Mathematical Statistics with Applications

Second Edition

Mendenhall
Scheaffer
Wackerly

Mathematical Statistics with Applications

SECOND EDITION

WILLIAM MENDENHALL
RICHARD L. SCHEAFFER
DENNIS D. WACKERLY
University of Florida

DUXBURY PRESS
Boston, Massachusetts

PWS PUBLISHERS
Prindle, Weber & Schmidt • Willard Grant Press • Duxbury Press •
Statler Office Building • 20 Providence Street • Boston, Massachusetts 02116

Mathematical Statistics with Applications, Second Edition, was prepared for publication by the following people:

Copy Editor: Carol Beal
Interior Designer: Dorothy Booth
Cover Designer: Oliver Kline

PWS PUBLISHERS
A Division of Wadsworth, Inc.

Library of Congress Cataloging in Publication Data

Mendenhall, William.
Mathematical statistics with applications.
Includes index.
1. Mathematical statistics. I. Scheaffer, Richard L., joint author. II. Wackerly, Dennis D., 1945– joint author. III. Title.
QA276.M426 1981 519.5 80-19393
ISBN 0-87150-410-3
(previously ISBN 0-87872-279-3)
ISBN 0-534-98019-8 International

ISBN 0-87150-410-3

(previously ISBN 0-87872-279-3)

ISBN 0-534-98019-8 International

Printed in the United States of America

6 7 8 9—85 84

Preface

This text was written for use with a one-year sequence of courses (9 quarter or 6 semester hours) on mathematical statistics for undergraduates. The intent of the text is to present a solid undergraduate foundation in statistical theory and, at the same time, to provide an indication of the relevance and importance of the theory in solving practical problems in the real world. We think a course of this type is suitable for most undergraduate disciplines, including mathematics, where contact with these applications may provide a refreshing and motivating experience. The only mathematical prerequisite is a thorough knowledge of first-year college calculus.

Talking with students taking or having completed a beginning course in mathematical statistics reveals a major flaw that exists in many courses. The student can take the course and leave it without a clear understanding of the nature of statistics. Many see the theory as a collection of topics, weakly or strongly related, but fail to see that statistics is a theory of information with inference as its goal. Further, they may leave the course without an understanding of the important role played by statistics in scientific investigations. Why this is true (assuming that you agree with us) is a matter for conjecture, but several reasons suggest themselves.

First, all mathematical statistics courses require, of necessity, a large amount of time devoted to the theory of probability. Since at least fifty percent of the total course time is devoted to this subject, and it occurs at the beginning of the course, it is not surprising that a student may think of probability and statistics as being synonymous.

A second reason is that the objective of statistics often is not defined clearly at the beginning of the course, and no attempt is made to relate the probabilistic half of the course to the ultimate objective, inference.

Third, the utility of statistics cannot be revealed until late in the course because of the large amount of material that must preface it.

A fourth and final possibility is that the interests of some instructors lead them to present the material as a sequence of topics in applied mathematics rather than as a cohesive course in statistics. In any case, regardless of the reasons why students fail to form a clear picture of their subject, our text is an attempt to cope with the problem.

We think this text differs from others in three ways. First, we have preceded the presentation of probability with a clear statement of the objective of statistics and its role in scientific research, and we hold this objective before the student throughout the text. As the student proceeds through the theory of probability (Chapters 2 through 7), he or she is reminded frequently of the role that major topics play in the objective of the course, statistical inference. We attempt to strongly emphasize statistical inference as the sole and dominating theme of the course. The second feature of the text is connectivity. We try to explain not only how major topics play a role in statistical inference but also how the topics are related one to another. These connective discussions most frequently appear in chapter introductions and conclusions. Finally, we think the text is unique in its practical emphasis throughout the text, by the exercises and by the useful statistical methodology presented at the end of the text. We hope to reinforce an elementary but sound theoretical foundation with some very useful methodological topics contained in the last five chapters.

The book can be used in a variety of ways and adjusted to the tastes of the students and instructor. The difficulty of the material can be increased or decreased by controlling the assignment of exercises, by eliminating some topics, and by varying the amount of time to be devoted to each. A stronger applied flavor can be added by the elimination of some topics, for example, some sections of Chapter 7, and by devoting more time to the applied chapters at the end.

The revisions introduced into the second edition of *Mathematical Statistics with Applications* were drawn from many suggestions given to us by instructors who used the first edition. These changes include revisions and additions to examples, exercises, and chapter discussions as outlined below.

This edition contains almost twice as many exercises as were contained in the first edition. They are placed at the ends of most major sections within the chapters. Review exercises are found at the end of each chapter. Many more worked-out examples and exercises, based on real experimental situations and data sets, are included. Challenging exercises are clearly marked with an asterisk (*).

Content changes are found in the following chapters:

Chapter 2: The topic of probability is introduced through a discussion of discrete sample spaces. Discussions of continuous analogues have been moved to Chapter 4.

Chapter 4: This chapter now contains a more thorough discussion of the properties of distribution functions for discrete and continuous random variables.

Chapter 6: Methods for finding distributions of functions of random variables have been reordered and synthesized.

Chapter 7: This chapter has been rewritten. It now contains a separate section on the inferential role of the sampling distribution of a statistic. Sampling distributions (normal, t, χ^2, F) related to sampling from normal populations are presented and discussed. This discussion facilitates use of these distributions in later chapters. The central limit theorem (with optional proof) and the normal approximation to the binomial distribution are presented. Discussion of the general topic of convergence in probability has been moved to Chapter 9 where the concept is required to establish the properties of estimators.

Chapter 8: This chapter contains all the material in the previous edition plus a more thorough discussion of the pivotal method for deriving confidence intervals. We then illustrate the use of this general method for finding confidence intervals by deriving some common large-and small-sample confidence intervals.

Chapter 9: The order of presentation has been altered to first define and motivate desirable properties of estimators and then present meth ods for deriving estimators and establishing whether or not the derived estimators possess these properties. Sections on sufficiency have been expanded and rewritten to include the ideal of minimal sufficiency and the role of minimal sufficient statistics in the construction of minimum-variance unbiased estimators.

Chapter 10: This chapter now contains a separate section on the philosophy of hypothesis testing. Minimax and Bayes testing procedures have been eliminated.

Chapter 11: The primary change in this chapter is the inclusion of a section with examples involving data from documented case studies. Each of the inferential methods is made more relevant through application to one or more of the case studies.

Chapter 13: Specific reference to the Latin square design has been eliminated from this chapter. A section has been added that introduces a method for finding simultaneous confidence intervals for more than one parameter. An introduction to the use of linear models in the analysis of variance is also included (new).

Chapter 15: This chapter has been revised, bringing the general discussion more in tune with recent developments in the field of nonparametric statistics.

The authors wish to thank the many colleagues, friends, and students who made helpful suggestions concerning the manuscript for this book. In particular, we are indebted to P. V. Rao, J. Devore, and J. J. Shuster for their technical comments on the writing, and to Cecily Noble for her excellent typing. We wish to thank E. S. Pearson, W. H. Beyer, I. Olkin, R. A. Wilcox, C. W. Dunnett, A. Hald, and John Wiley & Sons for their kind permission to use the tables reprinted in Appendix III. Thanks are also due to the reviewers, Richard J. Kryscio, Northern Illinois University; Jessica M. Utts, University of California,

Davis; Louis A. Guillou, Saint Mary's College, Minnesota; and Daniel G. Martinez, California State University, Long Beach. Finally, we wish to thank our wives for their patience and understanding throughout the time this work was in progress.

William Mendenhall
Richard L. Scheaffer
Dennis D. Wackerly

Note to the Student

As the title *Mathematical Statistics with Applications* implies, this text is concerned with statistics, both in theory and application, and only deals with mathematics as a necessary tool to give you a firm understanding of statistical techniques. The following suggestions for using the text will increase your learning and save your time.

The connectivity of the textbook is provided by the introductions and summaries in each chapter. These sections explain how each chapter fits into the overall picture of statistical inference and how each chapter relates to the preceding ones.

Within the chapters, important concepts are set off as definitions. These should be read and reread until they are clearly understood, because they form the framework on which everything else is built. The main theoretical results are set off as theorems. Although it is not necessary to understand the proof of each theorem, a clear understanding of the meaning and implications of the theorems is essential.

It is also essential that you work many of the exercises—for at least three reasons. First, you can be certain that you understand what you have read only by putting your knowledge to the test of working problems. Second, many of the exercises are of a practical nature and shed light on the applicability of probability and statistics. Third, some of the exercises present new concepts and thus extend the material covered in the chapter.

W. M.
R. L. S.
D. D. W

Contents

1

What Is Statistics?

1.1 Introduction

Statistical techniques are employed in almost every phase of life. Surveys are designed to collect early returns on election day to forecast the outcome of an election, and consumers are sampled to provide information for predicting product preference. The research physician conducts experiments to determine the effect of various drugs and controlled environmental conditions on humans in order to infer the appropriate method of treatment of a particular disease. The engineer samples a product quality characteristic along with various controllable process variables to assist in locating important variables related to product quality. Newly manufactured fuses are sampled before shipping to decide whether to ship or hold individual lots. The economist observes various indices of economic health over a period of time and uses the information to forecast the condition of the economy next fall. Statistical techniques play an important role in achieving the objective of each of these practical problems, and it is to the theory underlying this methodology that this textbook is devoted.

A prerequisite to a discussion of theory is a definition of "statistics" and a statement of its objectives. *Webster's New Collegiate Dictionary* defines statistics as "a branch of mathematics dealing with the collection, analysis, interpretation, and presentation of masses of numerical data." Kendall and Stuart [3] state: "Statistics is the branch of scientific method which deals with the data obtained by counting or measuring the properties of populations." Fraser [1], commenting on experimentation and statistical applications, states that "statistics is concerned with methods for drawing conclusions from results of the experiments or processes." Freund [2], among others, views statistics as encompassing "the entire science of decision making in the face of uncertainty," and Mood [5] defines statistics as "the technology of the scientific method" and adds that statistics is concerned with "(1) the design of experiments and investigations, (2) statistical inference." *A superficial examination of these definitions suggests a bewildering lack of agreement, but all possess common elements. Each implies collection of data with inference as the objective. Each requires the selection of a subset of a large collection of data, either existent or conceptual, in order to infer the characteristics of the complete set. Thus statistics is a theory of information with inference making as its objective.*

The large body of data that is the target of our interest is called a *population*, and the subset selected from it is a *sample*. The preferences of voters for a gubernatorial candidate, Jones, expressed in quantitative form (1 for "prefer" and 0 for "do not prefer") provide a real, finite, and existing population of great interest to Jones. Indeed, he may wish to sample the complete set of eligible voters in order to determine the fraction favoring his election. The voltage at a particular point in the guidance system for a spacecraft may be tested in the only three systems that have been built in order to estimate the voltage characteristics for other systems that might be manufactured some time in the future. In this case the population is *conceptual*. We think of the sample of three as being representative of a large population of guidance systems that could be built and would possess characteristics similar to the three in the sample. As another example, measurements on patients in a medical experiment represent a sample from a conceptual population consisting of all patients similarly afflicted today as well as those who will be afflicted in the near future. You will find it useful to clearly define the populations of interest for each of the statistical problems described earlier in this section and to clarify the inferential objective for each.

It is interesting to note that billions of dollars are spent each year by American industry and government for data from experimentation, sample surveys, or other collection procedures. Consequently, we see that this money is expended solely for information about a phenomenon susceptible to measurement in an area of business, science, or the arts. The implication of this statement provides a key to the nature of the very valuable contribution that statistics makes to research and development in all areas of society. Information useful in inferring some characteristic of a population (either existing or conceptual) can be purchased in a specified quantity and will result in an inference (estimation or decision) with an associated degree of goodness. For example, if Jones arranges for a sample of voters to be interviewed, the information in the sample can be used to estimate the true fraction of all voters favoring Jones's election. In addition to the estimate itself, Jones should also be concerned with the likelihood (chances) that the estimate provided is close to the true fraction of eligible voters favoring his election. Intuitively, the larger the number of eligible voters in the sample, the higher will be the likelihood of an accurate estimate. If a decision is made regarding the relative merits of two manufacturing processes based upon examination of samples of products from each process, we should be interested in the decision and the likelihood that the decision is correct. In general, we can say that *statistics is concerned with the design of experiments or sample surveys to obtain a specified quantity of information at minimum cost and the optimal utilization of this information in making an inference about a population. The objective of statistics is to make an inference about a population based on information contained in a sample and to provide an associated measure of goodness for the inference.*

Exercise

1.1 For each of the following situations discuss the nature of the population of interest, the inferential objective, and how you might go about collecting a sample.
   (a) A city engineer wants to estimate the average weekly water consumption for single-family dwelling units in the city.
   (b) The National Highway Safety Council wants to estimate the proportion of automobile tires with unsafe tread among all tires manufactured by a certain company in a specified year.
   (c) A political scientist wants to determine if a majority of adult residents of a state favor a unicameral legislature.
   (d) A medical scientist wants to estimate the average length of time until the recurrence of a certain disease.
   (e) An electrical engineer wants to determine if the average length of life of transistors of a certain type is greater than 500 hours.

1.2 Characterizing a Set of Measurements: Graphical Methods

In the broadest sense, making an inference implies the partial or complete description of a phenomenon or physical object. Little difficulty is encountered when appropriate and meaningful descriptive measures are available, but this is not always the case. For example, it is easy to characterize a person by using height, weight, color of hair and eyes, and other descriptive measures of one's physiognomy. Locating a set of descriptive measures to characterize an oil painting would be a comparatively more difficult task; characterizing a population, which consists of a set of measurements, is equally challenging. Consequently, a necessary prelude to a discussion of inference making is the