



LUI LAM *EDITOR*

INTRODUCTION TO
NONLINEAR
PHYSICS

非线性物理学导论



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A complex system (Charlene Lam, 1987).

Lui Lam
Editor

Introduction to Nonlinear Physics

With 264 Figures



Springer

Preface

A revolution occurred quietly in the development of physics—or, more accurately, of science—in the last three decades. The revolution touches upon *every* discipline in both the natural and social sciences. We are referring to the birth of a new science—nonlinear science—which, for the sake of presentation, may be divided into six parts: fractals, chaos, pattern formation, solitons, cellular automata, and complex systems.

Yet, in spite of all the excitement about this new science, there is not a single textbook covering all these topics. To remedy this situation and in view of the diversity of the subject, a number of pioneers and experts were invited to write about their own fields of research. The result is a textbook intended for advanced undergraduates and graduate students, which is also suitable for self-study. The materials contained in this book have been test taught in classrooms in universities, and in summer and winter schools. Examples and homework problems are included in most chapters.

Emphasis is placed on fractals, chaos, pattern formation, and solitons, which form Parts I to IV in the book. Special topics, including cellular automata, turbulence, and complex systems, are grouped in Part V. Part I to Part IV can be studied independently of each other, whereas Part V can be ignored in a first reading, except that Chapter 15 is a useful supplement to Part III.

Although most of the applications in this book are taken from examples in the physical sciences, the general principles and theories expounded are definitely applicable to other branches of science. The book is thus of use to students and researchers not just in physics but also in, for example, chemistry, biology, astronomy, meteorology, geology, mathematics, computer science, engineering, medicine, economics, and ecology. The multidisciplinary nature of nonlinear science makes it the ideal course for broadening the perspective and education of students.

I am grateful to the contributors and the publisher for their professional skills and patience in making this book possible. For discussion and encouragement I want to thank numerous colleagues, in particular, Armin Bunde, David Campbell, Patricia Cladis, James Crutchfield, Herman Cummins,

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San Jose

Lui Lam

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Introduction

Lui Lam

1.1 A Quiet Revolution

Quantum mechanics and relativity, the two important discoveries in physics developed at the beginning of this century, are well recognized as revolutions. These two revolutions present unexpected concepts and insights by going beyond the classical domains (Fig. 1.1). New results are obtained in quantum mechanics when one goes to the microscopic level ($<10^{-8}$ cm) and, in the case of relativity, the speed of the object has to be close to that of light ($\sim 10^{10}$ cm/s).

Here comes a new branch of science—nonlinear science—which, like quantum mechanics and relativity, delivers a whole set of fundamentally new ideas and surprising results. Yet, unlike quantum mechanics and relativity, nonlinear science covers systems of *every* scale, and objects moving with *any* speed; that is, the whole area displayed in Fig. 1.1. Then, by the same standard, nonlinear science is more than qualified to be called a revolution. The fact that nonlinear science delivers within the conventional system sizes and speed limits should not be counted as negative toward its novelty but, on the contrary, in view of its wide applicability, makes nonlinear science more important and powerful as a true revolution. In particular, nonlinear science can be studied with daily macroscopic systems with ordinary tools, such as a camera or a copying machine, making it accessible to almost everybody.

If nonlinear science appears to be a somewhat quiet revolution, it is perhaps due to its wide scope of coverage. The important works were done by so many researchers and accumulated over such a long period of time that it was hard for a single person or a group to call a press conference. Or, following a long scientific tradition, no one bothered to call a press conference.

For pedagogical purposes, nonlinear science may be divided into six areas of study, namely, fractals, chaos, pattern formation, solitons, cellular automata, and complex systems. The common theme underlying this diversity of subjects is the nonlinearity of the systems under study.