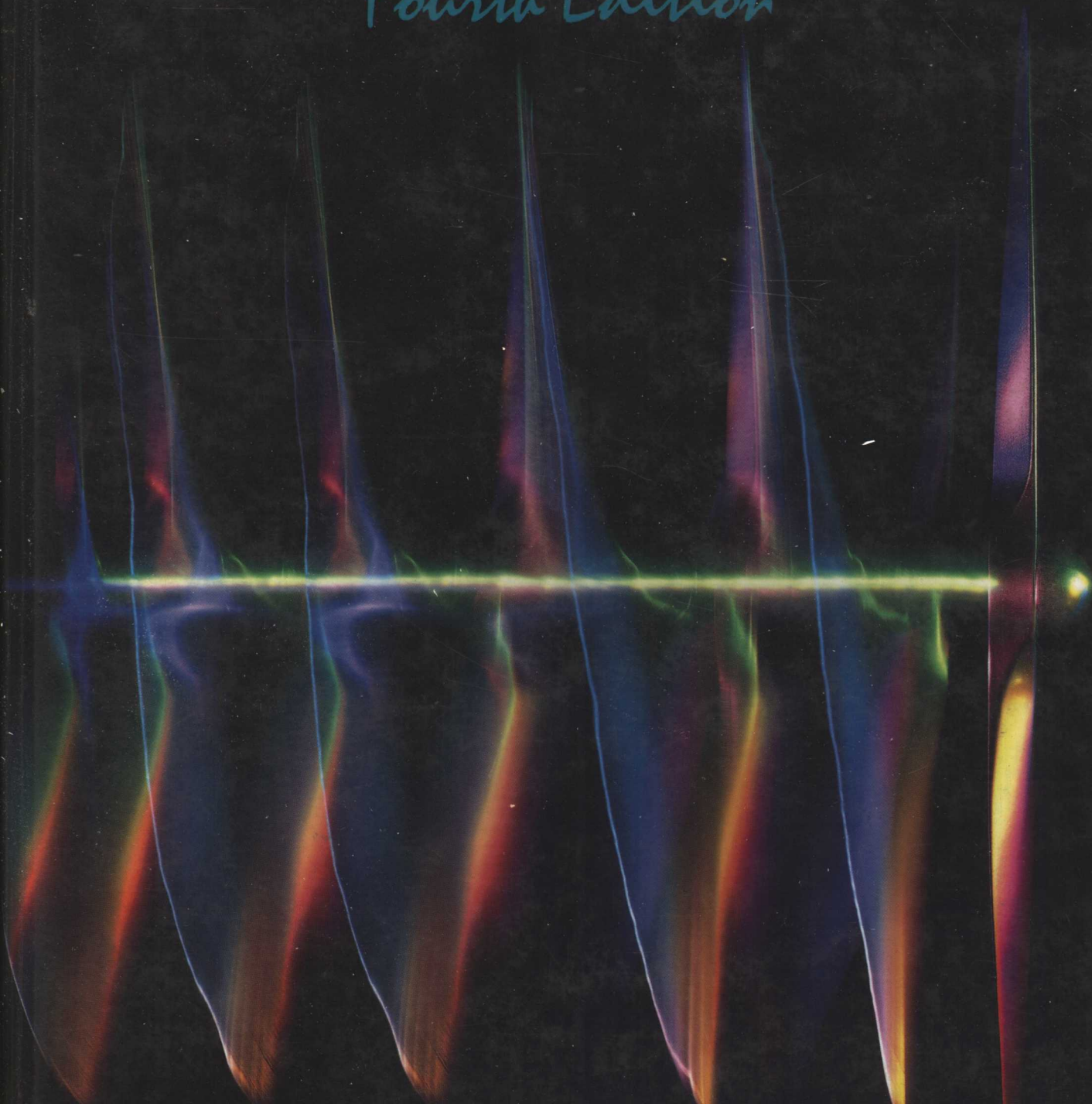


# CALCULUS

Larson / Hostetler / Edwards

*Fourth Edition*



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# CALCULUS

## WITH ANALYTIC GEOMETRY

*Fourth Edition*

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# Preface

*Calculus with Analytic Geometry, Fourth Edition*, is designed for use in a calculus course for students in engineering, economics, life sciences, mathematics, and the physical sciences. In writing the text, we were guided by two primary objectives that have crystallized over many years of teaching calculus. For the student, our objective was to write in a precise, readable manner and to clearly define and demonstrate the basic concepts of calculus. For the instructor, our objective was to design a comprehensive teaching instrument that uses proven pedagogical techniques, thus freeing the instructor to make the most efficient use of classroom time.

## Changes in the Fourth Edition

In the Fourth Edition, all examples, theorems, definitions, and prose have been revised—or at least considered for revision. Many new examples, exercises, and applications have been added to the text. The major changes in the Fourth Edition, listed by chapter, are as follows:

- The basic organization of Chapter 1 is the same as in the Third Edition. Section 1.4 contains a new summary of Equations of Lines, and Section 1.5 contains a new summary of the graphs of basic functions.
- Chapter 2 begins with a new introduction to the tangent line problem. Section 2.1 has a new example dealing with the  $\epsilon$ - $\delta$  definition applied to a quadratic function. Section 2.3 begins with a new summary of a strategy for finding limits.
- The “Four-Step Process” terminology has been eliminated from Section 3.1 in favor of simply applying the definition of the derivative. The introduction to the Chain Rule has been rewritten.
- Section 4.3 on the First Derivative Test has been reorganized. The section on business and economics applications has been moved to the end of the chapter.
- At the request of several users of the text, Chapter 5 has been reorganized. In an effort to get to the Fundamental Theorem of Calculus earlier in the

chapter, we moved the section on integration by substitution to Section 5.5. The material on sigma notation was condensed and now appears as the first part of Section 5.2. Numerical integration was moved from Chapter 8 to Section 5.6, and the introduction of the natural logarithmic function was postponed to Chapter 6.

- Chapter 6 has also been reorganized. It now begins with an introduction to the natural logarithmic function. There is a separate section on bases other than  $e$  and applications (Section 6.5) and a separate section on growth and decay (Section 6.7).
- In Chapter 7, we moved the material on arc length and surface area to Section 7.4, preceding the section on work.
- The section on numerical integration was moved from Chapter 8 to Chapter 5. In Section 8.2, the tabular method of repeated integration by parts was added to the text.
- At the request of many users of the Third Edition, the introduction to Taylor polynomials, which was in the first section of Chapter 9, has been moved to Section 9.7.
- In Chapter 10, the material on classifying conics was moved from Section 10.3 to Section 10.4.
- In Chapter 11, the section on area and arc length in polar coordinates now occurs before the section on polar equations for conics and Kepler's Laws.
- Although the table of contents for the five sections in Chapter 13 is the same as it was in the Third Edition, this chapter was substantially rewritten—especially the material introducing the arc length function and the material on curvature.
- New material on computer graphics in three dimensions was added to Section 14.1. A new discussion of continuity of a function of three variables was added to Section 14.2. The material on complex zeros of polynomial functions was deleted from Section 14.9, and a new example on least squares regression was added. In Section 14.10, the method of Lagrange multipliers was rewritten to agree with standard presentation.
- In Chapter 15, the introduction to Jacobians was rewritten and moved from the fourth section to the end of the chapter.
- As with Chapter 13, the table of contents for Chapter 16 is the same as in the Third Edition, but the chapter was substantially rewritten. Section 16.1 now formally defines inverse square fields, has five new examples, and introduces divergence of a vector field. In Section 16.2, the introduction to line integrals was rewritten, and two new examples were added. In Section 16.3, Theorem 16.7 is new. In Section 16.5, Examples 2 and 5 were rewritten, and material on Gauss's Law was added.

## Features

**Order of Topics** The seventeen chapters readily adapt to either semester or quarter systems. In each system both differentiation *and* integration can be introduced in the first course of the sequence. There is some flexibility in the order and depth in which the chapters can be covered. For instance, much of the precalculus material in Chapter 1 can be used as individual review. The  $\varepsilon$ - $\delta$  discussion of limits in Chapter 2 can be given minimal coverage. Sections 7.6 and 7.7 can be covered later in the course. Chapter 9 can be covered at

any time after Chapter 8. The coverage of Section 12.7 can be delayed until just before Section 15.8.

**Definitions and Theorems** Special care has been taken to state the definitions and theorems simply, without sacrificing accuracy.

**Proofs** We have chosen to include only those proofs that we have found to be both instructive and within the grasp of a beginning calculus student. Moreover, in presenting proofs, we have found that extensive detail often obscures rather than illuminates. For this reason, many of the proofs are presented in outline form, with an emphasis on the essence of the argument. (See the proof of the Product Rule in Section 3.4.) In some cases, we have included a more complete discussion of proofs in Appendix A. (See the proof of the Chain Rule in Section 3.5.)

**Graphics** The Fourth Edition has over 2325 figures. Of these, over 1080 are in the examples and exposition, over 650 are in the exercise sets, and over 590 are in the odd-numbered answers. The new art program in the Fourth Edition was computer generated for accuracy. Designed with additional colors used systematically for clarity, this improved art program will help students better visualize mathematical concepts, particularly in the presentation of complex, three-dimensional material. For example, axes and primary graphs are brown, while planes are blue and primary three-dimensional surfaces are red.

**Enhanced Presentation** As a new feature, the Fourth Edition has been designed with a functional use of four colors that strengthens the text as a pedagogical tool. Each color is used consistently to aid both reading and reference. For example, all theorems and definitions are highlighted by brown boxes, and equation side comments are given in red.

**Exercises** Over 1000 new problems have been added, so the text now contains nearly 8200 exercises. The exercises are graded, progressing from skill-development problems to more challenging problems involving applications and proofs. Many exercise sets begin with a group of exercises that provides the graphs of functions involved. Review exercises are included at the end of each chapter.

**Examples** The text contains nearly 1000 examples, each titled for easy reference. Many of the examples include red side comments that clarify the steps of the solution.

**Computer/Calculator Exercises** Another new feature in the Fourth Edition is the addition of exercises involving the use of a computer or graphics calculator. See, for instance, Exercises 69 and 70 in Section 1.3 or Exercises 79–82 in Section 1.6.

**Numerical Methods** With the increasing power and accessibility of computers, numerical techniques are becoming more widely used. This edition reflects this trend by introducing numerical integration earlier in the text, and by adding a tabular method of integration by parts. Calculators or computers are useful in these areas as well as with topics such as limits, Newton's Method, and Taylor polynomials.

**Chapter Introductions** As a new feature in the Fourth Edition, each chapter begins with a chapter overview and a special motivational application.

**Applications** We chose applications that offer variety and integrity, and that require a minimal knowledge of other fields. The Fourth Edition contains over 2000 different applications.

**Summaries** Many sections have summaries that identify core ideas and procedures—see Sections 3.4, 4.10, 6.7, 9.6, and 13.5. In some instances, an entire section summarizes the preceding topics—see Sections 4.6 and 8.1.

**Historical Notes** Throughout the text, we include several short biographical notes about prominent mathematicians. These are designed to help students gain an appreciation of both the people involved in the development of calculus and the nature of the problems that calculus was designed to solve.

**Remarks** The text contains many special instructional notes to students in the form of “Remarks.” These notes appear after definitions, theorems, or examples and are designed to give additional insight, help avoid common errors, or describe generalizations.

## Supplements

### For Students

- The *Study and Solutions Guide* by David E. Heyd contains detailed solutions to several representative problems from each exercise set. In the text, these exercises are identified in the exercise sets by blue numbers. The solutions to these exercises are given in greater detail than the examples in the text, with special care taken to show the algebra involved. In addition, the *Study and Solutions Guide* contains a review of algebra and a set of true-false questions for each chapter in the text.
- *The Algebra of Calculus* by Eric Braude provides additional calculus review. Keyed to the text, it offers a review of calculus topics, examples with solutions, related problems and their answers, and exercises with answers in the back of the book.

### For Instructors

- A *Complete Solutions Guide* by Dianna L. Zook is available in three volumes. This guide contains brief solutions to every exercise in the text, including exercises requiring proofs.
- An *Instructor's Guide* by Ann R. Kraus contains, for each chapter in the text, sample tests, concept reviews in the form of true-false questions, and suggestions for classroom instruction.
- A package of color transparencies of figures from the text is available.

### For Students and Instructors

- The *Calculus Applications in Engineering and Science* supplement contains solved examples and exercises (with answers to the odd-numbered ones) covering applications in engineering, physics, chemistry, biology, and other fields.

## Technology

### Software

A library of software products for this text is available for the IBM-PC, Macintosh, and Apple II. To make the most of the software products described below, instructors or students may choose to solve appropriate exercises from the text and those provided in a User Manual as well as analyze functions of the user's own selection. All of the following products can be used for classroom demonstration and have been class tested.

- *Math Lab Calculus*, The Math Lab (IBM-PC, Apple II)  
Offering both two-dimensional and three-dimensional computer graphics and numerical computations, the package consists of one disk and a User Manual with 80 lab assignments. These assignments are presented in worksheet format and are keyed to the text. Additionally, all computer/calculator exercises in the text can be solved using this software.
- *Math Utilities* (updated version 4.0), Bridge Software (IBM-PC)  
This is a program of three powerful graphing packages: CURVES, SURFS (Surfaces), and DIFFS (Differential Equations). Each package includes one disk and a User Manual. These enhanced graphing packages allow the user to save graphs and offer labelling and annotating capabilities for creating classroom handouts.
- *BestGrapher*, George Best (IBM-PC, Macintosh)  
This software graphs both two- and three-dimensional figures as well as offers some features that help teach concepts through simulation, such as rotation of a curve about an axis. A lab package that accompanies the software provides assignments in a worksheet format keyed to the text.
- *Computer Activities for Calculus*, Technology Training Associates (IBM-PC)  
This package includes three disks and a User Manual. Each easy-to-use disk has two units of tutorials for self-study and the Grapher, a graphing tool for two-dimensional figures that is designed to handle a wide variety of elementary functions.
- *TrueBASIC CALCULUS*, *TrueBASIC* (IBM-PC) and *TrueBASIC MULTIVARIATE CALCULUS: MACFUNCTION* (Macintosh)  
Each package includes one disk and a User Manual. *TrueBASIC CALCULUS* performs numerical routines and two-dimensional graphing for topics in the first two semesters of calculus. *MACFUNCTION* is a tool for plotting and examining three-dimensional graphs of functions.
- *Workout for Calculus*, Fourth Edition, Joseph Mazur (IBM-PC)  
This tutorial software offers exercises with on-line hints and guidance for student self-study. Keyed on screen to the text, the software generates problems and then adjusts the difficulty of the exercises with sensitivity to the student's level of accomplishment.
- *Derive*, Soft Warehouse (IBM-PC)  
Special discounts on this computer algebra software are available upon adoption of the text.



- *Calculus*, Broderbund (Macintosh)

Special discounts on this tutorial software are available upon adoption of the text.

#### Videotapes

- *Calculus Videotapes*, Video Tutorial Services, Inc.

Interactive videotapes combine thorough coverage of major topics with state-of-the-art computer graphics. Keyed to the text, these videotapes will help students practice, learn, and review calculus. A *Study Guide* and an *Instructor's Guide* are also available.

#### Calculators

- *HP28S*, *HP48SX*, *Casio FX7000G*, *Casio FX7500G*

Special discounts on these calculators are available upon adoption of the text.

### Computerized Testing

- *HeathTest Plus for Calculus* (IBM-PC, Apple II, Macintosh)

Instructors can produce chapter tests, mid-terms, and final exams easily and accurately. Instructors can also edit existing questions or add new ones as desired, or preview questions on screen and add them to a test with a single keystroke. The software supports graphics and offers both multiple-choice and open-ended questions. A User Manual and a printed test item file are available.

- *HeathTest for Calculus* (IBM-PC)

This is an algorithm-based program that generates tests, quizzes, or worksheets in a multiple-choice format. A User Manual with printed test items accompanies the software.

## Acknowledgments

We would like to thank the many people who have helped us at various stages of this project during the past seventeen years. Their encouragement, criticisms, and suggestions have been invaluable to us.

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Several other people also worked on this project: David E. Heyd wrote the *Study and Solutions Guide* and solved the exercises, Dianna L. Zook wrote the *Complete Solutions Manual*, Ann R. Kraus wrote the *Instructor's Guide*, Timothy R. Larson prepared the art, Linda L. Kifer proofread the galleys, Linda M. Bollinger proofread the galleys and typed the supplements, Helen Medley solved the exercises and performed an accuracy check for the text, and Kathleen Evanoff, Randall Hammond, and Paula Sibeto solved the exercises and assisted with the production of the supplement package.

A special note of thanks goes to the over 500,000 students who have used earlier editions of the text.

On a personal level, we are grateful to our wives, Deanna Gilbert Larson, Eloise Hostetler, and Consuelo Edwards, for their love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write to us. Over the past seventeen years we have received many useful comments from both instructors and students, and we value these very much.

Roland E. Larson, Robert P. Hostetler, Bruce H. Edwards

# Calculus

To accommodate the different methods of teaching calculus, D. C. Heath also offers the two texts described below. Each has its own supplements package. The following is a brief discussion of how each book differs from CALCULUS WITH ANALYTIC GEOMETRY, Fourth Edition.

**Calculus with Analytic Geometry, Alternate Fourth Edition with Late Trigonometry**  
**Larson/Hostetler**

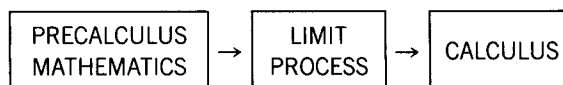
This text, with late trigonometry, is also designed for a three-semester course. Additionally, the text offers a different treatment of the following topics: limits, applications of integration, exponential and logarithmic functions, and vectors.

**Calculus with Analytic Geometry, Third Edition, Part I**  
**Larson/Hostetler**

This single-variable text is designed for a two-semester course. All six trigonometric functions are reviewed in Chapter 1, then used throughout the text.

# What Is Calculus?

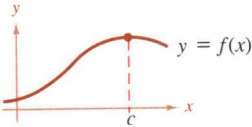
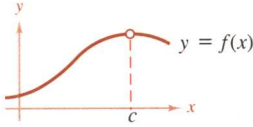
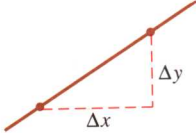
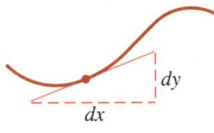






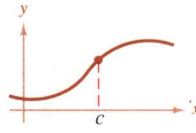
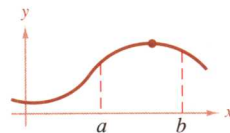
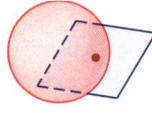
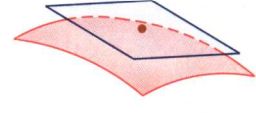


We begin to answer this question by saying that calculus is the reformulation of elementary mathematics through the use of a limit process. If limit processes are unfamiliar to you, then this answer is, at least for now, somewhat less than illuminating. From an elementary point of view, we may think of calculus as a “limit machine” that generates new formulas from old. Actually, the study of calculus involves three distinct stages of mathematics: *precalculus mathematics* (the length of a line segment, the area of a rectangle, and so forth), the *limit process*, and new *calculus* formulations (derivatives, integrals, and so forth).


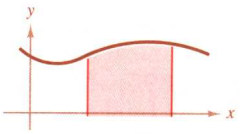
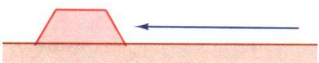
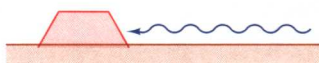
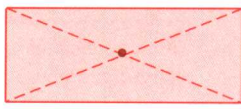
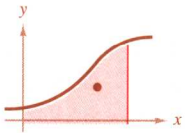


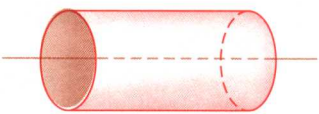
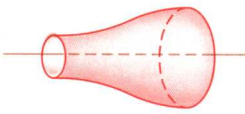
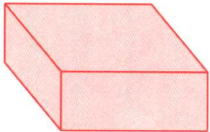
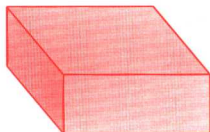
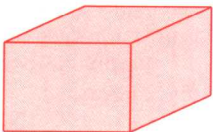
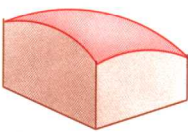


Some students try to learn calculus as if it were simply a collection of new formulas. This is unfortunate. When students reduce calculus to the memorization of differentiation and integration formulas, they miss a great deal of understanding, self-confidence, and satisfaction.

On the following two pages we have listed some familiar precalculus concepts coupled with their more powerful calculus versions. Throughout this text, our goal is to show you how precalculus formulas and techniques are used as building blocks to produce the more general calculus formulas and techniques. Don't worry if you are unfamiliar with some of the “old formulas” listed on the following two pages—we will be reviewing all of them.

As you proceed through this text, we suggest that you come back to this discussion repeatedly. Try to keep track of where you are relative to the three stages involved in the study of calculus. For example, the first three chapters break down as follows: precalculus (Chapter 1), the limit process (Chapter 2), and new calculus formulas (Chapter 3). This cycle is repeated many times on a smaller scale throughout the text. We wish you well in your venture into calculus.

WITHOUT CALCULUS	WITH DIFFERENTIAL CALCULUS
<p>value of <math>f(x)</math> when <math>x = c</math></p> 	<p>limit of <math>f(x)</math> as <math>x</math> approaches <math>c</math></p> 
<p>slope of a line</p> 	<p>slope of a curve</p> 
<p>secant line to a curve</p> 	<p>tangent line to a curve</p> 
<p>average rate of change between <math>t = a</math> and <math>t = b</math></p> 	<p>instantaneous rate of change at <math>t = c</math></p> 
<p>curvature of a circle</p> 	<p>curvature of a curve</p> 
<p>height of a curve when <math>x = c</math></p> 	<p>maximum height of a curve on an interval</p> 
<p>tangent plane to a sphere</p> 	<p>tangent plane to a surface</p> 
<p>direction of motion along a straight line</p> 	<p>direction of motion along a curved line</p> 

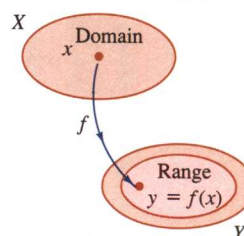
WITHOUT CALCULUS	WITH INTEGRAL CALCULUS
area of a rectangle 	area under a curve 
work done by a constant force 	work done by a variable force 
center of a rectangle 	centroid of a region 
length of a line segment 	length of an arc 
surface area of a cylinder 	surface area of a solid of revolution 
mass of a solid of constant density 	mass of a solid of variable density 
volume of a rectangular solid 	volume of a region under a surface 
sum of a finite number of terms $a_1 + a_2 + \cdots + a_n = S$	sum of an infinite number of terms $a_1 + a_2 + a_3 \cdots = S$

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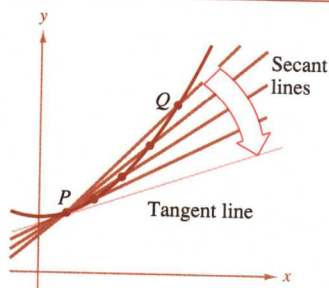
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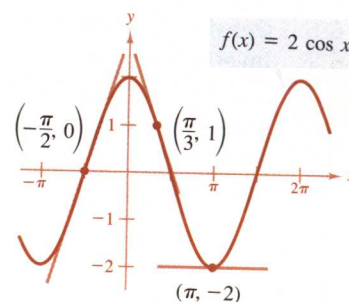


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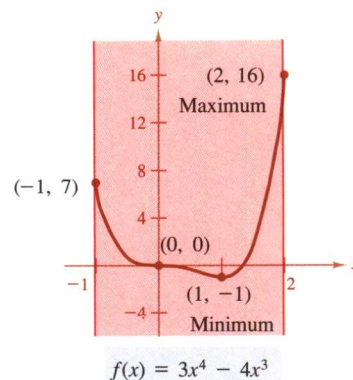
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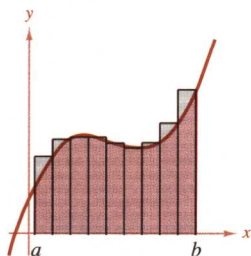


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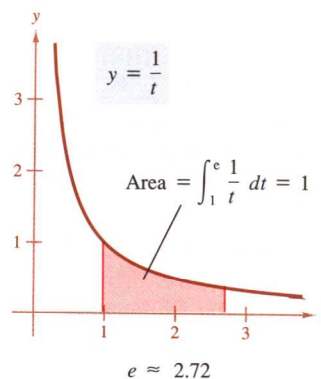
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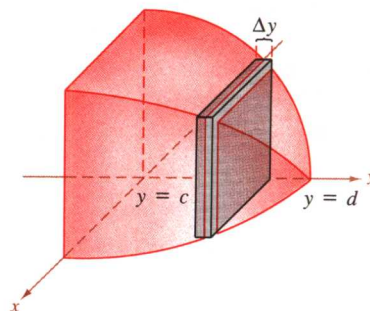
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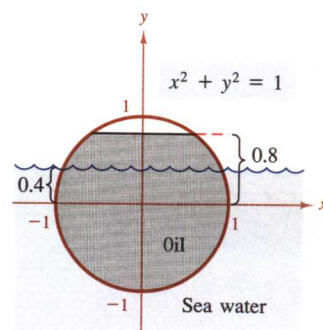
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