COLLEGE ALGEBRA WITH TRIGONOMETRY

Raymond A. Barnett

COLLEGE ALGEBRA WITH TRIGONOMETRY

Raymond A. Barnett
Merritt College

THIRD EDITION

McGRAW-HILL BOOK COMPANY
New York St. Louis San Francisco Auckland Bogotá Hamburg
London Madrid Mexico Montreal New Delhi Panama
Paris São Paulo Singapore Sydney Tokyo Toronto

COLLEGE ALGEBRA WITH TRIGONOMETRY

5 6 7 8 9 0 D O C D O C 8 9 8 7

ISBN 0-07-003864-3

This book was set in Melior.
The editor was Peter R. Devine;
the designer was Janet Bollow;
the cover was designed by Anne Canevari Green;
the production supervisor was Leroy A. Young.
The drawings were done by Carl Brown.
Project supervision was done by Phyllis Niklas.
R.R. Donnelley & Sons Company was printer and binder.

Cover Photo Credit

Nautilus shell, photo © by Anne Monk.

Chapter-Opening Photo Credits

Photo for Chapter 0 © by P. J. Bryant, University of California, Irvine/BPS. Photos for Chapters 1, 9, and 11 © by Peter Pearce. Photos for Chapters 2, 6, 7, 8, and 10 © by Anne Monk. Photo for Chapter 3 © by William C. Ferguson. Photo for Chapter 4 © by S. K. Webster, Monterey Bay Aquarium/BPS. Photo for Chapter 5 © by Glenn R. Steiner.

LIBRARY OF CONGRESS CATALOGING IN PUBLICATION DATA

Barnett, Raymond A.
College algebra with trigonometry.

"Includes chapters 0 to 8 and appendix A of College algebra" — Verso of t.p.
Includes index.
1. Algebra. 2. Trigonometry. I. Barnett, Raymond A. College algebra. II. Title.
QA154.2.B37 1984 512'.13 83-12083
ISBN 0-07-003864-3

Preface

This third edition of *College Algebra with Trigonometry* reflects experience and feedback from a large number of users of the earlier edition.

Principal Changes from the Second Edition

1. Chapter 2 on **graphs and functions** was completely rewritten. The treatment of function, composite function, and inverse function is more comprehensive and better motivated. The discussion of graphing techniques and the graphing of special functions such as polynomial functions and rational functions has been substantially expanded.
2. The **trigonometry chapters** (5, 6, and 7) were reviewed carefully and many parts were rewritten and/or expanded. The trigonometric functions are introduced through angle domains and are generalized for all real numbers. A special section on *circular functions* has been added (just prior to graphing) and is presented in such a way that a student will be able to shift back and forth between the two approaches with relative ease, using the approach that is most enlightening and productive for a given purpose. Calculator evaluation of trigonometric functions is emphasized. Table evaluation, for those who still desire that approach, can be found in Appendix B. Exact values for multiples of special angles or multiples of special real numbers still receive considerable attention, and these values are used extensively throughout all three chapters. Section 5-8 on inverse trigonometric functions has been rewritten to place more emphasis on inverse trigonometric functions rather than relations. Exact values using special angles and calculator evaluation are both emphasized. Chapter 6 on identities has been expanded and reorganized. Many new problems have been added. The treatment of polar coordinates and polar graphing has been expanded to two sections in Chapter 7.
3. Chapter 3 on **polynomial functions and the theory of equations** has been streamlined, and there is now a better focus on key topics and processes.
4. Chapter 8 on **systems of equations and inequalities** now includes solutions of linear systems using Gauss–Jordan elimination. The presentation is gradual, well motivated, and carefully done. (See Sections 8-1, 8-2, and 8-3.)

5. Chapter 9 on **matrices and determinants** has been reorganized and the material on matrices has been expanded. Inverses of square matrices and matrix equations are now treated in some detail. (See Sections 9-1, 9-2, and 9-3.)
6. Chapter 0 provides a comprehensive **review of basic algebraic operations** that are usually covered in intermediate algebra. This material may be treated systematically if a class is weak in basics; it may be briefly reviewed and the class tested to screen out students who need a more elementary course before this one; or it may be omitted altogether if time is a concern (students can then refer to the material as needed).
7. The material in Chapter 1 on **equations and inequalities** has been arranged to increase comprehension and motivation. For example, the real number line is introduced in conjunction with solving and graphing linear inequalities in one variable, and complex numbers are introduced just prior to solving quadratic equations.
8.  Much greater **use of scientific calculators** is made throughout the text both in examples and in exercise sets. Table use is included for logarithmic and trigonometric functions (for those who still desire that approach) but is optional. The author believes every student in the class should have a scientific calculator with a user's manual for that calculator.
9. Functional **use of a second color** is made throughout the text for increased clarity in topic presentation and graphing.
10. To **increase student support** and understanding, additional discussion, examples, and exercises have been added to almost every section, and some sections have been completely rewritten. These additions and changes have increased the length of the book, but have not added substantially to the number of topics covered. With this added student support, an instructor should be able to cover the same amount of material as before and with greater student comprehension.
11. There are now separate **chapter review sections** that include concise summaries of terms, formulas, and symbols discussed in each section; chapter review exercise sets keyed to specific sections through the answers in the back of the book; and practice tests, also keyed to specific sections through the answers in the back of the book.

- Important Features Retained from the Second Edition

1. The text is still **written for student comprehension.** Each concept is illustrated with an example, followed by a parallel problem with an answer so that a student can immediately check his or her understanding of the concept. These followup problems also encourage active rather than passive reading of the text.

2. An **informal style** is used for exposition, statements of definitions, and proofs of theorems.
3. The text includes **more than 4,000 carefully selected and graded problems.** The exercises are divided into A, B, and C groupings, with the A problems easy and routine, the B problems more challenging but still emphasizing mechanics, and the C problems a mixture of theoretical and difficult mechanics. In short, the text is designed so that an average or below-average student will be able to experience success and a very capable student will be challenged.
4. The subject matter is related to the real world through many carefully selected **realistic applications** from the physical sciences, business and economics, life sciences, and social sciences. Thus, the text is equally suited for students interested in any of these areas.
5. Following the recommendations of many national and international mathematical organizations and the author's own convictions, the **function concept is used as a unifying notion** from the second chapter onward.
6. **Answers** to all chapter review exercises and practice tests and to all odd-numbered problems from the other exercises are in the back of the book.

- Student Aids

1. **Common student errors** are clearly identified at places where they naturally occur (see Sections 0-2 and 1-2).
2. **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-2 and 1-4).
3. **Annotation** of examples and developments is found throughout the text to help students through critical stages (see Sections 1-2 and 1-4).
4. **Functional use of a second color** guides students through critical steps (see Sections 1-2 and 1-4).
5. **Chapter review sections** include a review of all important terms and symbols, a comprehensive review exercise, and a practice test. Answers to all review exercises and practice test problems are included in the back of the book and are keyed (with numbers in italics) to the corresponding text sections.
6. **Summaries** of formulas and symbols (keyed to the sections in which they are introduced) are inside the front and back covers of the text for convenient reference.
7. A **solutions manual** is available at a nominal cost through a bookstore. The manual includes detailed solutions to all odd-numbered problems, all chapter review exercises, and all practice test problems.

Instructor Aids

1. A **comprehensive test battery** is included in the instructor's manual, which can be obtained from the publisher at no cost. The test battery includes two regular and two multiple-choice tests for each chapter. All tests have easy-to-grade solution keys and sample student solution sheets. The format is $8\frac{1}{2}$ by 11 inches for ease of reproduction.
2. **Answers to even-numbered problems,** which are not included in the text, are given in the instructor's manual.
3. A **solutions manual** (see student aids) is available to instructors at no cost from the publisher.

Error Check

Because of the careful checking and proofing by a number of very competent people (acting independently), the author and publisher believe this book to be substantially error-free. For any errors remaining, the author would be grateful if they were sent to: Mathematics Editor, College Division, 27th Floor, McGraw-Hill Book Company, 1221 Avenue of the Americas, New York, New York 10020.

Acknowledgments

The preparation of a book requires the effort and skills of many people in addition to an author. I wish to thank the reviewers for their many helpful suggestions and comments. (It is this process of use, feedback, and adjustment that produces an increasingly effective book for both students and instructors.) In particular I wish to thank Thomas A. Atchison, Stephen F. Austin State University; Martin Broadwell, Jr., Florida College; Eddie J. Brown, St. Clair County Community College; Leland Fry, Kirkwood Community College; Lotus Hershberger, Illinois State University; Sidney Katoni, New York City Technical College; Peter Lindstrom, North Lake College; Gary Ling, Golden Gate University; Stanley Lukawecki, Clemson University; Francis E. Masat, Glassboro State College; Ronald Prielipp, Bethany College; Bruce Reed, Virginia Polytechnic Institute; Donald G. Spencer, Northeast Louisiana University; and George N. Trytten, Luther College. Special thanks go to Margaret Barnett-Burnette, Fred Safier (City College of San Francisco), and Ward A. Soper (Walla Walla College) for their careful checking of all examples, matched problems, and exercise sets.

Raymond A. Barnett

To the Student

The following suggestions are made to help you get the most out of this book and your efforts.

As you study the text we suggest a five-step process. For each section:

1. Read a mathematical development.
2. Work through the illustrative example.
3. Work the matched problem.

} Repeat the 1-2-3 cycle until the section is finished.

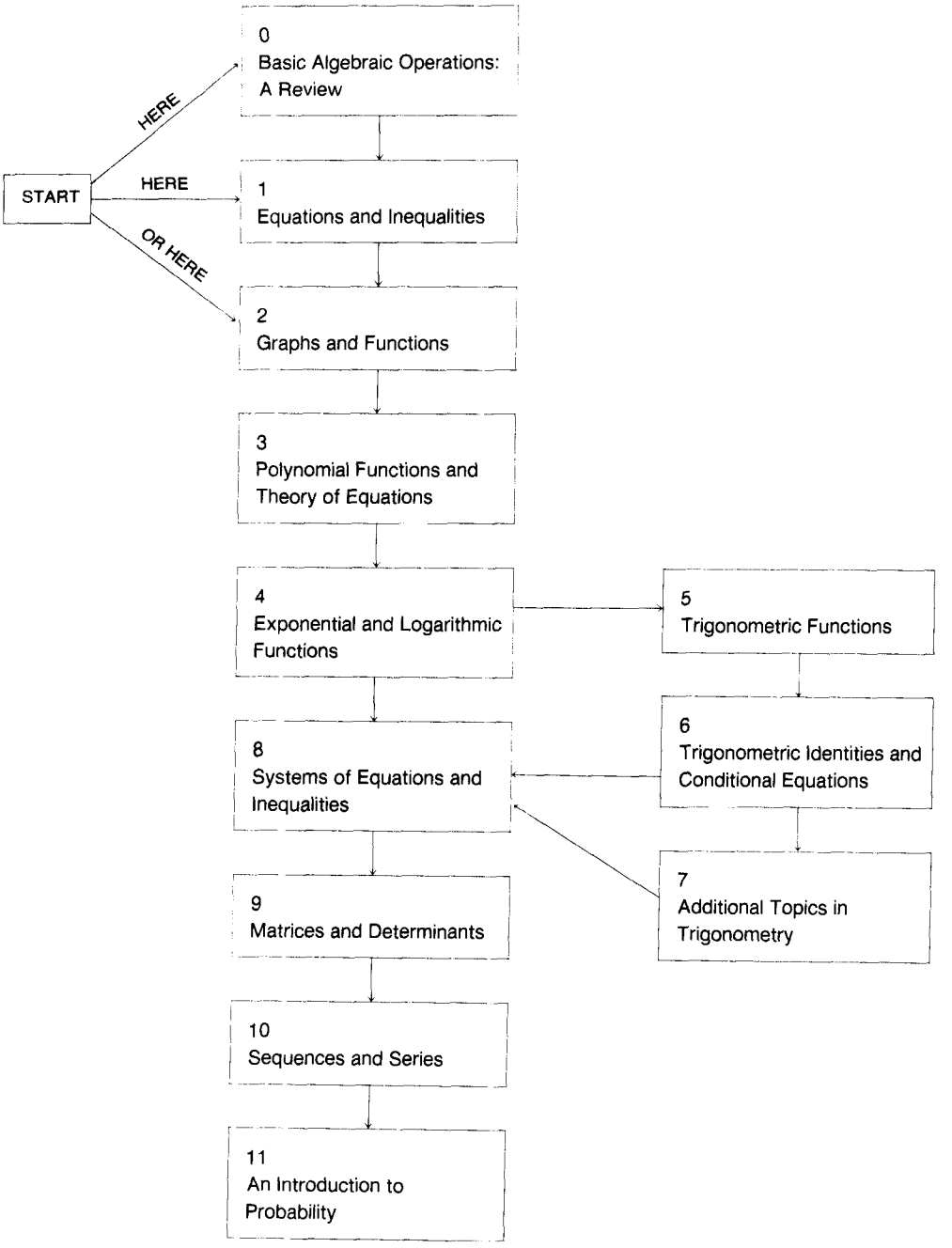
4. Review the main ideas in the section.
5. Work the assigned exercise at the end of the section.

All of this should be done with plenty of paper, pencils, and a wastebasket at hand. In fact, no mathematics text should be read without pencil and paper in hand; mathematics is not a spectator sport. Just as you cannot learn to swim by watching someone else swim, you cannot learn mathematics by simply reading worked examples—you must work problems, lots of them.

If you have difficulty with the course, then, in addition to doing the regular assignments, spend more time on the examples and matched problems and work more A exercises, even if they are not assigned. If the A exercises continue to be difficult for you, you probably should take an intermediate algebra course before attempting this one. If you find the course too easy, then work more C exercises, even if they are not assigned. If the C exercises are consistently easy for you, you are probably ready to start the calculus sequence.

Raymond A. Barnett

Possible Courses



Remarks on Calculator Use

Hand calculators are of two basic types relative to their internal logic (the way they compute): algebraic and reverse Polish notation (RPN). Throughout the book we will identify algebraic calculator steps with "A" and reverse Polish notation calculator steps with "P." Let us see how each type of calculator would compute

$$\frac{(5)(3)(2)-(7)(6)}{2(11)}$$

	PRESS	DISPLAY
A:	5 × 3 × 2 − 7 × 6 = ÷ 2 ÷ 11 =	−0.54545455
P:	5 ENTER 3 × 2 × 7 ENTER 6 × − 2 ÷ 11 ÷	−0.54545455

Some people prefer the algebraic logic and others prefer the Polish. Which is better is still being debated. The answer seems to rest with the type of problems one encounters and with individual preferences. The author owns both types and uses the one with Polish logic most frequently. However, he knows people who prefer the algebraic type, and they seem quite happy with their choice.

In any case, irrespective of the type of calculator that you own, it is essential that you read the user's manual for your own calculator. A large variety of calculators are on the market, and each is slightly different from the others. Therefore, it is important that you take the time to read the manual. Do not try to read and understand everything the calculator can do; this will only tend to confuse you. Read only those sections that pertain to the operations you are or will be using; then return to the manual as necessary when you encounter new operations.

In many places in the text calculator steps for new types of calculations will be shown (similar to those steps shown here). These are only aids. Try the calculation without the aid; then use the aid only if you get stuck.

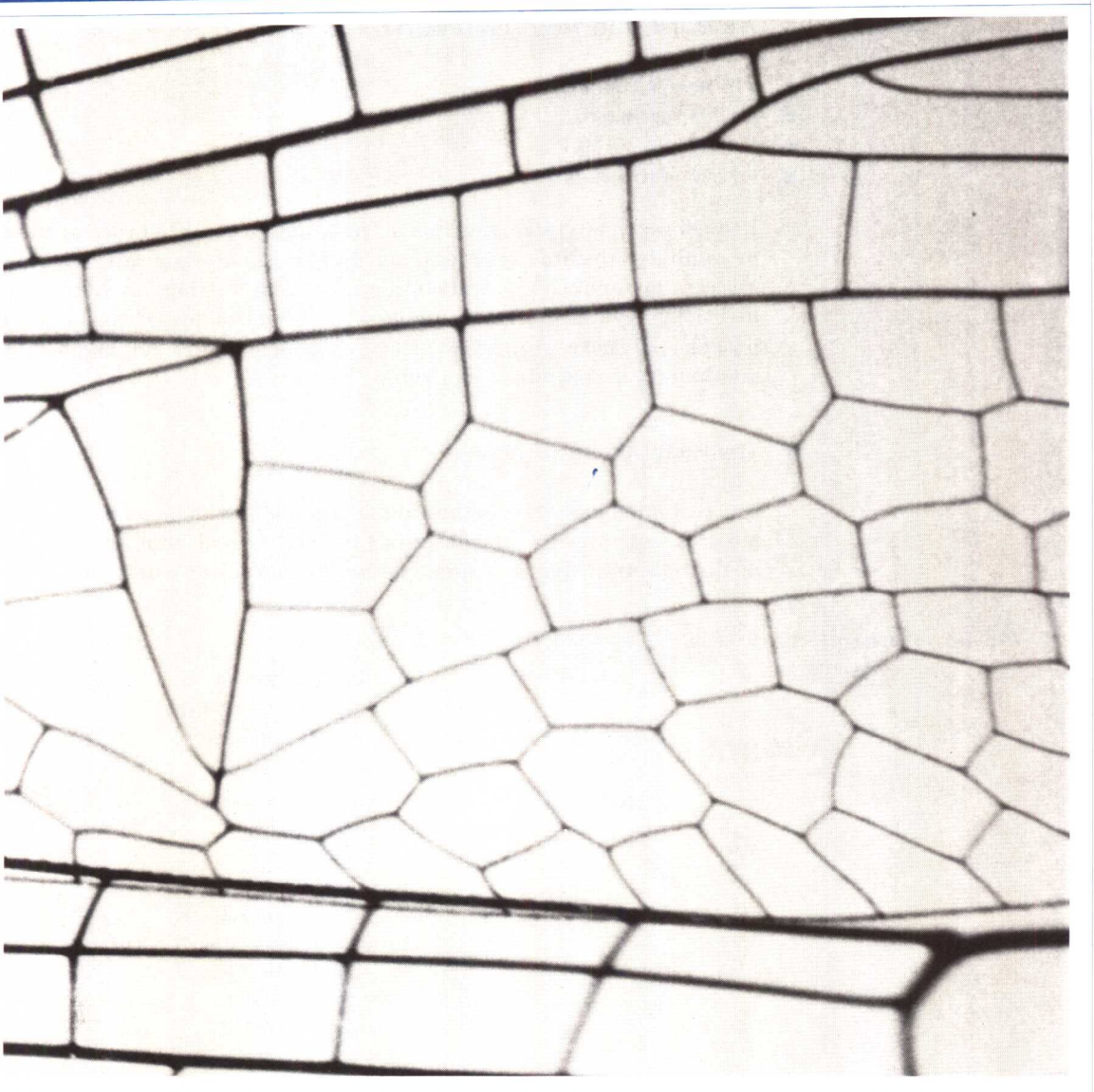
It is important to remember that *a calculator is not a substitute for thinking.* It can save you a great deal of time in certain types of problems, but you still must know how and when to use it.

Raymond A. Barnett

Contents

* *Note to instructor:* May be omitted, depending on time, need, and interest. (Some topics are review topics and some are new topics.)

Basic Algebraic Operations: A Review



A natural design of mathematical interest. Can you guess the source? See the back of the book.

Chapter 0 ■ Basic Algebraic Operations: A Review

Section 0-1 Algebra and Real Numbers

- The Real Number System
- Basic Properties
- Further Properties
- Fraction Properties

In algebra we are interested in manipulating symbols in order to change or simplify algebraic expressions and to solve algebraic equations. Because many of these symbols represent real numbers, it is important to briefly review the real number system and some of its important properties. These properties provide the basic rules for much of the manipulation of symbols in algebra.

■ The Real Number System

The real number system is the number system in which you have worked most of your life. Table 1 describes the set* of real numbers and some of the important types of numbers within the set of real numbers.

TABLE 1 The Set of Real Numbers

SYMBOL	NUMBER SYSTEM	DESCRIPTION	EXAMPLES
N	Natural numbers	Counting numbers (also called positive integers)	$1, 2, 3, \ldots$
Z	Integers	Set of natural numbers, their negatives, and 0	$\ldots, -2, -1, 0, 1, 2, \ldots$
Q	Rationals	Any number that can be represented as a/b, where a and b are integers and $b \neq 0$	$-4; \frac{-3}{5}; 0; 1; \frac{2}{3}; 3.67$
R	Reals	Set of all rational and irrational numbers (the irrational numbers are all the real numbers that are not rational)	$-4; \frac{-3}{5}; 0; 1; \frac{2}{3}; 3.67; \sqrt{2}; \pi; \sqrt[3]{5}$

* A **set** is a collection of objects. More will be said about sets in Chapter 1.

Figure 1 illustrates how these sets of numbers are related to one another.

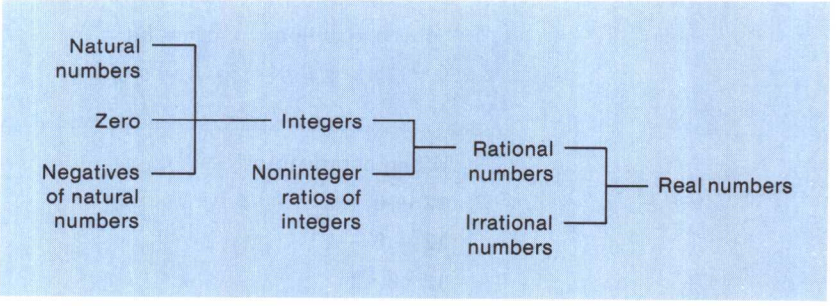


FIGURE 1 The real number system

The set of integers contains all the natural numbers and something else (their negatives and 0). The set of rational numbers contains all the integers and something else (noninteger ratios of integers). And the set of real numbers contains all the rational numbers and something else (irrational numbers).

Rational numbers have repeating decimal representations, whereas irrational numbers have infinite nonrepeating decimal representations. For example, the decimal representations of the rational numbers 2, $\frac{4}{3}$, and $\frac{5}{11}$ are, respectively,

$$2 = 2.0000\ldots \qquad \frac{4}{3} = 1.333\ldots \qquad \frac{5}{11} = 0.454545\ldots$$

whereas those of the irrational numbers $\sqrt{2}$ and π are, respectively,

$$\sqrt{2} = 1.41421356\ldots \qquad \pi = 3.14159265\ldots$$

Basic Properties

We now review (informally) a few basic real number properties. Other real number properties will be discussed as needed in other parts of the text. These properties of the real numbers become operational rules in the algebra of real numbers.

Real numbers can be added, subtracted, multiplied, and divided (except for division by 0). Does it matter in which order we perform addition, subtraction, multiplication, or division? In general, we may add in any order or multiply in any order [but this is not true for subtraction or division (for example, $4 - 2 \neq 2 - 4$ and $4 \div 2 \neq 2 \div 4$)]. This property is referred to as the **commutative property** (for addition and multiplication) for real numbers.