Fluid Flow, Heat and Mass Transfer at Bodies of Different Shapes

Numerical Solutions

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Preface

Fluid mechanics is one of the oldest branches of applied mathematics. It is also the foundation of the understanding of various aspects of science and engineering. A wide variety of mathematical problems, appearing in areas as diverse as fluid mechanics, mechanical engineering, chemical engineering, theoretical physics, and aerospace engineering, have been solved by means of analytical or numerical methods. Though analysis of different types of fluid flow and heat or mass transfer problems are available in the open literature, there are still a number of gaps that are to be filled up. There are several excellent books covering different aspects of fluid flow and heat or mass transfer. Yet one still looks for a systematic and sequential analysis that helps in understanding this particular area of interest.

To help students and researchers acquire a deeper understanding of the characteristics of fluid flow and heat and mass transfer, this monograph aims to present, in general, a study of transport phenomena. It is well known that for external flows, the shape of the object influences the flow over an object (i.e., a body) significantly. As a result, it affects the heat and mass transfer characteristics. In other words, the book aims to help readers develop their understanding in this particular field without spending huge time in searching the endless literature on this area. To help develop a clear insight, we discuss several flow features. By maintaining the applicability of the obtained results, we also discuss several cases of physical problems.

In selecting specific problems to work through, we have restricted our attention to the phenomena of fluid flow and heat or mass transfer as such problems introduce a wide variety of mathematical problems of interest. Hence, in order to illustrate various properties and tools useful in analyzing the problems, we have selected recent research in the area of fluid flow and heat or mass transfer.

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Introduction

Air and water are the most important constituents of the environment we live in, so that almost everything we do is connected to the science of fluid mechanics. For example, the flight of birds in the air and the motion of fish in the water can be explained from the perspective of fluid mechanics [1]. The designs of airplanes and ships are based on the theory of fluid mechanics. Fluid mechanics is one of the oldest branches of applied mathematics, and the foundation of the understanding of different aspects of science and engineering [2–5]. From the nineteenth century, the scope of fluid mechanics has steadily broadened, as the study of hydraulics was associated with the growth of the fields of civil engineering and naval architecture. In recent times, the development of the different branches of engineering, namely, aeronautical, chemical, and mechanical engineering, have given additional stimuli to the study of fluid mechanics. It now ranks as one of the most important basic subjects not only in applied mathematics but also in engineering [6]. Now, it is a subject of widespread interest in almost all fields of engineering as well as in astrophysics, meteorology, physical chemistry, plasma physics, geophysics, biology, and biomedicine [7–9].

In nature, fluid flow over bodies occurs frequently and gives rise to numerous physical phenomena, for example, drag force acting on trees, underwater pipelines, automobiles, the lift generated by airplane wings, upward draft of rains, dust particles in high winds, and transportation of red blood cells in blood flow (see, [6]). Sometimes, fluid moves over a stationary body, for example, wind blowing over a building, or a body moving through a quiescent fluid or a bus moving through air. Such motions are referred to as flows over bodies or external flows [10]. The shape of the object has profound influence on the flow over a body and thus affects significantly the heat and mass transfer characteristics. Flow past bodies can be classified into incompressible and compressible flows. Compressibility effects are neglected at velocities below 360 km/hour, and such types of flows are known as incompressible flows. In this book, we are concerned with incompressible fluid flows [11–17].

Because of the recent high demand in the need for understanding and analyzing the problems we come across in science and engineering, we feel that there is a need for a book of this kind. The underlying aim of this book is to present transport phenomena that will help students and researchers in the field of fluid mechanics in acquiring a deeper understanding of the characteristics of flow and heat and mass transfer (see, e.g., [18]). Obviously, part of the material in the book can be conveniently used as an introductory course material for researchers working in boundary layer theory, flow, and heat and mass transfer [19–45]. Also, the book is intended for graduate students in mathematics, engineering, and in the mathematical sciences. In addition, the material in the book may be of interest to researchers working in physical chemistry, soil physics, meteorology, and nanotechnology.

The book is designed to accommodate several topics of varying emphasis, and the chapters comprise fairly self-contained material from which one can make various coherent selections. There are several underlying themes that become apparent when one examines the literature on the subject. We hope to bring out clear insight by discussing several flow features. Also, we discuss several cases of physical problems in general.

The outline of the book will be as follows. Chapter 1 deals with the numerical method(s) adopted in these works. In Chapters 2–4, which comprise Part I of the book, we present the flow past surfaces of different types, namely, stretching, shrinking, and flat surfaces. This first set of chapters provides explanations intended for general readers and can be directly employed for problems in engineering, applied physics, and other applied sciences. We keep the discussion broad based so as to provide a framework for researchers. In order to motivate the reader and provide a good understanding of the subject matter, at the end of Chapters 2–4, we provide multiple examples of problems that have been solved numerically.

In Part II of the book, Chapters 5–6, we shift the focus to concrete examples and problems related to bluff bodies in fluid mechanics and heat and mass transfer. Here the governing equations of the problems are highly nonlinear differential equations. The problems considered in this Part will help the reader understand problems of physical relevance and apply it to the other physical fields of interest. We group such problems into three chapters: general fluid flow past a cylinder in Chapter 5, fluid flow over a sphere in Chapter 6, and finally problems related to flow past a wedge in Chapter 7.

References

[1] Yuan SW. Foundations of fluid mechanics. New Delhi: Prentice-Hall of India; 1969.
[2] Abbas Z, Wang Y, Hayat T, Oberlack M. Slip effects and heat transfer analysis in a viscous fluid over an oscillatory stretching surface. Int J Numer Meth Fluid 2009;59: 443–58.
[3] Andersson HI, Bech KH, Dandapat BS. Magnetohydrodynamic flow of a power law fluid over a stretching sheet. Int J Non-Linear Mech 1992;27:929–36.
[4] Ariel PD, Hayat T, Asghar S. The flow of an elastico-viscous fluid past a stretching sheet with partial slip. Acta Mechanica 2006;187:29–35.
[5] Bachok N, Ishak A, Pop I. Mixed convection boundary layer flow over a permeable vertical flat plate embedded in an anisotropic porous medium. Math Problems Eng 2010;659023:http://dx.doi.org/10.1155/2010/659023.
[6] Batchelor GK. An introduction to fluid dynamics. London: Cambridge University Press; 2006.
[7] Bhattacharyya K, Mukhopadhyay S, Layek GC. Slip effects on an unsteady boundary layer stagnation-point flow and heat transfer towards a stretching sheet. Chin Phys Lett 2011;28(9):094702.
[8] Bhattacharyya K, Mukhopadhyay S, Layek GC. MHD boundary layer slip flow and heat transfer over a flat plate. Chin Phys Lett 2011;28(2):024701.
[9] Bhattacharyya K, Vajravelu K. Stagnation-point flow and heat transfer over an exponentially shrinking sheet. Comm Nonlinear Sci Numer Simulat 2012;17(7):2728–34.

[10] Cebeci T, Bradshow P. Momentum transfer in boundary layer. Washington, DC: McGraw-Hill Hemisphere; 1977.
[11] Chamkha AJ, Aly AM, Mansour MA. Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects. Chem Eng Comm 2010;197:846–58.
[12] Cortell R. Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing. Fluid Dyn Res 2005;37:231–45.
[13] Cortell R. Effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet. Phys Lett A 2006;357:298–305.
[14] Crane LJ. Flow past a stretching plate. Z Angew Math Phys 1970;21:645–7.
[15] Dandapat BS, Santra B, Vajravelu K. The effects of variable fluid properties and thermocapillarity on the flow of a thin film on an unsteady stretching sheet. Int J Heat Mass Transfer 2007;50:991–6.
[16] Gupta PS, Gupta AS. Heat and mass transfer on a stretching sheet with suction or blowing. Can J Chem Eng 1977;55:744–6.
[17] Hayat T, Abbas Z, Sajid M. Series solution for the upper-convected Maxwell fluid over a porous stretching plate. Phys Lett A 2006;358:396–403.
[18] Ingham DB, Pop I, editors. Transport phenomena in porous media. Oxford: Elsevier; 2005, vol. III.
[19] Ishak A. Mixed convection boundary layer flow over a vertical cylinder with prescribed surface heat flux. J Phys A: Math Theor 2009;42:195501.
[20] Kumaran V, Banerjee AK, Kumar AV, Vajravelu K. MHD flow past a stretching permeable sheet. Appl Math Comput 2008;210:26–32.
[21] Liu IC, Andersson HI. Heat transfer in a liquid film on an unsteady stretching sheet. Int J Thermal Sci 2008;47:766–72.
[22] Mukhopadhyay S. Effect of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. Int J Heat Mass Transfer 2009;52:3261–5.
[23] Mukhopadhyay S. Effects of slip on unsteady mixed convective flow and heat transfer past a stretching surface. Chin Phys Lett 2010;27(12):124401.
[24] Mukhopadhyay S. Heat transfer analysis for unsteady MHD flow past a non-isothermal stretching surface. Nucl Eng Des 2011;241:4835–9.
[25] Mukhopadhyay S. Chemically reactive solute transfer in boundary layer slip flow along a stretching cylinder. Front Chem Sci Eng 2011;5:385–91.
[26] Mukhopadhyay S, Vajravelu K. Effects of transpiration and internal heat generation/absorption on the unsteady flow of a Maxwell fluid at a stretching surface. ASME J Appl Mech 2012;79:http://dx.doi.org/10.1115/1.4006260, 044508-1-6.
[27] Mukhopadhyay S, Vajravelu K, Van Gorder RA. Flow and heat transfer in a moving fluid over a moving non-isothermal surface. Int J Heat Mass Transfer 2012;55(23-24):6632–7.
[28] Prasad KV, Vajravelu K, Datti PS. Mixed convection heat transfer over a non-linear stretching surface with variable fluid properties. Int J Non-Linear Mech 2010;45:320–30.
[29] Prasad KV, Vajravelu K, Datti PS. The effects of variable fluid properties on the hydromagnetic flow and heat transfer over a non-linearly stretching sheet. Int J Thermal Sci 2010;49:603–10.
[30] Prasad KV, Sujatha A, Vajravelu K, Pop I. MHD flow and heat transfer of a UCM fluid over a stretching surface with variable thermo physical properties. Meccanica 2012;47:1425–39.

[31] Sajid M, Ahmad I, Hayat T, Ayub M. Unsteady flow and heat transfer of a second grade fluid over a stretching sheet. Comm Nonlinear Sci Numer Simulat 2009;14:96–108.
[32] Schlichting H. Boundary-layer theory. New York: McGraw-Hill; 2008.
[33] Sharidan S, Mahmood T, Pop I. Similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet. Int J Appl Mech Eng 2006;11:647–54.
[34] Siddiqui AM, Zeb A, Ghori QK, Benharbit AM. Homotopy perturbation method for heat transfer flow of a third grade fluid between parallel plates. Chaos Solitons Fractals 2008;36:182–92.
[35] Soundalgekar VM, Takhar HS, Vighnesam NV. The combined free and forced convection flow past a semi-infinite plate with variable surface temperature. Nucl Eng Des 1988;110: 95–8.
[36] Takhar HS, Gorla RSR, Soundalgekar VM. Radiation effects on MHD free convection flow of a gas past a semi-infinite vertical plate. Int J Numer Meth Heat Fluid Flow 1996;6:77–83.
[37] Tsai R, Huang KH, Huang JS. Flow and heat transfer over an unsteady stretching surface with a non-uniform heat source. Int Comm Heat Mass Tran 2008;35:1340–3.
[38] Vafai K, Tien CL. Boundary and inertia effects on flow and heat transfer in porous media. Int J Heat Mass Transfer 1981;24:195–204.
[39] Vajravelu K, Prasad KV, Chiu-On Ng. Unsteady flow and heat transfer in a thin film of Ostwald–de Waele liquid over a stretching surface. Comm Nonlinear Sci Numer Simulat 2012;17:4163–73.
[40] Van Gorder RA, Vajravelu K. Hydromagnetic stagnation point flow of a second grade fluid over a stretching sheet. Mech Res Comm 2010;37:113–8.
[41] Van Gorder RA, Sweet E, Vajravelu K. Nano boundary layers over stretching surfaces. Comm Nonlinear Sci Numer Simulat 2010;15:1494–500.
[42] Van Gorder RA, Vajravelu K, Akyildiz FT. Existence and uniqueness results for a nonlinear differential equation arising in viscous flow over a nonlinearly stretching sheet. Appl Math Lett 2011;24:238–42.
[43] Van Gorder RA, Vajravelu K. Convective heat transfer in a conducting fluid over a permeable stretching surface with suction and internal heat generation/absorption. Appl Math Comput 2011;217:5810–21.
[44] Vleggaar J. Laminar boundary-layer behavior on continuous accelerating surfaces. Chem Eng Sci 1977;32:1517–25.
[45] Wang CY. Flow due to a stretching boundary with partial slip—an exact solution of the Navier-Stokes equations. Chem Eng Sci 2002;57:3745–7.

Contents

Part I

Methods and applications

Numerical methods

1

The governing equations of fluid flow problems are generally of a nonlinear and boundary value type. Usually, the exact solutions of the boundary value problems (BVPs) are very difficult to obtain, and so we have to use numerical methods. For some special classes of flow problems, a set of partial differential equations are transformed into a set of ordinary differential equations with the help of similarity variables. The transformed (final) equations then can be solved analytically or numerically. The procedure for finding a numerical solution for a BVP is generally more difficult than that of an initial value problem (IVP). A number of methods can be used to solve linear BVPs. The method of differences is useful in such cases. But this method cannot be used for nonlinear equations. Other methods can be used to obtain linearly independent solutions, which can then be combined in such a way that they satisfy the boundary conditions (Mukhopadhyay [1]). For such problems, the difference method can be adapted. The most popular numerical method is the shooting method. The shooting method can be used for both linear and nonlinear problems. Because the convergence of the method depends on a good initial guess, there is no guarantee that the method will converge. But the method is easy to apply, and when it does converge, it is usually more efficient than other methods (Mukhopadhyay [1], Mukhopadhyay and Layek [2]). Moreover, the shooting method gives more accurate results if guess values (slopes) are chosen correctly.

The basic idea of a shooting method is to replace the BVP by some IVP where the slope at the initial point is obviously unknown. We can guess this unknown quantity, and then, using iteration, the guess value can be improved.

The shooting method consists of the following steps:

1. transformation of the given BVP to an IVP,
2. finding a solution of the IVP, and
3. finding a solution of the given BVP.

Let us consider a nonlinear second-order differential equation $y'' = f(x, y, y')$ with the boundary conditions $y(a) = y_0$ and $y(b) = y_1$.

At first, we set $y' = p$ and $p' = f(x, y, p)$ with $y = y_0$ at $x = a$. Actually, the equation is rewritten in terms of a first-order system of two unknown functions. Because these equations are nonlinear, we cannot get the solution by superposition principle. In order to integrate the above system as an IVP, we require a value for p at $x = a$ that is $y'(a)$. Generally, we take a guess for $p(a)$ and use it to obtain a numerical solution. Then comparing the calculated value for y at $x = b$ with the given boundary condition $y = y_1$ at $x = b$ and adjusting the guess value, $p(a)$, we get a better approximation for the solution. The derivative at $x = a$ gives the trajectory of computed solution. That is why, it is called shooting method. Basically, we seek a solution that also satisfies the relation $y(b) = y_1$. A suitable solution can be found by successively refining the interval. With the help of linear interpolation or other root-finding methods, it is also

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possible to improve the obtained solution. To start the integration, the initial slope, that is, $y'(a)$, is required, and the shooting method depends on the choice of the value of $y'(a)$.

To explain the method, we consider the equation

$$f'''(\eta) + \frac{1}{2} f(\eta) f''(\eta) = 0, \tag{1.1}$$

along with the boundary conditions

$$f(0) = 0,\ f'(0) = 0, \tag{1.2}$$

$$f'(\infty) = 1, \tag{1.3}$$

which is a third-order nonlinear BVP.

Now, the key factor is to choose an appropriate and suitable finite value of η as $\eta \to \infty$, say η_∞. Here $\eta = \eta_\infty$ corresponds to the edge of the boundary layer. For computational purposes, η_∞ is to be chosen arbitrarily larger than the boundary layer thickness. The most important factor of the shooting method is to choose an appropriate finite value of η_∞. In order to determine η_∞ for the BVP stated by equations (1.1)–(1.3), we start with some initial guess value α that must be determined so that the resulting solutions yield the prescribed value $f' = 1$ at $\eta = \eta_\infty$ for some particular set of physical parameters (Mukhopadhyay and Layek [2]). We therefore guess at the initial slope, and an iterative procedure is set up for convergence to the correct slope. A normally better approximation to α can now be obtained by the following linear interpolation formula:

$$\alpha_2 = \alpha_0 + (\alpha_1 - \alpha_0) \frac{f'(\eta_\infty) - f'(\alpha_0, \eta_\infty)}{f'(\alpha_1, \eta_\infty) - f'(\alpha_0, \eta_\infty)},$$

where α_0, α_1 are two guesses at the initial slope $f''(0)$ and $f'(\alpha_0, \eta_\infty), f'(\alpha_1, \eta_\infty)$ are the values of f' at $\eta = \eta_\infty$. We now integrate the differential equation using the initial values $f(0) = 0, f'(0) = 0$, and $f''(0) = \alpha_3$ to obtain $f'(\alpha_2, \eta_\infty)$. Using linear interpolation based on α_1, α_2, we can obtain a next approximation α_3. This process is repeated until convergence is obtained. The convergence depends on a good initial guess (Conte and Boor [3]).

The solution procedure is repeated with another large value of η_∞ until two successive values of $f''(0)$ differ only by the specified significant digit. The last value of η_∞ is finally chosen to be the most appropriate value of the limit η_∞. The value of η_∞ may change for another set of physical parameters, if involved in the problem. Once the finite value of η_∞ is determined, then the integration is carried out. We compare the calculated value for f' at $\eta = 15$ (say) with the given boundary condition $f'(15) = 1$, and the estimated value is adjusted using the secant method to get a better approximation for the solution (Mukhopadhyay [4]).