VIBRATION OF STRUCTURES

Applications in civil engineering design

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Preface

This book is intended as a concise guide to vibration theory, sources of dynamic loading and methods of vibration control for civil and structural engineers. It has been written primarily for final year students specializing in advanced structural engineering, and for practising engineers needing some revision or updating in the area of vibrations. The book should also be useful for research engineers who are interested in practical applications of their work.

An elementary knowledge of differential equations and matrices is assumed but an effort has been made to keep the mathematical content to a minimum. The book endeavours to develop a sound physical understanding of the nature of vibration. This is particularly important in the case of finite element modelling which is readily available in the form of sophisticated commercial programs running on machines ranging from desk-top microcomputers to large mainframes. It is now possible to model complex structural problems realistically without having to understand the advanced mathematics which is embedded in the finite element program. However, the responsibility for correct modelling of the *physical* problem remains with the engineer.

Numerical examples are included in the text. In many cases these are modified versions of practical problems that the author has encountered as a vibrations consultant. They have been devised to illustrate the various principles outlined in the book.

A large number of references is listed at the back. This is partly to give credit where it is due. It also gives the reader the opportunity of following up certain areas of vibration knowledge more deeply. An attempt has been made to trace back to original papers as far as possible. This should benefit specialists and researchers since it is important to read the work of the originator of any concept. Inevitably, the works of some worthy authors have been omitted. No offence is intended. Liberal references are made to codes of practice and design regulations of various countries, in particular those of the United Kingdom, Europe and North America. Extracts from British Standards are reproduced by permission of BSI. Complete copies can be obtained from them at Linford Wood, Milton Keynes, MK14 6LE.

In writing the book, I am indebted to my wife and family who have supported me enthusiastically throughout. I would like to acknowledge the support of my colleagues at the University of Bristol, especially Professor Roy Severn, leader of the earthquake engineering research team. I am particularly grateful to Professor A. Heidebrecht and Dr Tom Lawson for their helpful reviews of certain chapters. I would also like to thank Professor Sir Alfred Pugsley who first suggested that I should write such a book.

Finally, I should emphasize that no creative work is possible without the presence of the living God. 'The Son is the radiance of God's glory and the exact representation of his being, sustaining all things by his powerful word' (Hebrews, 1:3). Praise Him!

Notation

a, a_i, a_f, a_c	half length of a crack, initial, final, critical
$\boldsymbol{A}$	dynamic matrix
C_L, C_D	lift, drag coefficients
C_S	seismic coefficient
c_R	Rayleigh wave velocity
$C(r, r'; n)$	covariance of wind velocity
C_z, C_ϕ, C_x, C_ψ	soil coefficients of uniform compression, non-uniform compression, uniform shear, non-uniform shear
c, $\boldsymbol{C}$	damping coefficient, matrix
D	flexural stiffness of a thin plate
EI	flexural stiffness of a beam
f	frequency
g_B, g_D	peak factors for non-resonant, resonant response
G	geometry factor in seismic risk analysis
G	Lamé's constant
h	focal depth of an earthquake
$H(n)$	mechanical admittance
i	$\sqrt{(-1)}$
I	Modified Mercalli intensity
I	$P(t)dt$, impulse
I	importance factor (seismic design)
k, $\boldsymbol{K}$, K_m	stiffness – coefficient, matrix, generalized
k	frequency number in wave equation
K	configuration factor for footbridges
K_1, K_{1c}	stress intensity factor, fracture toughness
m, m_s, m_u, $\boldsymbol{M}$, M_m	mass or mass per unit length, sprung, unsprung, matrix, generalized
m	slope of S–N fatigue curve or index in Paris equation
$M(x)$	bending moment

M, M_0, $\bar{M}$	Richter magnitude for earthquakes, smallest of interest, average
n	frequency (wind engineering)
N	isoparametric interpolation functions
$p(x,t)$, $P(t)$, $\boldsymbol{P}$	distributed load, loading function, vector
$q(x)$, $\boldsymbol{q}$, Q_m	distributed load, load vector, generalized force
$\boldsymbol{Q}$	matrix of subspace eigenvectors
r, R	radial distance, focal distance
R	factor to take account of ductility
R_e	Reynolds number
$R(\Delta r)$	cross-correlation coefficient
S	seismic response factor
$S(r,r'; n)$	cross spectral density
S	Strouhal number
$S_x(n)$, S_{Q_m}, S_{Y_m}	spectral density of horizontal gustiness, generalized force, modal amplitude
S_a, S_d, S_v	spectral acceleration, displacement, velocity (strong ground motion)
t, T	time, period of vibration
u, $\dot{u}$, $\ddot{u}$	horizontal displacement, velocity, acceleration
u_g, u_r	ground displacement, relative displacement
$\boldsymbol{u}$	displacement vector
U	strain energy
v, $\dot{v}$, $\ddot{v}$	vertical displacement, velocity, acceleration
v_x, v_y, v_z	wind velocity components
$v(t)$, $\bar{V}$, v_*	wind speed, mean speed, friction velocity
$V(x)$, V	shear force, base shear
V	vehicle speed
$\boldsymbol{V}$	matrix of Lanczos vectors
V, V_x, V_y, V_z	peak particle velocity and components
w, $\dot{w}$, $\ddot{w}$	vertical displacement, velocity, acceleration
W_e, W_i	virtual work – external, internal
Y_m, $\dot{Y}_m$, $\ddot{Y}_m$, $\boldsymbol{Y}$	modal amplitude of displacement, velocity, acceleration, vector
z_0	roughness length (wind)
α	frequency constant
α	damping matrix coefficient
α	speed parameter $\pi V/L\omega$ (moving load)
α	stress intensity geometry factor
α	compression wave velocity
β	Newmark weighting factor

β	shear wave velocity
β	influence coefficient
β	bounce parameter (moving sprung mass)
γ_{xy}	shear strain
γ_n	mode participation factor
$\gamma^2(r, r'; n)$	coherence (wind)
δ	Newmark weighting factor
δ_c	midspan displacement
$\varepsilon_x, \varepsilon_y, \varepsilon_z, \boldsymbol{\varepsilon}$	strain components, vector
ζ	surface wave number
η	surface wave number
η	eddy viscosity
$\theta_x, \theta_y, \theta_z$	mass moments of inertia
λ	wavelength
λ	Lamé's constant
$\boldsymbol{\Lambda}$	matrix of subspace eigenvalues
μ	viscosity
μ	mass ratio
ν	Poisson's ratio
ξ, ξ_s, ξ_a	damping ratio, structural, aerodynamic
ρ	density
ρ	frequency ratio
$\sigma_x, \sigma_y, \sigma_z, \boldsymbol{\sigma}$	stress components, vector
$\sigma_x^2, \sigma_B^2(E), \sigma_D^2(E)$	variance of horizontal gustiness, non-resonant load effect, resonant load effect
τ	time
τ_{xy}	shear stress
ϕ	phase lag
$\phi(x), \boldsymbol{\phi}_n, \boldsymbol{\Phi}$	mode shape, vector, matrix of eigenvectors
$\chi_a(n)$	aerodynamic admittance
ψ	dynamic response factor (footbridges)
$\boldsymbol{\Psi}$	matrix of transformed eigenvectors
$\omega, \omega', \bar{\omega}$	natural circular frequency, damped, limiting frequency
Ω	forcing frequency

Contents

1

Introduction

1.1 EXAMPLES OF VIBRATION IN EVERYDAY EXPERIENCE

It is a common experience to observe a tall street lighting standard oscillating in a steady wind. It may be noticed that the phenomenon usually occurs in strong winds, and careful observation often reveals that the motion takes place at right angles to the direction of the wind. This is due to vortex shedding and is caused by small eddy currents as the wind flows around the pole. The natural frequency of vibration of the lighting standard happens to coincide with the rate at which the eddies are created. The Tacoma Narrows Bridge in the USA collapsed in 1940 partly because of vortex shedding. All long span bridges are now designed to resist the dynamic components of wind loading.

The throbbing of the deck of a passenger ship under one's feet has pleasant associations of holidays and travel. It is caused by the unbalanced forces of powerful engines being transmitted through the relatively flexible structure of the ship's hull. On the other hand, heavy industrial machinery can give rise to unpleasant vibrations in factories such that working conditions may be impaired, or even to the extent that damage may occur to the buildings themselves. Therefore the foundations of industrial machinery have to be designed to keep vibrations within acceptable limits.

Traffic rumble is often blamed for cracks in plaster and minor damage to buildings close to a busy road. The problem tends to be complicated by the accompanying noise together with emotional reactions stemming from ownership of the properties. However, where a road repair has been carried out, resulting in an abrupt discontinuity in the road pavement profile, heavy vehicles travelling at speed can generate a shock that is transmitted through the ground to the foundations of adjoining buildings. Similar problems can be caused by pile driving and blasting.

If not an everyday experience, in certain parts of the world earthquakes occur very frequently and affect the lives of the population. Countries such as the USA, Mexico, Chile, China and Japan have experienced many severe and devastating earthquakes during this century. In these countries the design of buildings is dominated by the need to provide earthquake resistance. Even in countries that are often thought to be aseismic, such as Great Britain, there is a small but finite risk of significant earthquake motion. This risk has to be considered in the design of nuclear plant or major industrial installations where the consequences of damage could be far reaching.

1.2 THE PHYSICAL NATURE OF VIBRATION

Vibration differs from static behaviour in two important respects. First, the applied forces vary with time. Fluctuating wind pressure and blast forces are obvious examples. Reciprocating machines produce pulsating loads. Moving vehicles and locomotives bounce on their suspension in addition to being time variant in terms of position on the structure. In the case of earthquakes we have ground motion imposed on buildings that are initially at rest.

Secondly, the motion of a structure gives rise to inertia forces. Motion of a fixed structure implies a to-and-fro oscillation. During the motion the various parts of the structure possess momentum which tends to cause the structure to overshoot its natural position of deflection, decelerate and come to rest momentarily before returning in an oscillatory manner. Inertia forces correspond to the changing momentum and are distributed along the structure in proportion to its mass. The applied loads, the inertia forces and the elastic resistance are in a continually changing state of dynamic equilibrium.

A good example is the classical problem of soldiers marching over a light footbridge. Pulsating forces are applied in time with the marching frequency causing motion of the structure. According to Newton's second law there must be inertia forces acting on each element of the bridge in proportion to its acceleration. These inertia forces will be distributed along the bridge and tend to make it overshoot as it approaches its point of maximum deflection. The elasticity of the structure acts as a restoring force that makes the bridge spring back, resulting in oscillation. This tendency to vibrate is aggravated if the marching frequency coincides with the natural frequency of the structure. Very large motions can quickly build up and hence the custom has arisen of breaking step.

1.3 SOURCES OF DYNAMIC EXCITATION

In practical engineering design the first requirement is to identify the sources of dynamic excitation and to assess their magnitude and significance compared with the static loads. Structural calculations for static loads are generally much easier than for dynamic excitation, and that is why structural engineers prefer to adopt equivalent static forces as far as possible for the analysis. However, most forms of loading have dynamic components and some forms of structure, especially if they are slender, are susceptible to the dynamic effects. Furthermore, the use of the structure, e.g. as a laboratory housing sensitive instruments, may require that vibration be considered. Thus there is an interrelation between the sources of dynamic excitation, the structural form and the purpose of the structure.

The different types of dynamic loading considered in this book are as follows: earthquakes; wind; industrial machinery; human forces; moving vehicles; and blasting and pile-driving. It is standard practice to use equivalent static horizontal forces when designing buildings for earthquake and wind resistance. This is the simplest way of obtaining the dimensions of structural members. Dynamic calculations may follow to check and perhaps modify the design. Human forces, in the form of crowd loading, are almost always treated as static distributed loads. But observations at a pop concert or football ground will demonstrate the highly dynamic nature of the loading, though the worst effects are restricted to a particular frequency range. Moving vehicles can be designed for by adding an allowance for impact to their static weight. This has proved to be satisfactory for the design of highway and railway bridges. But the procedure may not be justified for loads moving at ultra-high speeds. Vibrations caused by industrial machinery, blasting and pile-driving have to be assessed by methods of dynamical analysis or by experiment.

Some examples of dynamic loading are shown in Fig. 1.1. The first is a record of fluctuating wind velocity. Corresponding fluctuating pressures will be applied to the structure. The random nature of the loading is evident and it is clear that statistical methods are required for establishing an appropriate design loading. The next figure shows the regular pulsating force applied to the foundations of a reciprocating engine. In practice the fluctuation may not be perfectly sinusoidal but it will be at constant frequency and of known amplitude. The third figure shows the characteristic shape of the air pressure impulse caused by a sonic boom or blasting, in construction or open-cast mining. The shapes of air blast curves are usually quite similar, having an initial peak followed by an almost linear decay and often followed by some suction. The lengths of the pulses and their amplitudes depend on many factors, e.g. distance from blast, nature of rock being blasted or size, shape and altitude of aircraft.

Some forms of loading are quite well defined and may be quantified by observation or experiment. Many forms of loading are not at all well defined and require judgement on the part of the engineer. Data on certain types of dynamic loading, e.g. earthquakes and wind, are readily available in many design codes. Other types of loading are less well covered, though much data may be available in published research papers. One of the aims of this book is to discuss the nature of the most important types of dynamic loading and to direct the reader to relevant literature for further information.

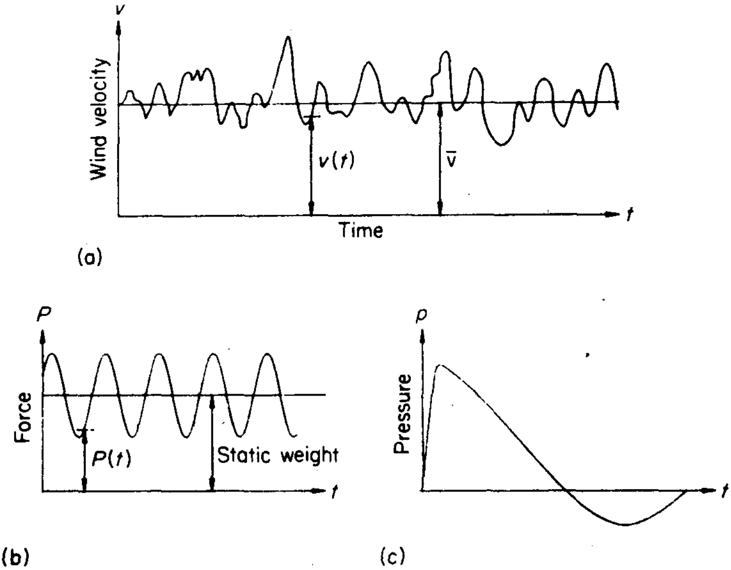

Fig. 1.1 Examples of dynamic loading. (a) Fluctuating wind velocity; (b) pulsating force due to reciprocating engine; (c) air pressure impulse due to blasting or sonic boom.

1.4 DYNAMIC ANALYSIS OF STRUCTURES

Dynamic analysis is similar to static analysis except that there is the extra dimension of time to take into account. Static analysis, using elastic theory, requires the solution of equilibrium equations while taking account of compatibility which relates the elastic deformations of different parts of the structure to each other. The formal theory usually reduces to a set of simultaneous stiffness equations. This is the basis of slope deflection, moment distribution and the stiffness matrix method. Dynamic analysis essentially involves solving for equilibrium in the same way but with variation in time. Consequently the stiffness equations have additional terms proportional to velocity and acceleration and form sets of second-order differential equations. Thus it is evident that detailed mathematical analysis of structural vibrations requires more calculation than is the case for static problems.

Fortunately, in many design problems it is not necessary to obtain detailed stress distributions at all instants of time. Furthermore, when structures vibrate they tend to adopt smoother deflected shapes than under static loads and the corresponding stresses are more evenly distributed. These factors make it possible to model structures in less fine detail than for static analysis and still obtain accurate results.

It is even possible to obtain useful practical information for many structures when they are reduced to *single-degree-of-freedom* systems. An example is shown in Fig. 1.2(a), where a single-storey dwelling is subjected to air blast loading. The structure may be idealized as shown in Fig. 1.2(b) where the pitched roof is reduced to a rigid beam and the wall framing is treated as two vertical cantilevers built in at the base. This system may be further reduced to the simple mass and spring of Fig. 1.2(c). The mass can only move horizontally, hence the single degree of freedom. The product of the pressure and the area over which it is effectively distributed resolves to a point load varying with time. A simplified model of this kind has, in fact, been used for calculating the vibrations of buildings under blast loading (Dowding, Fulthorpe and Langan, 1982).

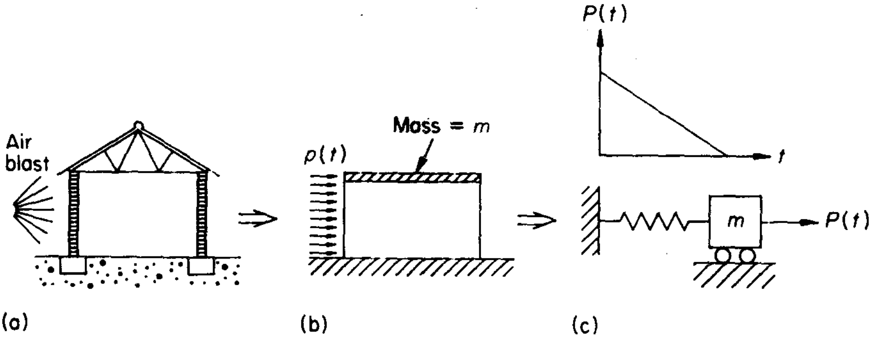

Fig. 1.2 Modelling of a structure for dynamic analysis.

A larger, more important structure may demand more detailed modelling. But even so a judicious engineer will start with a simple model to estimate the vibration approximately before proceeding to a more realistic model. An example of an offshore oil production structure is shown in Fig. 1.3. The varying-diameter tubular members are represented by thin rods connected together while the flexible soil foundation is represented by a system of springs. The stiffness properties of the members and the springs have to be assigned rather carefully, but the resulting simplified model is capable of being analysed on a computer of modest size.

With mention of computers, this is the right point to state that one of the features of this book will be its emphasis on computer analysis using the finite element method. Perhaps the most useful aspect of the finite element method is that it frees engineers from advanced mathematical analysis and enables them to concentrate on choosing appropriate and realistic models of structures using the excellent graphical facilities that are now available.

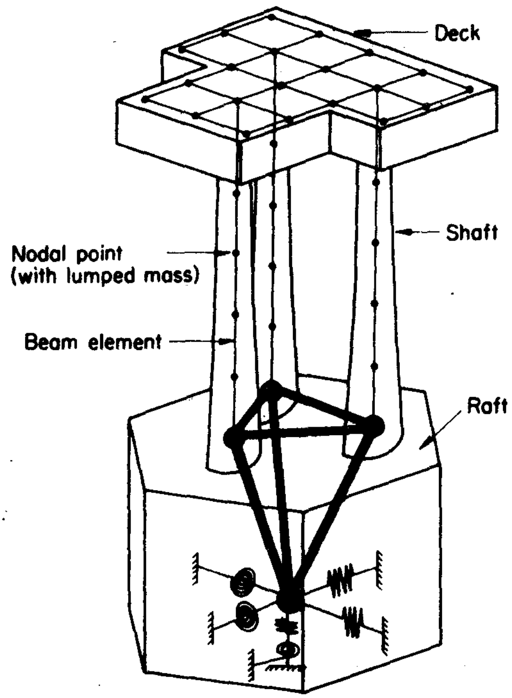

Fig. 1.3 Simplified modelling of a three-dimensional structure (with permission of *Offshore Engineer*).

1.5 CONSEQUENCES OF VIBRATION

Vibration of structures is undesirable for a number of reasons, as follows:

(a) overstressing and collapse of structures,
(b) cracking and other damage requiring repair,
(c) damage to safety-related equipment,
(d) impaired performance of equipment or delicate apparatus,
(e) adverse human response,
(f) fatigue fracture.

With modern forms of construction it is feasible to design buildings to resist the forces arising from major earthquakes. The essential requirement is to prevent total collapse and consequent loss of life. For economic reasons, however, it is the accepted practice to absorb the earthquake energy by ductile deformation, therefore accepting that repair might be required. Blasting, pile-driving and sonic booms can cause superficial damage in the form of cracked plaster and broken window panes. Minor structural damage is also