COLLEGE ALGEBRA

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P R E F A C E

Textbooks in mathematics must address a variety of interests and concerns if they are to be successful tools for the teacher and the student.

Intent/Purpose

As co-author of a calculus text and a professor at an urban public university, I am aware of the needs of the student who has completed 2 years of high school algebra and intends to go on to study calculus. At the same time, though, the needs of the student who intends to study finite mathematics, business calculus, or discrete mathematics, or who is not planning further study in mathematics, have not been neglected. A conscious effort has been made to serve each constituency through suitable motivation and encouragement.

Contents and Organization

The topics contained in this book were selected and organized with the different types of students who take this course in mind. Thus, throughout the book, a deliberate attempt is made to preview topics and techniques that will be used in later courses.

The first part of the book, Chapters 1–3, provides a detailed presentation of material needed for the later, essential, topics. Chapter 1, in the main, consists of review material. Although complex numbers are introduced in Chapter 1, this section may be postponed at the discretion of the instructor. If the instructor chooses to cover complex numbers later, Section 2.4 (Quadratic Equations with a Negative Discriminant) would also be postponed and then covered with Section 1.9. Whether complex numbers are covered early or late, all exercises dependent upon complex numbers are clearly identified in the directions for the exercise.

The coverage of geometry in Chapter 1 should also be noted. No review would be complete without a discussion of the Pythagorean Theorem and its converse, and the formulas for perimeter and area of a rectangle, area and circumference of a circle, and volume of a sphere.

For some students, Chapters 2 and 3 will represent new material; for others, they will be review. The large majority, of course, fall somewhere in between. Chapter 2 presents an example of careful presentation of topics for the benefit of both instructor and student. In Section 2.3, quadratic equations are treated solely from the real-number point of view; in Section 2.4, quadratic equations with complex-number solutions are discussed. This ensures flexibility of coverage for the instructor and clarity of understanding for the student.

The middle part of the book, Chapters 4–7, covers the essential topics of college algebra. In Chapter 4, special emphasis is placed on functions and the graphs of functions. Graphing is usually done in steps, all of which are illustrated. The graphing techniques introduced in Chapter 4 are utilized and reinforced using the new functions introduced in Chapters 5 and 6.

Because of the need to evaluate polynomial functions, both synthetic division and the nested form of a polynomial are utilized in Chapter 5 as alternatives to substitution. The nested-form technique will be especially appreciated by students of computer science. Precision-type results for finding the zeros of a polynomial, such as the Rational Root Theorem and Descartes' Rule of Signs, are introduced to facilitate graphing of polynomial and rational functions. Approximating results, such as upper and lower bounds on zeros and partition techniques, are discussed in a separate section to provide a clear sense of the difference between numerical and nonnumerical methods.

Chapter 6 treats the exponential and logarithmic functions in the detail necessary and with language consistent for subsequent use in calculus. Applications to both compound interest and growth and decay are given.

Chapter 7 discusses a variety of methods (four in all) for solving systems of linear equations. Sections on systems of nonlinear equations and systems of linear inequalities are also included.

The last part of the book covers topics that may be selected by determining the specific needs of the students. For example, Chapter 8 includes an introduction to matrices and linear programming, topics of use to students intent on taking finite mathematics. Also included is a section on vectors that would be of help to students who will go on to study calculus. Note that partial fraction decomposition is presented in Section 8.3, after systems of equations rather than after rational functions. In this way it provides not only an application for solving systems of linear equations, but also allows for a full and detailed discussion. This should prove useful to students going on to calculus.

Chapter 9 presents mathematical induction, the binomial theorem, and sequences, and Chapter 10 discusses sets, counting, and probability. Each of these chapters contains applications of interest to the student in computer science, business, and mathematics.

The final chapter in the book provides a detailed look at conics. Emphasis is on graphing, analyzing equations, and applications.

Writing Style

The writing style in this book is directed toward the reader; every effort has been taken to be clear, precise, and consistent. Whenever it seemed appropriate, encouragement has been offered; and whenever necessary, warnings have been given.

Applications

Every opportunity was taken to present understandable, realistic applications consistent with the abilities of the student, drawing from such sources as tax rate tables, the *Guinness Book of World Records*, and newspaper articles. For added interest, some of the applied exercises have been adapted from textbooks the students may be using in other courses (for example, economics, chemistry, physics, etc.).

Historical Notes

William Schulz of Northern Arizona University has provided historical context and information in anecdotes that appear as introductory material and at the ends of many sections. In some cases, these comments also include exercises and discussion of comparative techniques.

Examples, Exercises, and Illustrations

The text includes 530 examples and 4000 exercises, of which 500 are word problems. The examples are worked out in appropriate detail, starting with simple, reasonable problems and working gradually up to more complicated ones.

Exercises are numerous, well-balanced, and graduated. They include a number of exercises where the student will need a calculator for their solution; these are clearly marked. A few computer problems also have been included.

Illustrations are abundant, numbering over 450. Full use is made of a second color to help clarify and highlight.

A Word about Format and Design

Each chapter-opening page contains a table of contents for that chapter and an overview—often historical—of the contents.

New terms appear in boldface type where they are defined throughout. The more important definitions are shown in color. New terms always appear in the margin for easy reference and are listed in the Chapter Review at the end of every chapter.

Theorems are set with the word "Theorem" in the margin for easy identification; if it is a named theorem, the name also appears in the margin. When a theorem has a proof given, the word "Proof" appears in the margin to mark the beginning of the proof clearly, and the symbol ■ is used to indicate the end of the proof.

All important formulas are enclosed by a box and shown in color.

Examples are numbered within each section and identified clearly with the word "Example" in the margin. The solution to each example appears immediately following the example with the word "Solution" in the margin to identify it. The symbol ■ indicates the end of each solution.

Each section ends with an exercise set. Each chapter ends with a Chapter Review containing a vocabulary list, a selection of fill-in-the-blank questions (to test vocabulary and formulas), and a set of review exercises. Answers are given in the back of the book for all of the odd-numbered exercises.

Supplementary Material

Student's Solutions Manual to accompany College Algebra, by Christopher Lattin, consists of complete step-by-step solutions for the odd-numbered exercises.

Instructor's Solutions Manual to accompany College Algebra is the complement to the above volume. It contains step-by-step solutions to the even-numbered exercises.

DellenTest is an ipsTest which uses computer software to randomly generate tests using an IBM PC.

Transparency Acetates, prepared by Roger Carlsen, consist of approximately 150 transparencies and overlays that duplicate important illustrations in the text. Another 25 are supplementary to those found in the text.

Acknowledgments

Textbooks are written by an author, but evolve from an idea into final form through the efforts of many people. Before initial writing began, a survey was conducted which drew nearly 250 instructor responses. The manuscript then underwent a thorough and lengthy review process, including class-testing at Chicago State University. I would like to thank my colleagues and students at Chicago State, who cooperated and contributed to this text while it was being class-tested.

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CONTENTS

CHAPTER 1

PRELIMINARIES

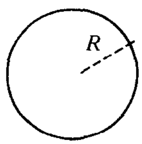
The word *algebra* is derived from the Arabic word *al-jabr.* This word is a part of the title of a ninth century work, "Hisâb al-jabr w'al-muqâbalah," written by Mohammed ibn Mûsâ al-Khowârizmî. The word *al-jabr* means "a restoration," a reference to the fact that if a number is added to one side of an equation, then it must also be added to the other side in order to "restore" the equality. The title of the work, freely translated, means "The science of reduction and cancellation." Of course, today, algebra has come to mean a great deal more.

1.1 REAL NUMBERS

Algebra can be thought of as a generalization of arithmetic in which letters take the place of numbers and symbols are used to indicate the common arithmetical operations of addition, subtraction, multiplication, and division. The laws of algebra are therefore based on the laws of arithmetic.

VARIABLE
CONSTANT

We will use letters such as x, y, a, b, and c to represent numbers. If the letter represents *any* number, it is called a **variable**; if the letter represents a *particular* number, it is called a **constant**. For example, in the formula for the area of a circle, $A = \pi R^2$, R is a variable representing the radius of the circle, A is a variable representing the area of the circle, and π (the Greek letter pi) is a constant, with a value of approximately 3.14. See Figure 1.



$A = \pi R^2$

FIGURE 1

EQUATION
EQUAL SIGN

The formula $A = \pi R^2$ is called an **equation**, and the symbol =, called an **equal sign**, is used to express the idea that the number A on the left of the equal sign is the same as the number π multiplied by R squared on its right. Let's review three important properties of equality; in what follows, a and b represent any number.

REFLEXIVE PROPERTY

1. The **reflexive property** states that a number always equals itself; that is, $a = a$. Although this result seems obvious, it forms the basis for much of what we do in algebra.

SYMMETRIC PROPERTY

2. The **symmetric property** states that, if $a = b$, then $b = a$. This, too, is an often-used property.

PRINCIPLE OF SUBSTITUTION

3. The **principle of substitution** states that, if $a = b$, then we may substitute b for a in any expression containing a.

The familiar operations of addition, subtraction, multiplication, and division of two numbers a and b are symbolized in algebra by

Addition: $a + b$	Subtraction: $a - b$
Multiplication: $a \cdot b$ or ab	Division: $\frac{a}{b}$ or a/b

Thus, in algebra we avoid using the multiplication sign $\times$ and the division sign $\div$ so familiar in arithmetic. Notice also that, when two expressions are placed next to each other without an operation symbol, it is understood that the expressions are to be multiplied. Thus, $2x$ means 2 times x.

Sets

When it is preferred to treat a collection of similar objects as a whole, we use the idea of a **set**. For example, the set of *digits* consists of the collection of numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. If we use the symbol D to denote the set of digits, then we can write

SET

$$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

where the braces { } are used to enclose the objects in the set.

In listing the objects of a set, we do not list an object more than once because the reason for listing the objects is to tell which objects belong to the set. Also, the order in which the objects are listed does not matter. Thus, for example, {2, 3} and {3, 2} both represent the same set.

If every object of a set A is also an object of set B, then we say that A is a **subset** of B. If two sets A and B have the same objects, then A **is equal** to B. For example, {1, 2, 3} is a subset of {1, 2, 3, 4, 5}; {1, 2, 3} equals {2, 3, 1}.

SUBSET

EQUAL SETS

Classification of Numbers

It is helpful to classify the various kinds of numbers we deal with in algebra. The **counting numbers** or **natural numbers** are the numbers 1, 2, 3, 4, ... (The three dots, called an **ellipsis**, indicate that the pattern here continues indefinitely.) As their name implies, these numbers are used to count things. For example, there are 26 letters in our alphabet; there are 100 cents in a dollar.

COUNTING NUMBERS

NATURAL NUMBERS

ELLIPSIS

The **integers** or **whole numbers** are the numbers . . . , −3, −2, −1, 0, 1, 2, 3, ...

INTEGERS

These numbers prove useful in certain situations. For example, if your checking account has \$10 in it and you write a check for \$15, you can represent the current balance as −\$5.

Notice that the counting numbers are included among the integers. Each time we expand a number system, such as from the counting numbers to the integers, we do so in order to be able to handle new, and usually more complicated, problems. Thus, the integers allow us to solve problems requiring both positive and negative counting numbers, such as profit/loss, height above/below sea level, temperature above/below zero, and so on.

But integers alone are not sufficient for *all* problems. For example, they do not answer questions such as "What part of a dollar is 38 cents?" or "What part of a pound is 5 ounces?" To answer such questions, we enlarge our number system to include rational numbers. For example, $\frac{38}{100}$ answers the question "What part of a dollar is 38 cents?" and $\frac{5}{16}$ answers the question "What part of a pound is 5 ounces?"

RATIONAL NUMBER
NUMERATOR
DENOMINATOR

A **rational number** is a ratio a/b of two integers; the integer a is called the **numerator** and the integer b, which cannot be zero, is called the **denominator**.

Examples of rational numbers are $\frac{3}{4}, \frac{5}{2}, \frac{0}{4}, -\frac{2}{3}$, and $\frac{100}{3}$. Since $a/1 = a$ for any integer a, it follows that the rational numbers contain the integers as a special case.

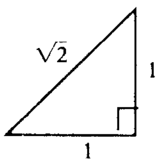


FIGURE 2

Although rational numbers occur frequently enough in applications, numbers that are not rational do also. For example, consider an isosceles right triangle whose legs are each of length 1; the number that equals the length of the hypotenuse, namely $\sqrt{2}$, is not rational. See Figure 2. Also, the number π, referred to earlier in the formula $A = \pi R^2$, is not a rational number.

IRRATIONAL NUMBERS
REAL NUMBERS

Numbers that are not rational are called **irrational**; together, the rational numbers and irrational numbers form the **real numbers**.

See Figure 3.

Decimals

DECIMALS

To represent a real number, we can use **decimals**. For example, the rational numbers $\frac{3}{4}, \frac{5}{2}, -\frac{2}{3}$, and $\frac{7}{66}$ are represented as decimals merely by carrying out the division:

$$\frac{3}{4} = 0.75 \qquad \frac{5}{2} = 2.5 \qquad -\frac{2}{3} = -0.666\ldots \qquad \frac{7}{66} = 0.1060606\ldots$$

TERMINATING DECIMALS
REPEATING DECIMALS

It can be shown that the decimal associated with a rational number is always one of two types: **terminating**, or ending, as in $\frac{3}{4} = 0.75$ and $\frac{5}{2} = 2.5$; or **repeating**, as in $-\frac{2}{3} = -0.666\ldots$, where the 6 repeats, and $\frac{7}{66} = 0.1060606\ldots$, where the block 06 repeats.

Although it may seem that these two types of decimals would be the only types, there are, in fact, infinitely many decimals that neither repeat nor terminate. The decimal 0.12345678910111213 . . . , in which we write down the positive integers one after the other, for example, will neither repeat (think about it) nor terminate. In fact, every irrational number is represented by a decimal that neither

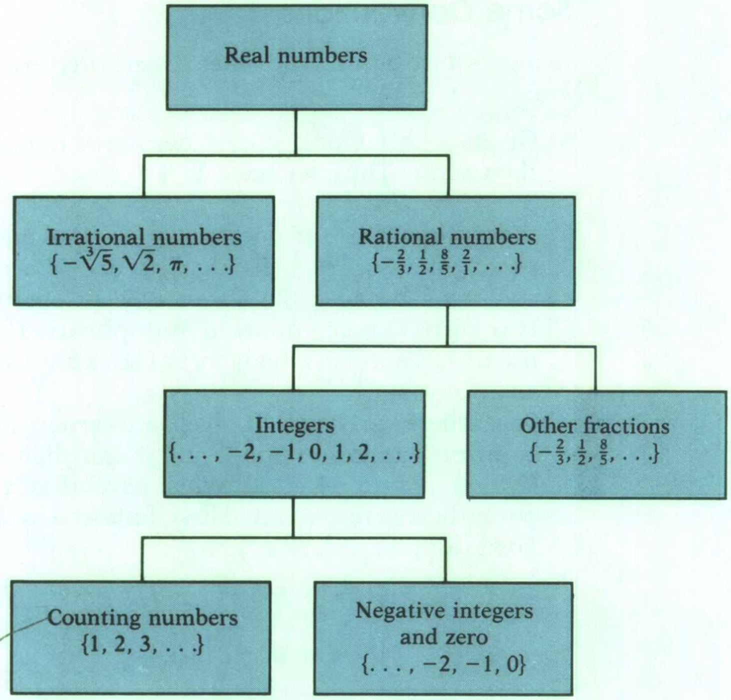


FIGURE 3

repeats nor terminates. For example, the irrational number $\sqrt{2}$ and π have decimals that neither repeat nor terminate:

$$\sqrt{2} = 1.414213\ldots \qquad \pi = 3.14159\ldots$$

Thus, every decimal can be associated with a real number and, conversely, every real number can be represented by a decimal. It is this feature of real numbers that gives them their practicality. In the physical world, many changing quantities like the length of a heated rod, the velocity of a falling object, and so on, are assumed to pass through every possible magnitude from the initial one to the final one as they change. Real numbers as decimals provide a convenient way to measure such quantities as they change.

In practice, it is usually necessary to represent real numbers by approximations. For example, using the symbol ≈ (read "approximately equal to"), we can write

$$\sqrt{2} \approx 1.4142 \qquad \pi \approx 3.1416$$

We shall discuss the idea of approximation in more detail in Section 1.10.