# STRENGTH AND DEFORMATION OF DETERMINATE STRUCTURES

PRABHAT KUMAR

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# Preface

Engineering education has been mushrooming in India through numerous private engineering colleges and universities. Availability of competent teaching faculties is not proportional to the number of technical institutions. Nobody becomes a qualified and competent engineer just by attending limiting number of lectures and by securing a degree after four years. Student must input huge amount of work at home. A good book which describes all the theories in simple language helps a student at this stage. This book is written with this motivation.

The salient features of this book are as follows:

- It is written in a simple language.
- Entire subject is divided into chapters, sections and subsections.
- Each topic is described in reasonable detail and no aspect is bypassed.
- All technical presentation is serial and repetition is avoided.
- Each concept is illustrated with suitable illustrations known as free body diagrams.
- All the relevant material is included.
- Topics covered are common to syllabus of numerous technical universities of India.
- Technical contents of the book are also useful for students of Architecture.

One very special feature of this book is that this book shall be useful for the students as well as to the teachers of this subject. The detailed derivations and descriptions of underlying concepts make life of a teacher very much easy. Conventionally, a typical textbook presents all theories in one place followed by a section having plethora of solved problems. Present book presents theory followed by solved problems. It educates a student on the application of theory in arriving at a solution. When a problem can be solved by several methods, the same problem is solved by different methods to highlight salient features of method.

Attempt is made to keep text error-free. However, some mistakes in typing and in derivations or solutions may have escaped my attention. I appeal to all teachers and students who choose to read this book to locate all types of mistakes and communicate to me. All suggestions to improve this book are welcome and shall be gracefully accepted. Please feel free to communicate with me.

This book is divided in 14 chapters. Chapter 1 defines structural analysis, why it is done and explains what output is expected from this exercise. Chapters 1 and 2 revise the fundamental concepts of engineering mechanics. Chapter 3 deals with truss structures.

Chapters 4 and 5 describe beams and frames. Drawing of shear force and bending moment diagrams is fundamental in structural analysis. These are needed in design and deflection computation. A student should be able to draw shear force and bending moment diagrams quickly after reading Chapters 4 and 5. Chapter 6 deals with three-hinge arches. Chapter 7 describes methods of deflection computation where expertise of drawing bending moment diagram is put to use. Chapter 8 is devoted to applications of energy methods in structural analysis. Chapter 9 is devoted to rolling loads and construction of influence lines. Use of influence lines in handling rolling loads is delegated to Chapter 10. Chapter 11 deals with simple cases of unsymmetric bending. Chapter 12 describes calculation of shear center in complex structural shapes. Chapter 13 considers analysis of beams which are curved in elevation. Chapter 14 applies vector method to solve three-dimensional problems. This treatment of space structures will not be found in many books on this subject which are currently available in the market. Each chapter of this book ends with a section termed "High Alert" in which salient and important points of the theory of that chapter are summarized. These points make the subject of objective type of questions. Student while reading this book can add more high alert points to the given list. Finally, appendices are given which contain useful information of this subject.

I wish to thank all my teachers, colleagues, friends and students who directly or indirectly helped me in the compilation of this book.

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# Contents

CHAPTER 1

# Basic Fundamentals of Structural Analysis

## 1.0 INTRODUCTION

The title of this subject consists of two terms: Structure and Analysis. It is appropriate to begin with the meaning of each of these terms.

### 1.0.1 Structure

A "structure" is a creation which is conceived and designed to fulfill a particular need. It obeys designated fundamental laws of nature (such as mass, momentum and energy conservation) and at the same time meets all the functional requirements. It performs all its expected functions with assurance of stability and safety throughout its life span.

All the items which we see around us are structures. Majority of structures with which an engineer may have to deal with may be classified as follows.

- Structures which remain attached to the ground all the time. Examples are buildings and bridges.
- Structures, which remain on the ground, and which can move around the ground as well. All simple household gadgets, motorized vehicles, construction equipment and lifting equipment fall in this category. Mechanical machinery falls in between the above two categories in that the unit remains fixed on ground but it may contain some moving parts.
- Structures, which remain on the ground, move around and fly as well but remain within the gravitational field of earth. Aircrafts fall in this category.
- Structures, which fly, and which can go into outer space. Missiles and rockets fall in this category.
- Structures which float over water and can go below the free surface also. Ships and submarines fall in this category.
- Structures like International Space Station (ISS), which float in the outer space. The concepts for design of above mentioned structures must be thoroughly modified before these can be applied for the design of structures like ISS.
- A new category of structures may come up in near future in the form of habitat on planets like moon and mars. Design of such structure is a challenge of 21st century.
- Human body and its parts were considered to be nature made. Now it is possible to apply principles of Engineering Mechanics and analyze human body. It is also possible to manufacture artificial limbs to replace defective body parts, which are damaged either due to some accident or due to natural decay.

- Robots and remotely controlled structures, which can perform some human-like functions.
- Microprocessor controlled and artificial intelligence based structures which can take intelligent decisions based on a built-in logic. This procedure is used in washing machines in which machine can determine the cleaning cycle depending upon the fabric and its condition.

The design of buildings which remain attached to the ground is of primary concern in this course. Classification of buildings follows in Section 1.1. Physical realization of a structure is multi-disciplinary in that it involves various services like electricity, plumbing, carpentry, mechanical operations, and building.

### 1.0.2 Analysis

The second term "analysis" in the title of this subject implies that the mechanism of transfer of external loads, which act on the structure, to the foundations or supports, is to be identified and quantified. Force systems arise in engineering profession and these are of primary concern to any structural designer.

Force is an "external action" which tends to change the current state of rest or motion of an object or body. A body at rest may start to move whereas a moving body may come to rest with the application of force of a suitable magnitude and direction. A force is characterized by magnitude, direction, orientation (or sense) and a point of application. A force is graphically represented as an arrow in which the length is proportional to its magnitude and the arrowhead gives orientation of force. Inclination of line gives direction of force. The tail point of the arrow makes the point of application. A force is sometimes known as load particularly in the terminology of Civil Engineering. When an external agency creates a force and this force acts on the structure, it is more appropriate to call it a load. Force system consists of a number of forces of various magnitudes which act in different directions, at different time, and at different points of application. All components of a concurrent force system meet at a common point. But components of a non-concurrent force system is variable.

In addition to external loads, a structure may be subjected to secondary effects such as temperature, support settlement and lack of fit.

A typical building can be broken down into several individual components. Each component such as column, beam, slab, etc., performs a particular type of load transfer function. All the structural components together combine to transfer external loads to the supports. This actual mechanism of load transfer may involve any combination of structural actions which are described in Section 1.2. Structural analysis is done to determine magnitude and type of structural action for each component of a structure under all forms of external actions and secondary effects.

## 1.1 CLASSIFICATION OF BUILDINGS

NATIONAL BUILDING CODE OF INDIA (NBC–2005) gives most exhaustive classification of buildings. A typical feature of this classification is that the design concept involves several separate and exclusive considerations in addition to safety, stability, economy, functionality and aesthetics. The classification and additional special design considerations are as follows.

### *Residential Buildings*

Most persons are familiar with these. Design concept includes provision of all amenities which are needed in carrying out human activities. Safety, privacy, security and protection must be provided to all occupants of buildings. Residential buildings remain occupied on 24/7 basis. Sleeping is also involved. It is necessary for these buildings to resist emergency for sufficiently long time so that a person wakes up from sleep and escapes to safety.

### *Educational Buildings*

These buildings are occupied mostly during the daytime and large gathering is involved. No sleeping is involved. However, the wood furniture in the classrooms constitutes fire load. So precaution against fire is a design criterion. Live load is present only during daytime or working hours.

### *Institutional Buildings*

These buildings include hospitals, sanatoriums, nursing homes, orphanages, prisons, mental hospitals and reformatories. The occupants may be disabled people or people with restricted liberties so that they may not be able to move at their own. Sleeping accommodation is provided. Occupants must have sufficient time to safely escape in any emergency situation. Gathering however may not be large.

### *Assembly Buildings*

These include buildings where groups of people congregate or gather for amusement and recreation, and social and religious ceremonies. Theater, cinema hall, auditorium, sports ground, exhibition hall, restaurant, club and dance halls come in this category. There must be sufficient provision for all the people to vacate the building in a very short time to meet emergency. Precaution against stampede is necessary.

### *Business Buildings*

Courts, libraries, city hall, town hall and municipal buildings fall in this category. These buildings keep records of accounts and money transactions. Danger of fire breakout and spread must be a minimum. There must be extensive automatic fire safety mechanism.

### *Mercantile Buildings*

Shop, departmental store and super market, etc., are included in this category. Wholesale or retail business is transacted in such buildings. The loss on account of fire may be substantial due to destruction of goods. Even the firefighting operation may damage goods of substantial value. There must be sensitive fire detection mechanism and provision for a large crowd to escape safely.

### *Industrial Buildings*

Buildings in which products or materials of all kind and properties are fabricated, assembled or processed such as refinery, gas plants, mills, etc., come in this category. Some potentially dangerous operations like welding and gas cutting may be taking place. Besides, loud noise may also be produced in some industrial operations. Such buildings are not allowed to operate in populated residential areas. Loading of finished items and unloading of raw materials are involved so that large maneuvering space for commercial vehicles must be provided.

### *Storage Buildings*

Cold storage, transit shed, garage, hangers, grain elevators, warehouse, etc., are included in this category. Although occupancy is not large but loss of goods on account of fire may be substantial. The storage items must not be dangerous. Loading and unloading of materials are involved so enough open space must be allocated.

### *Hazardous Buildings*

Buildings which are used for storage, handling, manufacture or processing of highly combustible explosive materials and products, and dangerous chemicals come in this category. Chemicals may be toxic or highly corrosive and noxious. These materials burn with rapid rate and also produce dangerous fumes and gases. Explosion may also occur which may damage adjacent structures. Such buildings must not be allowed to operate in populated residential areas. Special firefighting procedures are required.

Classification of civil engineering structures can also be based on its use. In addition to above the following types must be classified as structures.

| | |
|---|---|
| Bridges/culverts/drains | To cover a gap without blocking the way underneath |
| Dams and barrage | To store water |
| Canals and irrigation channels | To transport water |
| Roads, airfields and pavements | To facilitate smooth and safe transport and movement of vehicles |

## 1.2 CLASSIFICATION OF STRUCTURAL ACTIONS

The purpose of any structure is to safely transmit all the external forces to the supports. In doing so the load passes through various components of the structure. The primary requirement of a safe design is that no component of a structure must be overloaded under its share of load. This implies that the internal forces must never exceed the available strength of components. Alternatively, the component must be made strong enough to accommodate its share of load without overloading itself. There are four basic internal structural actions through which external forces are transmitted. The overall action of all the above-mentioned structures is some combination of these basic structural actions. Analysis determines this combination of load transfer mechanisms.

### 1.2.1 Axial Action or Column Action

The force (or load) in this structural action moves through the axis of the member. Such members are usually termed columns. Therefore, this action is also called column action. The force which travels along the axis of member may be tensile or compressive. A structural member carrying axial tension is termed a tie and this mode of load transfer becomes tie action. In compressive force, the member is called a strut and this action is termed strut action.

These actions are graphically illustrated in Fig. 1.1. Member under column action either elongates (tensile force) or shortens (compressive force). The changes in length are also axial and take place in the direction of member axis. Columns near the center of floor are subjected to axial action.


**Fig. 1.1** Column action which involves load application and load transfer along column axes which are parallel. Such members are called axially loaded. (a) Strut action (b) Tie action.

### 1.2.2 Bending Action or Beam Action

The beam action is also known as flexural action which arises in a typical beam shown in Fig. 1.2. Forces, which act laterally or perpendicular to the beam axis, cause bending. The structural component (beam) bends either downward or upward as shown in Fig. 1.3, when it is loaded in lateral direction. The upward bending is termed hogging (Fig. 1.3a) while downward bending is called sagging (Fig. 1.3b). Actions known as bending moments produce hogging or sagging deformations.

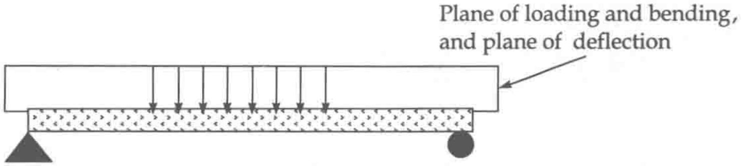


**Fig. 1.2** Direction of load movement along the beam axis is perpendicular to the direction of load application in bending. In a two–dimensional case plane of loading and plane of bending are identical. The beam shown here is simply supported as it is supported at either end by a hinge (shown as a black triangle) and a roller (shown as a black circle) respectively.

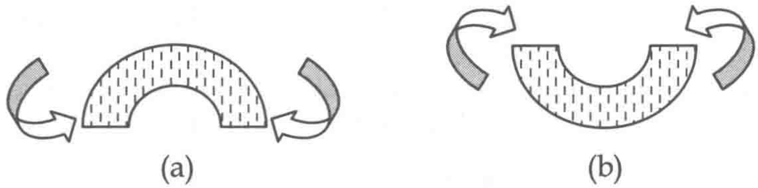


**Fig. 1.3** Plane of bending and plane of loading are same. (a) Bending displacements are upwards in hogging beam action (b) Bending displacements are downwards in sagging beam action.

One end of a cantilever is fixed while the other end is free. Figure 1.4 shows flexural action in cantilever which may be of hogging or sagging type. The plane of moments in bending of beam and cantilever coincides with the plane of deflection. This flexure action is an in-plane or coplanar action.


**Fig. 1.4** Bending takes place at one end only in in-plane flexural (cantilever) action. One end of a cantilever is fixed against all movements. (a) Hogging cantilever action (b) Sagging cantilever action.

The difference between a beam and a cantilever lies in the type of support system. Various types of supports are presented in Section 1.6. The load in flexural actions always moves towards the support. It is shared when more than one support is provided.

### 1.2.3 Shear Action or Slip Action

This action is caused by two equal and opposite parallel forces the lines of action of which are separated by a small distance. Plane of slip lies somewhere in between the lines of action of shearing forces. This action is very relevant in Soil Mechanics as most of the soils fail in shear. The shear action is illustrated in Fig. 1.5. Various components in steel structures and machines are joined together by rivets or bolts. Such rivets and bolts are subjected to shear when the machine components deform under the action of external load.

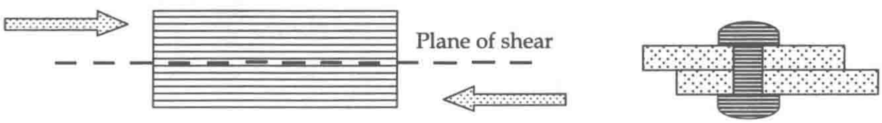


**Fig. 1.5** Shear action in which parts of same body slide in different directions. Shearing takes place along the middle plane shown by dotted line. A rivet is subjected to shearing in a riveted joint.

### 1.2.4 Torsion Action or Shaft Action

Torsion action is caused by torque which acts in a plane perpendicular to the structural plane. This action twists the member. A shaft transfers power from one location to