

The Fourier transform is so powerful that people tend to apply it everywhere without noticing one fundamental difference between the mathematical prism and a real prism. The spectrum produced by the prism in the morning is different from that in the evening. Using a fancy word, we may say that the prism gives *instantaneous spectra*. Using a prism to examine spectra of light, there is no need for the information about light that existed a million years ago and the light that will be there tomorrow. However, this is not the case for the Fourier transform. To compute the Fourier transform, we not only need previous information, but also information that has not yet occurred. The spectrum computed by the Fourier transform is the spectrum averaged over an infinitely long time before the present to an infinitely long time after the present!

Figure 1-1 illustrates two linear chirp signals. Each is a time reversed version of the other. While frequencies of the signal on the left plot increase with time, frequencies of the signal on the right decrease with time. Although the frequency behavior of the two signals is obviously different, their frequency spectra computed by the Fourier transform, as shown in Figure 1-2, are identical. The Fourier transform preserves all information about the time waveform (if it did not, we could not reconstruct the signal from the transform), but information about time or space is buried deep within the phases, which is beyond our comprehension.

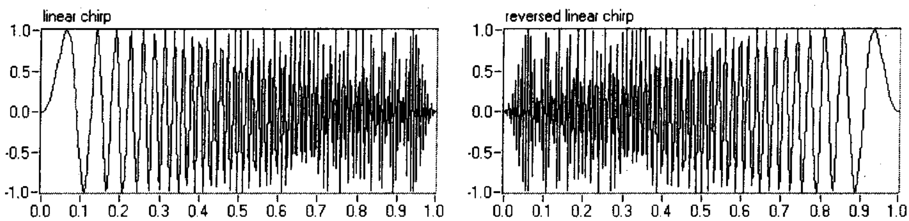

Figure 1-1 The linear chirp signal in the plot on the right is a time reversed version of the signal in the plot on the left. While frequencies of the chirp signal on the left increase with time, frequencies of the signal on the right decrease with time.

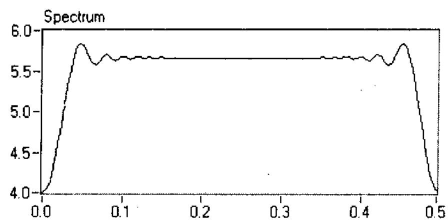

Figure 1-2 Although the two linear chirp signals in Figure 1-1 have completely different time waveforms, their frequency spectra are identical. The Fourier transform preserves all information about the time waveform (if it did not, we could not reconstruct the signal from the transform), but information about time or space is buried deep within the phases, which is beyond our comprehension.

Figure 1-3 depicts the spectrum of an engine sound (the corresponding time waveform is illustrated in the top plot of Figure 1-4). When listening to this signal, we can clearly identify several knocking sounds caused by out of phase firing inside the engine. As indicated by the wavelet transform, the second plot in Figure 1-4, the knocking sound is actually quite strong. To compute the Fourier transform, we have to include the signal before knocking takes place and also the signal after the knocking ends. What the spectrum computed by using the Fourier transform tells us are the frequencies contained in the entire time waveform, not the frequencies at a particular time instant. The Fourier transform provides the signal's average characteristics. Although the amplitude of engine knock sounds could be rather large in a very short time period, the energy of the sound, compared to the entire background noise, is negligible. Consequently, there will be no obvious signatures in the spectrum to show the presence of engine knocking. The Fourier transform smears the signal's local behavior globally. "The Fourier transform is poorly suited to very brief signals, or signals that change suddenly and unpredictably; yet in signal processing, brief changes often carry the most interesting information" [22].

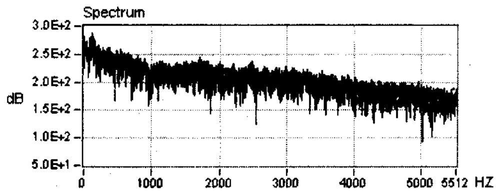

Figure 1-3 Because the energy of the engine knocking sound is relatively small, the presence of engine knocks is completely overwhelmed in the averaged spectra computed by the Fourier transform.

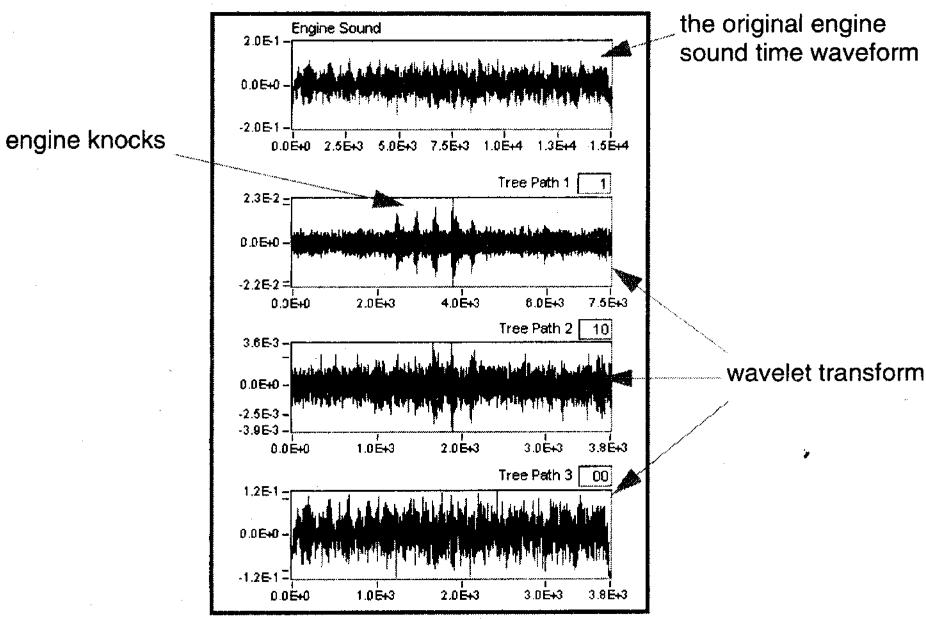

Figure 1-4 While engine knock sounds are completely concealed in the background noise in the time waveform plot (top), the wavelet transform (second from the top) clearly indicates the existence of engine defects.

Although most natural spectra are time dependent (for example, the light during the morning is different than that during the evening), the Fourier transform makes "changing frequency" unthinkable. As Gabor wrote, "even experts could not at times conceal an uneasy feeling when it came to the physical interpretation of results obtained by the Fourier method" [113].

The shortcoming of the Fourier transform has been recognized for a long time. The development of Fourier's alternatives, involving a great many individuals, started at least a half century ago. The first two important articles, dealing with the limitation of the Fourier transform, appeared right after World War II: one by *Dennis Gabor* in 1946 (who later received the Nobel Prize for the invention of holography) [113] and the other by *J. Ville* in 1948 [219]. Since the result obtained by Ville was similar to the one introduced by *Eugene Wigner* (who received the Nobel Prize for the discoveries concerning the theory of the atomic nucleus and elementary particles) in the area of quantum mechanics in 1932 [226], traditionally Ville's method is named the *Wigner-Ville distribution*.

The initial reaction to neither Gabor nor Ville's work was enthusiastic. The difficulty associated with the *Gabor expansion* was that the sets of elementary functions that are suitable for time-frequency analysis in general do not form orthogonal bases. The problem with the Wigner-Ville distribution has been the co-called cross-term interference that makes the resulting presentation difficult to be interpreted. It was two sets of papers, *Claasen* and *Mecklenbräuker* [91] and *Bastiaans* [72], which both appeared in the early 1980's, that triggered great interest in revisiting Gabor and Ville's pioneer work. Since then, there has been a tremendous amount of activities and numerous developments. Some of them appear to be reaching a level of maturity for real applications.

The recognition of the wavelets transform is much more recent, though a similar methodology can be traced as early as the beginning of the twentieth century [305]. *Wavelets* are not a "bright new idea" but concepts that have existed in other forms in many different fields. For instance, the numerical implementation of the wavelet transform is nothing more than the well-established filter banks. As *Stéphane Mallat* wrote, "This wavelet theory is truly the result of a dialogue between scientists who often met by chance, and were ready to listen... this is a particularly sensitive task (mentioning who did what), risking aggressive replies from forgotten scientific tribes" [27]. "Tracing the history of wavelets is almost a job for an archaeologist" [22].

The set of basis functions employed by Fourier, sine and cosine functions, not only is the mathematical model of the most fundamental natural phenomena – the *wave*, but also is a solution of differential equations.[2] Unfortunately, this is not the case for time-frequency and wavelet transforms. Neither time-frequency nor wavelet transforms will likely have the revolutionary impact upon science and engineering that the Fourier transform has had. However, the time-frequency and wavelet transforms do offer many interesting features that the Fourier transform does not possess.

In addition to detecting engine knocks, for example, the wavelet transform is also successfully used for train wheel diagnosis. It has been found that one of the main causes of train accidents was the result of defective wheels and bearings. Hence, on-line train wheel and bearing diagnoses are exceptionally important for avoiding potential catastrophes. The parameter that engineers believe can be used to effectively detect hidden flaws in wheels and bearings is variations of railroad track. Defective wheels and bearings usually will generate an impulse like noise

2. It would be hard to exaggerate the significance of the differential equation to the history of civilization. It was the differential equation that enabled *Sir Issac Newton* to use the current information, such as position, velocity, and acceleration, to predict the future. Newton's discovery led the French mathematician and astronomer *Pierre Simon Laplace* to believe, "Nothing would be uncertain to it, and the future, like the past, would be present before its eyes." Because of such successful application of the differential equation, Laplace imagined a single formula that would describe the motion of every object in the universe for all time.

as the train moves on the track, making abnormal track variations. Such noise could be effectively filtered out by the wavelet transform.

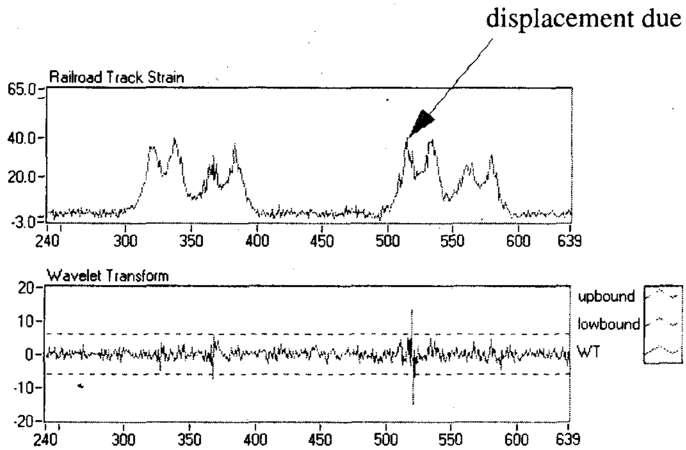

Figure 1-5 Displacement of the railroad track during the time that eight wheels pass over a strain gauge, and the corresponding wavelet transform. There is no clear signature between the normal and abnormal wheels in the time waveform (the upper plot). However, in the wavelet transform domain (the lower plot), we can readily identify a potential problem at the fifth wheel (between $x = 500$ and $x = 550$).

Figure 1-5 illustrates a typical train wheel on-line testing result. When a wheel is far away from the strain gauge mounted beneath the track, the corresponding track displacement is small. It increases as the train wheel approaches the strain gauge. The displacement reaches a maximum when a wheel is right above the strain gauge. The plot on the top of Figure 1-5 shows the displacement history during the time that eight wheels pass the strain gauge. While the X-axis describes the time index, the Y-axis indicates the magnitude of track displacement. Each bump corresponds to one wheel passing over the strain gauge mounted underneath the railroad track. Obviously, there is no clear signature between the normal and abnormal wheels in the time waveform. However, in the wavelet transform domain, the plot on the bottom of Figure 1-5, we can readily identify a potential problem at the fifth wheel (between $x = 500$ and $x = 550$). The wavelet transform-based on-line diagnosis system is expected to substantially reduce potential train accidents caused by defective wheels or bearings.

Another interesting application of the wavelet transform is for detecting oil leakage. One of the most challenging tasks in an oil field is on-line pipeline leakage monitoring. This is particularly true concerning incidents directly caused by organized oil theft. This is not only an economic loss for the oil company, but also environmental pollution, a public issue.

When a leakage incident occurs, the oil pressure in the vicinity of the leakage point drops rapidly. Such a drop is presumably propagated in all directions along the pipeline. Consequently:

1. Oil pressure decreases at both the inlet and outlet
2. The oil flow rate at the outlet decreases, while the oil flow rate at the inlet increases

Based on the time difference of the pressure drops observed at the inlet and outlet, conceptually, the leakage location can then be determined by

$$\frac{length\ of\ pileline + pressure\ wave\ velocity \times time\ difference}{2} \tag{1.1}$$

Figure 1-6 depicts the layout of the pressure and flow meters. Note that when both conditions 1 and 2 mentioned previously are simultaneously satisfied, all other combinations can be excluded from the leakage. For instance, the decrease of pressure and flow rate at both ends can be considered as the result of the inlet pump slowing down. Conversely, increased pressure and flow rate indicates the pump is speeding up.

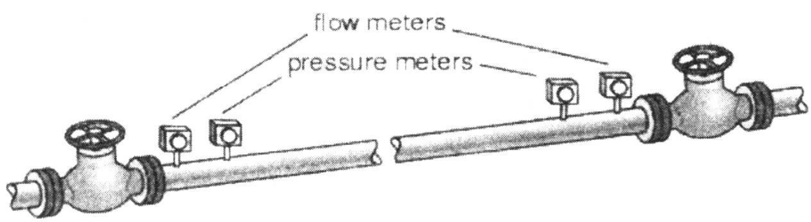

Figure 1-6 Oil leakage will cause pressures at both the outlet and inlet to decrease, the flow rate at the outlet to decrease, and the flow rate at the inlet to increase.

It is said that one invents with intuition and one proves with logic. This is certainly true in this application. The idea is straightforward but the implementation is very challenging. The main difficulties include:

1. Synchronization of all pressure and flow meters that typically are 60 km apart.
2. Variation of pressure wave velocity. The pressure wave velocity is related to temperature, the density of the medium, as well as the elasticity of the pipe material. To facilitate oil movement, the raw oil is often heated at each station, especially in cold weather. Due to the non-uniform temperature distribution, the pressure wave velocity is not constant. Consequently, the actual formula for estimating the location of the leakage is much more involved than that which one may anticipate (e.g., Eq. (1.1)).
3. Background noise. Compared to the main oil flow, the leakage usually is negligible. Therefore, the rapid changes of pressure caused by incidents of leakage often are not noticed.

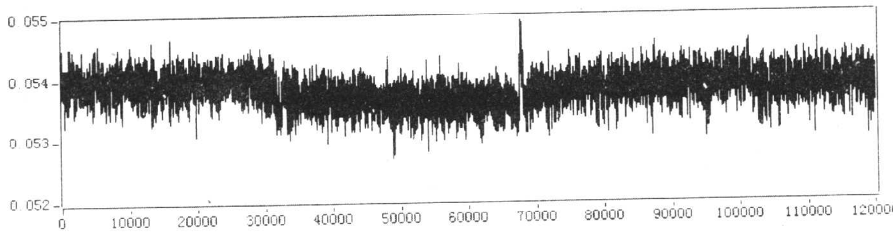

Figure 1-7 A typical pressure signal. Although there is a drop caused by a leakage in the vicinity of 31,000, there is no obvious indication in the time waveform (Data provided by Zhuang Li, College of Engineering, Tianjin University, China).

Figure 1-7 illustrates a typical oil pressure signal. Although there is a drop caused by leakage in the vicinity of 31,000, there is no obvious indication in the time waveform. However, by applying the wavelet transform,[3] one can accurately determine the time instant of the pressure drop.

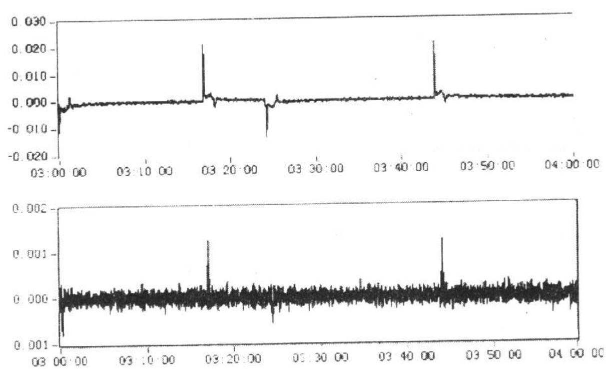

Figure 1-8 The upper plot is the wavelet transform of the signal at the pipeline inlet, whereas the lower plot shows the wavelet transform of the signal at the pipeline outlet. There were two incidents of leakage from the pipeline. The first one occurred between 3:00 and 3:18 am and the second was between 3:24 and 3:44 am (Data provided by Zhuang Li, College of Engineering, Tianjin University, China).

Figure 1-8 depicts the wavelet transforms of signals recorded between 3:00 to 4:00 am on April 13, 2001. The upper plot shows the wavelet transform of the signal at the pipeline inlet, whereas the lower plot is the wavelet transform of the signal at the pipeline outlet. As indicated in the wavelet transform domain, there were two incidents of leakage from the pipeline. The first one occurred between 3:00 and 3:18 am and the second was between 3:24 and 3:44 am. For the

3. Due to the time accuracy required in this application, the wavelet transform must be time-shift invariant.