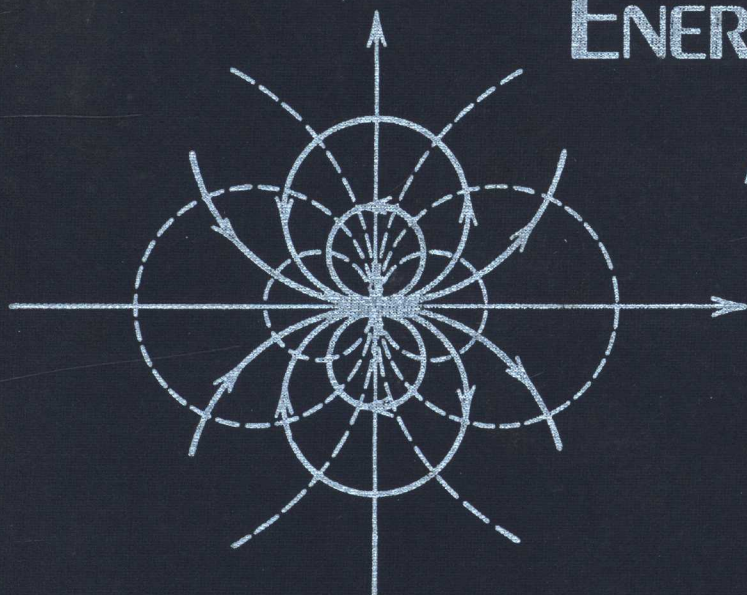


ELECTROMAGNETIC FIELDS, ENERGY, AND WAVES



LEONARD M. MAGID

ELECTROMAGNETIC FIELDS, ENERGY, AND WAVES

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Brooklyn, New York

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ELECTROMAGNETIC FIELDS, ENERGY, AND WAVES

TO
BRUCE, LISA
AND BARBARA

PREFACE

This book develops the fundamentals of electromagnetic field theory in a form designed to have sufficient physical (and mathematical) meaning to the average undergraduate in electrical engineering to motivate him towards further study and work in this area.

As originally planned, this book was to be a revision of some of the more basic material contained in two M.I.T. undergraduate electrical engineering textbooks by Professors Richard B. Adler, Lan Jen Chu, and Robert M. Fano.¹ What eventually developed, however, is a separate new book that offers a much simpler and more understandable presentation of electromagnetic field theory, with a greater emphasis placed on basic topics to make this book suitable for general use outside of M.I.T.

This book is similar to the previous ones in that it preserves their basic format. It begins with the basic postulates of electromagnetism and then proceeds to deduce, hand in hand, the physical and the mathematical properties of electromagnetic fields within the classical (nonrelativistic) range, from statics to radiation. Where it differs, however, is in bringing this approach down to the level of the average electrical engineering undergraduate. In this regard, I have combined the results of many long and fruitful discussions with one of the earlier authors (Professor L. J. Chu) with my own experiences in teaching this subject to all levels of students, both at M.I.T. and at Brooklyn Poly. This has led to an emphasis on the more fundamental concepts of fields and field theory, coupled with an extensive use of physically motivated illustrative examples to clarify all major theoretical points. On this latter point, I may be accused of being too verbose. To this I readily plead guilty. It is, however, all in the good cause of bringing the reader to a fuller understanding of and appreciation for the elegance and beauty of field theory.

Most of the subject matter in this book has been found by experience to be well suited to a standard two-term, three-credit undergraduate sequence in fields for either electrical engineers or physicists. The only prerequisites are the standard calculus and physics sequences typically taken by these students during their freshman and sophomore years. A course in vector analysis is specifically *not* required. All necessary concepts of vectors and vector analysis are developed within this text.

The specific details of the material presented are discussed extensively in the *Note to The Reader* that follows and need not be restated here. It is hoped

¹R. M. Fano, L. J. Chu, and R. B. Adler, *Electromagnetic Fields, Energy, and Forces*, John Wiley and Sons and The M.I.T. Press, New York and Cambridge, 1960; and R. B. Adler, L. J. Chu, and R. M. Fano, *Electromagnetic Energy Transmission and Radiation*, John Wiley and Sons and The M.I.T. Press, New York and Cambridge, 1960.

that that detailed outline will serve as a continuing guide or *roadmap*, enabling the reader to keep the proper perspective as he uses this book.

To achieve maximum flexibility, the very basic material on the general theory of electromagnetic fields is developed first, in the initial four chapters. The remaining eight chapters are devoted mainly to the development and illustration of the many diverse applications of electromagnetic fields, from statics to dynamics and radiation. Each of these chapters is also organized along this same principle. Basic concepts are developed first and are followed directly by several simple illustrated examples. The more advanced aspects of each chapter follow in turn in the order of their progressive difficulty. Accordingly, many of the more advanced sections appearing near the ends of Chapters Five to Twelve (particularly those sections marked with an arrow head ►) may safely be omitted without any loss in continuity.

Instructors who wish to emphasize waves and radiation early, before statics, can do so by beginning (immediately after the introductory chapter) with the discussion of the scalar waves on lossless transmission lines, as presented (in Chapter Ten) from the viewpoint of distributed circuits. This discussion should then be followed, in turn, by: The Basic Field Laws in Integral and Differential Form (Chapters Two to Four); Electromagnetic Power (Chapter Eight); T.E.M. Fields, Plane Waves, and Radiation (Chapters Ten to Twelve); and finally Static Fields, Fields in Matter, and Low-Frequency (or Quasi-Static) Fields (Chapters Five to Seven and Nine).

On the other hand, those able to devote only one term to a fields course could do so by covering the first four chapters of general theory and then concentrating on only the initial sections of the remaining chapters on the specific applications of fields. A listing of the particular sections that may safely be omitted under these circumstances is given at the end of the *Contents*.

I wish to express my profound indebtedness and thanks to Professors Richard B. Adler, Lan Jen Chu, and Robert M. Fano of the Electrical Engineering Department of M.I.T., both for their continued support and encouragement in this project and for their and their present publisher's (The M.I.T. Press) permission to draw freely on many of their concepts, ideas, and even, in a few cases, some well-chosen words from their abovementioned textbooks. I owe a special debt of gratitude to Professor Chu for the innumerable hours that he unselfishly devoted to assisting me in organizing this book. Thanks are also due to my many colleagues in the Department of Electrical Engineering and Electrophysics at the Polytechnic Institute of Brooklyn for their many helpful suggestions, and particularly to Professor Philip E. Serafim. Finally, I wish to also thank Myrna Singer, Esther Cytryn, Florence Schiff, and Anne Nattboy without whose cooperation and assistance this book could never have been completed.

L. M. Magid

Brooklyn, New York
March 1972

Note To The Reader

A detailed outline of the subjects covered in this text is presented below, with particular emphasis given to the location of each of the basic concepts as they appear within each chapter. This outline is designed as a guide or roadmap for use on a continuing basis, to give the reader a means of keeping the proper perspective as he travels through this book.

Following a brief introduction to fields and field theory in *Chapter One*, the basic postulates of electromagnetic field theory are presented and discussed in *Chapter Two*. There, *Maxwell's equations in integral form for free space* are taken as the net result of many years of experimental study on the behavior of electromagnetic fields. The electric and magnetic fields themselves are defined in terms of the *Lorentz Force law*, another of the basic postulates. After several examples of the application of the basic field laws to highly symmetric systems, it is found that a corresponding set of *differential field laws* would be more useful.

To facilitate the derivation of and the appreciation for the differential field laws, the basic properties of vectors and vector analysis are given in *Chapter Three*. Of particular importance here are the concepts of the spatial derivatives of scalar and vector fields in the form of the *gradient*, *divergence*, and *curl* operations.

The reduction of the integral fields laws into corresponding differential equations follows in *Chapter Four* from the basic curl and divergence definitions. It is found that the differential laws must be supplemented by *boundary conditions* along surfaces where sudden jumps in the basic field values cause some terms of the differential equations to become infinite (that is, singular). We find, however, that basic field laws, now in differential form, can be directly integrated only for the same restricted class of highly symmetric systems that were also amenable to solution using the integral laws. Thus, to be able to treat non-symmetric systems in particular, and to better understand the behavior of fields in general, we need a more detailed study of the basic field laws. Such a study, based on the field approach outlined in Section 1.2, begins in Chapter Five with the simplest form of the field laws, as found in static or time-independent systems.

Chapter Five begins our discussion of *static fields* in free space and uniform conductors. Starting with *electrostatic fields*, we are led to the concept of *electric potential* and thereby to *Poisson's* and *Laplace's equations*. At this

point, considerable time and effort is spent on the properties and general solutions of Laplace's equation in Cartesian coordinates. We soon find that it is just such general Laplacian solutions that serve as the key to a full understanding of static field behavior.

Chapter Six extends our discussion of static fields to include the solutions of Laplace's equation in *cylindrical* and *spherical* coordinates. We next show that *static magnetic fields* in current-free space are also governed by the solutions of Laplace's equation for *scalar magnetic potentials*. The concept of a *vector (magnetic) potential* is introduced to help in the derivation of magnetic fields in current-carrying regions, but only at a theoretical level. For practical purposes, the *Biot-Savart law* (derivable from the vector potential) and the scalar magnetic potential are found to be much simpler and more direct to apply. This chapter concludes with discussions on the *multipole expansion* and its use in leading to reasonable approximations to the fields far from arbitrary source distributions, and the *dipole-layer analog* that enables one to treat the magnetic fields surrounding solenoids in terms of a more readily visualized electric-charge analog.

Up to this point in the book, the study of the basic field laws and their solutions are limited to free space (that is, vacuum) and uniform conductors. *Chapter Seven* remedies this with a discussion of the *macroscopic* behavior of electromagnetic fields in *dielectric* and *magnetic material*. Dielectric materials, treated first, are considered in terms of the *polarization density* representing an equivalent number of dipoles induced per unit volume. The basic field laws in free space are then modified to apply to fields in matter. In particular, it is *postulated* that the *sole effects of polarized matter on the field laws is the addition of the polarization charges and currents to the free-charge and current sources already present in the basic field laws*. In analogous fashion, magnetism is studied in terms of an equivalent number of magnetic dipoles per unit volume (that is, the *magnetization density*). We now find, however, that there are two equally valid *mathematical* models by which we can introduce the additional magnetic sources into Maxwell's equations: the *magnetic charge-model* and the *Amperian-current model*. Both models are developed and their differences analyzed. Although the magnetic-charge model lacks a physical basis (that is, such charges have never been observed), nevertheless it yields perfectly valid results in *free space*, and completely consistent results within matter. As a purely mathematical model, moreover, it also offers the advantages of a complete analogy to electric-charge systems, often allowing one to find many magnetic-field solutions almost by inspection. It is in good part for this latter reason, in fact, that we introduce it here. The *constituent relations* relating the effects of the applied fields on matter are also discussed, and the concepts of *permittivity* and *permeability* are developed in detail.

Chapter Eight serves to bridge our study of static and time-varying fields by developing the concept of *electromagnetic power*. *Poynting's theorem*, giving

the energy balance in a general time-varying electromagnetic system, is derived from the Lorentz Force law and Maxwell's equations. The consequences of the resulting *electromagnetic energy and power expressions* are developed and illustrated in several static field examples.

Chapter Nine formally begins our investigation *time-varying* fields. The time-derivative terms that were omitted from the basic field laws in our study of static systems are now included. The direct coupling that results between time-varying electric and magnetic fields offers the new complication here. But, by limiting our study in this chapter to *slowly-changing fields only*, we are able to minimize this complication and make a smooth transition from purely static to *low-frequency* (or *quasi-static*) systems. The two significant features of quasi-statics that result are: (1) that *time-varying fields are, in general, derivable from the simpler static solutions* (and thus smoothly and continuously related to them), and (2) that *quasi-static (or low-frequency) field theory includes all of lumped circuit theory*. The development of parasitic elements in circuits and the eventual formation of distributed circuits at higher frequencies are also found to result directly from field theory, but at frequencies beyond the low-frequency limit of quasi-statics. A formal *power series approach* to the derivation of electromagnetic fields in terms of the frequency ($\omega = 2\pi f$) is developed to handle just such problems. This is an area that is becoming more significant these days with the increasing demand for more basic and more innovative circuit designs for integrated circuitry.

Our study of *electromagnetic waves* formally begins in *Chapter Ten* with the development of *Transverse Electromagnetic (TEM) Waves* in lossless media. The TEM fields are found to be separable into a purely *transverse, static vector component* that varies *only* with the transverse (x, y) coordinates and *not* with z or t , and a completely independent *scalar-wave component* that depends *solely* on z and t . With the transverse vector components of each TEM field governed by familiar *static* field laws, only the latter scalar-wave dependence on z and t is new. We soon find that the structures capable of supporting TEM fields all belong to the general class of *perfectly-conducting transmission line systems* and that the scalar-wave components of TEM fields are simply the *voltage and current waves* that propagate on such lines. The development of *lossless transmission line theory* at this point offers a simple introduction to *traveling and standing waves* (in both the time and frequency domains) and their *reflection and transmission* properties (in the frequency domain) that serves as the basis of our study of waves throughout the remainder of this text.

Our study of electromagnetic waves continues in *Chapter Eleven* with the development of *uniform plane waves* in lossless media. As in the case of static fields, general wave solutions are developed first, followed by examples that result from various boundary reflection and transmission conditions, both for *normal and oblique incidence* of the uniform plane waves. The *reflection and transmission (or refraction)* of plane waves at oblique incidences to plane

boundaries leads us to the discovery of the TE and TM *nonuniform* (or *slow*) plane waves in lossless media. A more direct study of the properties of *non-uniform* (and *uniform*) plane waves is also given in terms of the most general solutions of Maxwell's equations having *linear-exponential spatial dependence*. This chapter concludes with a derivation of the general $TE_{m,n}$ and $TM_{m,n}$ modes found in *lossless rectangular wave guides*. The wave-guide fields are shown first to result from the multiple reflections of plane waves off of the walls of the guide. A second, more direct, approach to the determination of rectangular wave-guide modes is also given, based on the *general solutions of Maxwell's equations* in a form that is *almost identical to that used earlier* (in Chapter Five) *to present the static-field solutions of Laplace's equation in rectangular coordinates*. Once again, then, we see the inherent similarity present in *all* solutions of Maxwell's equations. A brief discussion of the behavior of uniform plane waves in *lossy (dissipative) media* is given, for completeness, in *Appendix Four* (at the end of this book).

Chapter Twelve concludes this text with a study of *radiation fields* and the properties of *physical dipole antennas*. The radiation field of a dipole is derived from the concept of *retarded potentials* and the effects of the delay time that arise from the finite velocity of wave propagation. At this point, we now come full circle when we realize that the very same *scalar electric* and *vector magnetic potentials* used earlier in our development of *static field theory* now return to assist us in the analysis of the fully-dynamic radiation fields, and that the low-frequency (and d.c.) behavior of the *dipole radiation field* reduce to the *quasi-static* (and *static*) fields with which we began our study of electromagnetism. Finally a brief discussion of *dipole arrays* and their *radiation and diffraction* characteristics concludes this chapter and our study of classical electromagnetic field theory.

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