SOIL MECHANICS

CALCULATIONS PRINCIPLES AND METHODS

Victor N. Kaliakin

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ELSEVIER
Butterworth-Heinemann
An imprint of Elsevier
elsevier.com

Butterworth-Heinemann is an imprint of Elsevier
The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, United Kingdom
50 Hampshire Street, 5th Floor, Cambridge, MA 02139, United States

Library of Congress Cataloging-in-Publication Data
A catalog record for this book is available from the Library of Congress

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library

ISBN: 978-0-12-804491-9

Publisher: Joe Hayton
Acquisition Editor: Ken McCombs
Editorial Project Manager: Peter Jardim
Production Project Manager: Mohanapriyan Rajendran
Designer: Maria Ines Cruz

Typeset by TNQ Books and Journals

Preface

The first course in soil mechanics typically proves to be challenging for undergraduate students. This is due to the fact that soils are three-phase particulate materials, and thus must be treated differently than other engineering materials that undergraduates are introduced to as part of their curriculum. The situation is further complicated by the need to account for the presence of pore fluid, both under hydrostatic and transient conditions, as well as the subject of shear strength.

One of the biggest difficulties in teaching soil mechanics is the lack of lecture time in which to present a sufficient number of example problems, with varying degrees of difficulty, that illustrate the concepts associated with the subject. This book has been written to address the aforementioned shortcoming. It presents worked example problems that will facilitate a student's understanding of topics presented in lecture. This book is not meant to replace existing soil mechanics textbooks but to serve as a supplementary resource.

Victor N. Kaliakin

Acknowledgments

Professor Namunu (Jay) Meegoda from the New Jersey Institute of Technology first suggested the idea for the present book and encouraged me to undertake the task of writing it. The example problems presented in the book have been developed over several years of teaching soil mechanics. Some of the more challenging problems are patterned after similar ones that were provided by my former University of Delaware colleague, Dr. Dov Leshchinsky. Finally, special thanks goes to my current colleague, Dr. Kalehiwot Nega Manahiloh for critically reviewing select chapters of the book and for providing some ideas for example problems.

Cheers,
Victor N. Kaliakin

Contents

Chapter 1

Example Problems Involving Phase Relations for Soils

1.0 GENERAL COMMENTS

Soils are prime examples of complex engineering materials, whereas in elementary physics, solid, liquid, and gaseous states are distinguished. Soils are not simple bodies that can be placed in one of these three groups. Soils are generally composed of *solid*, *liquid*, and *gas*, with the solid part being a porous medium made up of numerous particles. Soils are thus *particulate* materials.

The behavior of soils is largely determined by the *relative* amounts of the aforementioned constituents. To quantify these relative amounts requires knowledge of the "mass—volume" or "weight—volume" relations. These relations quantify a soil's *aggregate properties*.

1.1 GENERAL DEFINITIONS

The volume of the various constituents of a soil is quantified by following quantities:

V = total volume of a soil. In some books V_t denotes the total volume.
V_v = volume of the voids (pores).
V_s = volume of the solid phase.
V_a = volume of the gas in the voids.
V_w = volume of the liquid in the voids.

Thus, for all soils

$$V = V_v + V_s = (V_a + V_w) + V_s \tag{1.1}$$

The mass of the various constituents of a soil is quantified by following quantities:

M = total mass of a soil. In some books M_t denotes the total mass.
M_a = mass of the gas in the voids (pores) = 0.
M_w = mass of the liquid in the voids.
M_s = mass of the solid phase.

Soil Mechanics. http://dx.doi.org/10.1016/B978-0-12-804491-9.00001-X

The weight of the various constituents of a soil is quantified by following quantities:

W = total weight of a soil. In some books W_t denotes the total weight.
W_a = weight of the gas in the voids (pores) = 0.
W_w = weight of the liquid in the voids.
W_s = weight of the solid phase.

Thus, for all soils

$$W = W_w + W_s \tag{1.2}$$

Remark: If $V_v = V_w (\Rightarrow V_a = 0)$ and $W_w \neq 0$, the soil is said to be *saturated*; otherwise it is *unsaturated*.

A very convenient, although somewhat idealized, way in which to visualize the mass–volume and weight–volume relations is through the use of phase diagrams. A phase diagram depicts the three phases of a soil as being segregated. For example, Figure 1.1 shows a phase diagram that relates the volume and mass of the three phases.

Figure 1.2 shows a similar phase diagram that relates the volume and weight of the three phases.

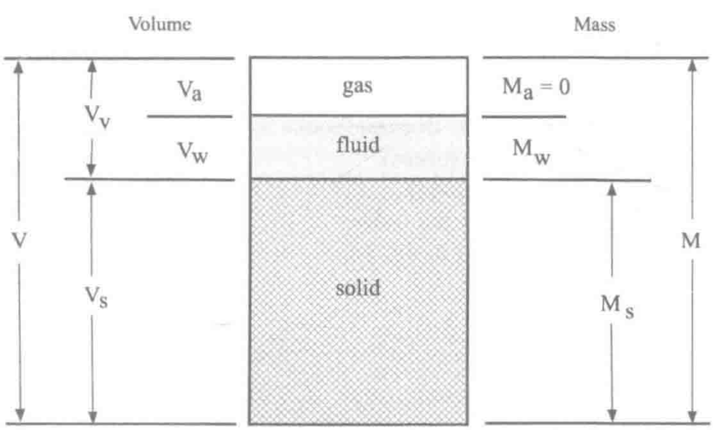

FIGURE 1.1 Phase diagram showing the relationship between volume and mass of gas, fluid, and solid phases in a soil.

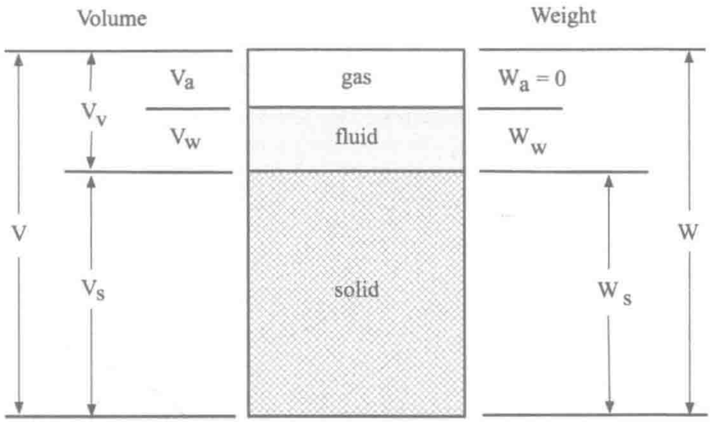

FIGURE 1.2 Phase diagram showing the relationship between volume and weight of gas, fluid, and solid phases in a soil.

1.2 MASS DENSITIES

The following mass densities are used to quantify the relative amounts of a soil's constituents:

- Soil (moist) mass density:

$$\rho = \frac{M}{V} \tag{1.3}$$

- Solid mass density:

$$\rho_s = \frac{M_s}{V_s} \tag{1.4}$$

- Dry mass density:

$$\rho_d = \frac{M_s}{V} \tag{1.5}$$

- Mass density of water:

$$\rho_w = \frac{M_w}{V_w} \tag{1.6}$$

At 4°C, $\rho_w = \rho_0 = 1000\ \text{kg/m}^3 = 1\ \text{g/cm}^3 = 1\ \text{Mg/m}^3 = 1.941\ \text{slug/ft}^3$. For ordinary engineering applications at other temperatures, $\rho_w \cong \rho_0$.

1.3 UNIT WEIGHTS

The following unit weights are used to quantify the relative amounts of a soil's constituents:

- Soil (moist) unit weight:

$$\gamma = \frac{W}{V} = \frac{Mg}{V} = \rho g \tag{1.7}$$

- Solid unit weight:

$$\gamma_s = \frac{W_s}{V_s} = \frac{M_s g}{V_s} = \rho_s g \tag{1.8}$$

- Dry unit weight:

$$\gamma_d = \frac{W_s}{V} = \frac{M_s g}{V} = \rho_d g \tag{1.9}$$

- Unit weight of water:

$$\gamma_w = \frac{W_w}{V_w} \tag{1.10}$$

At 4°C, $\gamma_w = \gamma_0 = 9810\ \text{N/m}^3 = 9.81\ \text{kN/m}^3 = 62.4\ \text{lb/ft}^3$. For ordinary engineering applications at other temperatures, $\gamma_w \approx \gamma_0$. In the equations, $g = 9.81\ \text{m/s}^2 = 32.2\ \text{ft/s}^2$ is the gravitational acceleration.

1.4 DEFINITION OF FUNDAMENTAL QUANTITIES

The specific gravity of solids is defined as follows:

$$G_s = \frac{\gamma_s}{\gamma_0} \approx \frac{\gamma_s}{\gamma_w} = \frac{W_s}{V_s \gamma_w} \tag{1.11}$$

Remark: G_s normalizes the solid unit weight of a material.

The volume of voids is defined by two quantities, namely the porosity n, and the void ratio e, where,

$$n = \left(\frac{V_v}{V}\right) * 100\% \tag{1.12}$$

and

$$e = \frac{V_v}{V_s} \tag{1.13}$$

The *relative* weight and volume of the pore fluid is quantified by the moisture content (w) and the degree of saturation (S), where,

$$w = \left(\frac{W_w}{W_s}\right) * 100\% \tag{1.14}$$

and

$$S = \left(\frac{V_w}{V_v}\right) * 100\% \tag{1.15}$$

For a *saturated* soil, $V_w = V_v$ and $S = 100\%$.

1.5 RELATIONS DERIVED FROM FUNDAMENTAL QUANTITIES

The basic quantities G_s, n, e, w, and S can be suitably combined to form relations that are particularly useful for particular types of problems. These relations *do not*, however, constitute any new definitions of quantities used to describe the phase relations for soils. Some specific examples of such relations are given in the following section.

1.5.1 Case 1.1: Relation Between Void Ratio and Porosity

Rewriting the void ratio definition in terms of the volume of voids (V_v) and then dividing through by the total volume (V) gives the following relation:

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{V_v/V}{1 - (V_v/V)} = \frac{n}{1-n} \tag{1.16}$$

where n is understood to be a decimal number.

1.5.2 Case 1.2: Relation Between Porosity and Void Ratio

Rewriting the porosity definition by expanding the total volume (V) and then dividing through by the volume of solids (V_s) gives the following relation:

$$\begin{aligned} n &= \left(\frac{V_v}{V}\right) * 100\% = \left(\frac{V_v}{V_s + V_v}\right) * 100\% = \left(\frac{V_v/V_s}{1 + V_v/V_s}\right) * 100\% \\ &= \left(\frac{e}{1+e}\right) * 100\% \end{aligned} \tag{1.17}$$

This result could likewise have been obtained by solving the equation derived in Case 1.1 for porosity in terms of void ratio.

1.5.3 Case 1.3: Relation Between Moisture Content, Specific Gravity of Solids, Void Ratio, and Degree of Saturation

The weight of the solid phase is written in terms of G_s as follows:

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{W_s}{V_s \gamma_w} \Rightarrow W_s = G_s V_s \gamma_w \tag{1.18}$$

Next, the weight of the pore fluid is written in terms of γ_w.

$$\gamma_w = \frac{W_w}{V_w} \Rightarrow W_w = V_w \gamma_w \tag{1.19}$$

Substituting Eqs. (1.18) and (1.19) into the definition of the moisture content (Eq. 1.14) gives,

$$w = \left(\frac{W_w}{W_s}\right) * 100\% = \left(\frac{V_w \gamma_w}{G_s V_s \gamma_w}\right) * 100\% = \left(\frac{V_w}{G_s V_s}\right) * 100\% \tag{1.20}$$

The volume of pore fluid is next written in terms of the degree of saturation; i.e.,

$$S = \left(\frac{V_w}{V_v}\right) * 100\% \Rightarrow V_w = \left(\frac{S}{100\%}\right) V_v = \left(\frac{S}{100\%}\right) e V_s \tag{1.21}$$

where the definition of the void ratio has been used. Substituting Eq. (1.21) into Eq. (1.20) gives the desired relation; i.e.,

$$w = \frac{Se}{G_s} \quad \text{or} \quad Se = G_s w \tag{1.22}$$

where w and S are understood to be decimal numbers.

The aforementioned expression shows that the moisture content (w) is thus a function of *three* quantities, namely e, S, and G_s. The upper bound on w corresponds to the case of full saturation (i.e., $S = 100\%$), when $w \equiv w_{sat} = e/G_s$. The lower bound on w is zero, which corresponds to a completely dry soil for which $S = 0\%$.

1.5.4 Case 1.4: Relation Between Dry Unit Weight, Specific Gravity of Solids, and Void Ratio

Beginning with the definition of the dry unit weight given by Eq. (1.9), substituting for W_s in terms of G_s gives,

$$\gamma_d = \frac{W_s}{V} = \frac{G_s V_s \gamma_w}{V_s + V_v} \tag{1.23}$$

Dividing through the resulting expression by V_s gives the desired relation; i.e.,

$$\gamma_d = \frac{G_s \gamma_w}{1 + e} \tag{1.24}$$

1.5.5 Case 1.5: Relation Between Moist Unit Weight, Specific Gravity of Solids, Moisture Content, and Void Ratio

Beginning with the definition of the moist unit weight given by Eq. (1.7), and representing the weight of the pore fluid in terms of the w and W_s gives

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v} = \frac{W_s(1+w)}{V_s + V_v} \tag{1.25}$$

where w is understood to be a decimal number. Substituting for W_s in terms of G_s (i.e., $W_s = G_s V_s \gamma_w$) and diving through the resulting expression by V_s gives the desired relation; i.e.,

$$\gamma = \frac{G_s \gamma_w (1+w)}{1+e} \tag{1.26}$$

1.5.6 Case 1.6: Relation Between Moist Unit Weight, Dry Unit Weight, and Moisture Content

In light of Eq. (1.24), the relation for γ derived in of Case 1.5 becomes

$$\gamma = \gamma_d (1+w) \quad \text{or} \quad \gamma_d = \frac{\gamma}{(1+w)} \tag{1.27}$$

where w is understood to be a decimal number.

1.5.7 Case 1.7: Relation Between Moist Unit Weight, Specific Gravity of Solids, Degree of Saturation, and Void Ratio

Replacing the moisture content in Eq. (1.26) with the relation derived in Case 1.3 (i.e., $w = Se/G_s$) gives

$$\gamma = \frac{\gamma_w (G_s + Se)}{1+e} \tag{1.28}$$

1.5.8 Case 1.8: Unit Weight of Submerged Soil and Its Relation to Moist Unit Weight

Consider a saturated soil that is submerged in water. According to Archimedes' principle, the buoyancy force acting on a body is equal to the weight of the fluid displaced by the body.

Since the soil is saturated, $S = 100\%$ and $V_w = V_v$. The buoyant unit weight is thus

$$\gamma_b = \frac{(W_s - V_s \gamma_w) + (W_w - V_v \gamma_w)}{V_s + V_v} \tag{1.29}$$

Writing W_s in terms of G_s and W_w in terms γ_w gives

$$\gamma_b = \frac{(G_s V_s \gamma_w - V_s \gamma_w) + (V_v \gamma_w - V_v \gamma_w)}{V_s + V_v} = \frac{\gamma_w V_s (G_s - 1)}{V_s + V_v} \tag{1.30}$$

Dividing through the equation by V_s gives the final expression for the buoyant unit weight; i.e.,

$$\gamma_b = \frac{\gamma_w (G_s - 1)}{1 + e} \tag{1.31}$$

For a saturated soil the expression for moist unit weight given by Eq. (1.28) reduces to

$$\gamma = \gamma_{sat} = \frac{\gamma_w (G_s + e)}{1 + e} \tag{1.32}$$

Manipulating this expression gives the relationship between the saturated and buoyant unit weights; i.e.,

$$\gamma_{sat} = \frac{\gamma_w (G_s + e)}{1 + e} = \frac{\gamma_w (G_s - 1)}{1 + e} + \frac{\gamma_w (1 + e)}{1 + e} = \gamma_b + \gamma_w \tag{1.33}$$

or

$$\gamma_b = \gamma_{sat} - \gamma_w \tag{1.34}$$

EXAMPLE PROBLEM 1.1

General Remarks

Knowing the definitions of the basic quantities e, n, w, S, and G_s, it is relatively straightforward to derive more specific relations than those presented in Cases 1.1–1.8.

Problem Statement

Derive an expression for void ratio (e) in terms of the total weight (W), total volume (V), the unit weight of water (γ_w), the degree of saturation (S), and the specific gravity of solids (G_s).

Solution

Recall the relation for moist unit weight derived in Case 1.7 (Eq. 1.28); i.e.,

$$\gamma = \frac{W}{V} = \frac{\gamma_w (G_s + Se)}{1 + e} \tag{1.1.1}$$

Solving for the void ratio leads to the following results:

$$e + 1 = \left(\frac{V \gamma_w}{W}\right)(G_s + Se) \Rightarrow e\left(1 - \frac{V}{W}\gamma_w S\right) = \frac{V}{W}\gamma_w G_s - 1 \tag{1.1.2}$$