Signal Processing and Pattern Recognition in Nondestructive Evaluation of Materials

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Edited by C.H.Chen

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Preface

The NATO Advanced Research Workshop on Signal Processing and Pattern Recognition in Nondestructive Evaluation (NDE) of Materials was held August 19-22, 1987 at the Manoir St-Castin, Lac Beauport, Quebec, Canada.

Modern signal processing, pattern recognition and artificial intelligence have been playing an increasingly important role in improving nondestructive evaluation and testing techniques. The cross fertilization of the two major areas can lead to major advances in NDE as well as presenting a new research area in signal processing. With this in mind, the Workshop provided a good review of progress and comparison of potential techniques, as well as constructive discussions and suggestions for effective use of modern signal processing to improve flaw detection, classification and prediction, as well as material characterization.

This Proceedings volume includes most presentations given at the Workshop. This publication, like the meeting itself, is unique in the sense that it provides extensive interactions among the interrelated areas of NDE. The book starts with research advances on inverse problems and then covers different aspects of digital waveform processing in NDE and eddy current signal analysis. These are followed by four papers of pattern recognition and AI in NDE, and five papers of image processing and reconstruction in NDE. The last two papers deal with parameter estimation problems. Though the list of papers is not extensive, as the field of NDE signal processing is very new, the book has an excellent collection of both tutorial and research papers in this exciting new field. While most signal processing work has not yet been integrated into practical NDE systems, as pointed out by Dr. L.J. Bond at the Workshop discussion session, the future direction clearly shows greatly increased use of signal processing in NDE.

I am grateful to all participants for their active participation that made the Workshop very productive, and to NATO Scientific Affairs Division for support. The Workshop format is indeed ideal for a research meeting like this that brings together an interdisciplinary group of researchers. I am confident that this publication can be equally successful in helping to foster continued research interest in NDE signal processing.

C.H. Chen
Workshop Director

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RESEARCH ON INVERSE PROBLEMS IN MATERIALS SCIENCE AND ENGINEERING

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Abstract

The role of inverse problems in the characterisation of materials is discussed. Four such problems are described in detail: deconvolution for acoustic emission, tomographic reconstruction of temperature distribution, electrical-conductivity profiling and inverse scattering. Each exploits a priori information in a different way to mitigate the ill-conditioning inherent in most inverse problems.

Introduction

The importance of inverse problems in the characterisation and processing of materials has increased considerably with the recent growth of advanced sensor technology. Frequently, the quantitative information of interest must be extracted from a physical measurement (or more typically, a set of physical measurements) that by itself may be only indirectly related to the information desired and thus difficult to interpret. For instance, sensor measurements often yield some form of spatial and/or temporal average of the desired information; such an average may, for example, be expressed mathematically in the form of an integral equation (e.g., a convolution) or a system of linear equations (e.g., as in tomography), where the mathematical relationship is derived from a knowledge of the physics governing the measurement.

In an inverse problem one attempts to extract the desired information from measurements containing noise on the basis of an idealised model of the measurement process. The problem is made more difficult since inverse problems are characteristically ill-conditioned; that is, small errors in the measurement typically lead to large errors in the solution. However, nowadays we know that the key to mitigating such ill-conditioning is the judicious use of a priori information. The incorporation of such a priori information often takes the form of constraining the solution to a class of physically reasonable possibilities, or it may take the form of incorporating a priori probabilistic information about the solution or the statistical distribution of measurement errors. The use of a priori information necessarily introduces an element of subjectivity into the problem, since often the choices of a priori constraints (or how they are best incorporated) are not clear cut; such choices are usually decided by experience derived from real problems.

This paper emphasises the point that inverse problems in materials science often offer an unusual abundance of physically-motivated a priori constraints; certainly the possibilities appear greater than in many other fields where inverse problems have traditionally played an important role, such as in medical

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imaging and geophysical prospecting. As a result, the nondestructive characterisation of materials based on ultrasonic and electromagnetic sensors offers an unusually fertile area for innovation in inverse-problem development and application. In this paper we will see several examples of the use of a priori information in problems that have arisen in our work on acoustic emission, ultrasonic and electromagnetic nondestructive evaluation.

In the analysis of acoustic emission signals, a problem of central importance is the deconvolution of the acoustic-emission source signal from the transducer response (characterised by the transducer impulse response) and propagation effects (characterised by the temporally-dependent elastic Green's function of the material). With this problem in mind, a new and robust approach to deconvolution was developed that is particularly well suited for deconvolving causal signals [1]. This approach is described in the next section. In the area of ultrasonics, we describe a technique based on time-of-flight tomography for reconstructing two-dimensional temperature distributions in hot metallic bodies [2]. In this problem, a priori heat-flow information is utilised to help mitigate the effects of severe ill-conditioning in the inversion. The third example is drawn from the area of electromagnetic NDE, in which we describe the problem of reconstructing one-dimensional conductivity profiles from variable-frequency impedance measurements [3]. We conclude with a description of a new iterative approach to the exact, nonlinear inverse-scattering problem [4]. A significant result reported here is the use of an exact expression for the gradient of the measurements with respect to the scattering model. The exact gradient leads to a mean-square-error minimisation algorithm with better stability and a higher rate of convergence compared with most other proposed iterative inverse-scattering schemes.

1. Deconvolution for Acoustic Emission

Acoustic emission may be regarded as naturally generated ultrasound produced by sudden, localized changes of stress in an elastic body. The analysis of acoustic emission signals is complicated by the fact that the observed signal is the two-fold convolution between the source signal, the elastic Green's function characterising the propagating medium, and the detecting transducer's impulse response. In principle, the latter two response functions can be calculated or measured. The problem then reduces to deconvolving the source signal from the transducer and material response functions in the presence of noise.

A wide variety of numerical deconvolution schemes have been proposed over the years by researchers in disciplines ranging from seismology to astronomy. Most modern deconvolution methods exploit some form of regularisation to reduce the sensitivity to measurement errors of an inherently ill-posed inversion problem. A widely-used regularisation approach is to impose some generalised form of smoothing constraint, of which Tikhonov regularisation is the prototype [5]. The latter approach has the undesirable side effect of destroying the causality of the deconvolved signal. The algorithm described below, however, not only preserves causality, but may be thought of as yielding the "best" causal estimate of the original (deconvolved) signal in a least-squares sense [1]. The method exploits the fact that the roots of the Z

transform (or the related Y transform defined below) of a discrete (sampled) signal are preserved under convolution. Recent progress in the development of polynomial root-finding algorithms has now made this powerful approach practical for time series exceeding several thousand samples.

Consider the deconvolution of two discrete-time (i.e., sampled) waveforms represented by the finite time series $\{a_k, k = 0, 1, \ldots, N-1\}$, $\{b_k, k = 0, 1, \ldots, N-1\}$, and their convolution $\{c_k, k = 0, 1, \ldots, 2N-1\}$, where

$$c_k = \sum_{j=0}^{k} a_{k-j}\, b_j. \tag{1.1}$$

Note in particular that the time series we are concerned with here are causal, i.e., are zero for negative k.

One way of representing convolution utilizes a simple modification of Z transforms, which we shall call the Y transform. We define the Y transform $a(y)$ of an infinite causal time series $\{a\}$ by the formal power series

$$a(y) = \sum_{n=0}^{\infty} a_n\, y^n. \tag{1.2}$$

For any finite segment of a causal time series, the Y transform is a polynomial. Here, we want to examine the convolution equation (1.1) in terms of Y transforms, where it can be shown to take the form

$$c(y) = a(y) \cdot b(y), \tag{1.3}$$

that is, the convolution of two time series becomes multiplication of their Y transforms. If we wish to deconvolve $\{b\}$ from $\{c\}$, when $\{a\}$ is known, the formal solution should, in principle, then be

$$b(y) = c(y)/a(y). \tag{1.4}$$

Unfortunately, the division algorithm seldom works in practice because of noise in the data. Due to noise, $a(y)$ does not exactly divide $c(y)$, and the division process magnifies the errors exponentially with increasing terms in the time series.

A second approach is to divide the fast Fourier transforms (FFT's) of the two functions. This idea may also be explained in complex function language as follows. The well-known Cauchy theorem, applied on the unit circle, gives

$$b_k = \frac{1}{2\pi i} \oint \frac{c(y)\, dy}{a(y) y^{k+1}}, \tag{1.5}$$

which is the Taylor's series (i.e., causal time series) for $\{b\}$. If we evaluate this integral numerically by sampling on the unit circle at the points

$$y = e^{2\pi i \ell / N} \qquad \text{for} \qquad \ell = 0, 1, \ldots, N-1, \tag{1.6}$$

equation (1.5) leads directly to the FFT division formula.

In the deconvolution problem, we shall assume that $a(y)$ is given and that $c(y)$ is measured in the presence of noise. We shall further assume that $a(y)$ and $c(y)$ are both causal, and hence $b(y)$ is causal, where $b(y)$ is to be determined.

The difficulty in using the FFT division method for determining the series $\{b\}$ stems from the fact that $a(y)$ often has roots inside the unit circle (typically about $N/2$ such roots). In the integral (1.5) the roots of $a(y)$ become poles and consequently, from complex function theory, equation (1.5) will not in general give the wanted Taylor's series, but rather a Laurent series (a non-causal series which is only valid within an annular region of convergence bounded by the nearest poles bracketing the unit circle). Only if all the roots of $a(y)$ happen to lie outside the unit circle is the method exact.

We cannot do anything about the location of the roots of $a(y)$, since they are characteristic of the series $\{a\}$. The transform of $c(y)$, which is formally the product of $a(y)$ and (the unknown) $b(y)$, should have among its roots all those roots of $a(y)$, including those lying inside the unit circle. In the absence of noise and if the calculation were perfect, these roots of $c(y)$ would exactly cancel the roots of $a(y)$ [the poles of $1/a(y)$] in the division (1.4). The result would be a causal $b(y)$ with a Taylor's series expansion. The reason this approach generally fails is that noise in the measurement of $c(y)$ perturbs the location of the roots so that they are not exactly divided by those of $a(y)$. This suggests that a new and robust deconvolution method could be developed based on a procedure for adjusting $c(y)$ so that its roots include all of those of $a(y)$ inside the unit circle; in this case the FFT division [equations (1.5) and (1.6)] should give a stable, and causal, result.

To adjust $c(y)$, let y be any complex number and consider the N-dimensional vector

$$\underline{y} = (1, y, y^2, \ldots, y^{N-1})^T,$$

where T denotes transpose. Similarly, we can represent the series $\{a\}$, $\{b\}$ and $\{c\}$ as the vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ of appropriate dimension. Then the dot product of $\underline{a}$ and $\underline{y}$ is $a(y)$, and if y_1 is a root of $a(y)$ [i.e., $a(y_1) = 0$], this means that $\underline{y}_1$ is orthogonal to $\underline{a}$. Therefore, if we can find all the roots of $a(y)$ inside the unit circle, we can use powerful least-squares projection methods to adjust the series $\underline{c}$ to a new series $\tilde{\underline{c}}$. The new series $\tilde{\underline{c}}$ can be selected to be the closest one to $\underline{c}$, in a least-squares sense, which is orthogonal to all the geometric root vectors $\underline{y}$, where $\underline{y}$ are the roots of $a(y)$ inside or on the unit circle. To put this another way, we select the new series $\tilde{\underline{c}}$ which minimizes the distance between $\tilde{\underline{c}}$ and $\underline{c}$, i.e., $(\underline{c} - \tilde{\underline{c}})^T(\underline{c} - \tilde{\underline{c}})$, subject to the constraints

$$\tilde{\underline{c}}^T \underline{y}_k = 0, \qquad k = 1, 2, \ldots, K,$$

where $\underline{y}_k$ are the roots of $a(y)$ in and on the unit circle. The latter problem can also be interpreted as selecting $\tilde{\underline{c}}$ as the projection of $\underline{c}$ onto a subspace orthogonal to the space spanned by the geometric root vectors $\underline{y}_k$. This approach can easily be generalized so that the new series $\tilde{\underline{c}}$ can be selected with time

or frequency weighting to take advantage of a priori information about the signal and noise statistics. Now the common roots of $\tilde{c}(y)$ and $a(y)$ will divide exactly, and the resulting series obtained by FFT division, i.e., by using $\tilde{c}(y)/a(y)$ in equation (1.5), will be the "best" causal estimate of $\{b\}$. We call $\{\tilde{c}\}$ the root-projected series and the resulting $\{\tilde{b}\}$ the root-projection deconvolution (RPD) estimate of $\{b\}$.

We have previously developed a singular-value matrix method (SVD) as an alternative approach for solving the deconvolution problem, and this method is quite powerful [6]. However, it requires selecting a best guess filtered answer and frequently that is difficult to do. Also, the frequency transform of the estimated answer often has unnecessary errors, even in those frequency bands where there is much information, because the eigenfunctions which are built by the method to represent the answer do not quite reflect the exponential functions used in a frequency representation. Because the particular decomposition of the answer differs for SVD (singular-vector representation) and RPD (frequency representation), the information (signal) and noise are distributed differently over the orthogonal "channels" corresponding to the particular basis functions utilised in that representation. For example, typically the SVD estimate will show some of the most prominent high frequency features, but will have reduced low frequency fidelity. The RPD estimate, on the other hand, will tend to have good low frequency features, but will have reduced high frequency features and greater end noise in the time representation. This suggests that the two "complementary" approaches, SVD and RPD, can be combined to exploit the best features of each. One strategy that has been successfully demonstrated on numerous simulation problems consists of the following. SVD and RPD are each applied independently to produce a first estimate to the inverse. The data residuals generated by each algorithm, conservatively filtered to avoid extraneous features, are then fed into the other algorithm. What one of these algorithms may discard as noise can contain useful signal when decomposed using the other algorithm. Taking the average of the final two estimates yields an estimate that is not only more accurate, but more robust than the result of using either separately. The process of combining SVD and RPD in this manner we call the cross-cut deconvolution algorithm, which has been successfully applied to a variety of extremely ill-conditioned deconvolution problems, employing both simulated and experimental signals [1].

2. Ultrasonic Measurement of Internal Temperature Distributions

The development of a sensor for measuring the internal temperature distribution in hot metallic bodies has long been identified by the American Iron and Steel Institute (AISI) as a fundamental goal in improving productivity and quality and optimising energy consumption in metals processing. As a consequence, the AISI and the National Bureau of Standards initiated a joint research program to develop such a temperature sensor based on ultrasonic velocity measurements. Potential applications include measuring the internal temperature distribution in steel ingots during reheating and monitoring the temperature profiles of steel strands produced by continuous casting. The temperature sensor is based on the tomographic reconstruction of sound velocity from ultrasonic time-of-flight measurements — a particularly ill-conditioned inverse problem.

The operation of the sensor relies on measuring changes in the velocity of sound through a hot metallic object and exploiting the strong, almost linear, dependence of ultrasonic velocity on temperature [2]. For example, 304 stainless steel exhibits a change of longitudinal velocity of about -0.68 m/sec per degree Celsius. If the relationship is sufficiently linear, we may write in general,

$$v(\underline{r},t) = v_{ref} + \beta(T(\underline{r},t) - T_{ref}), \tag{2.1}$$

where the space and time dependence, $(\underline{r},t)$, of the velocity v has been explicitly indicated since the temperature T is in general a function of $\underline{r}$ and t. The constants v_{ref}, T_{ref} and β are presumed known from prior measurement.

Ultrasonic velocity is measured by recording the time-of-flight (TOF) of transmitted ultrasonic pulses through the sample. This provides a measure of the average velocity along the propagation path, which, in turn, can be converted to a measure of the average temperature along the path using a previously calibrated, velocity-temperature relationship of the form (2.1). Moreover, an actual image, or map, of the temperature distribution can be derived using tomographic reconstruction algorithms if a sufficient number of TOF measurements are made along multiple overlapping paths.

The TOF of an ultrasonic pulse along a ray path through an object is the line integral of the reciprocal sound velocity along that path, i.e.,

$$\tau_m = \int_{L_m} \frac{dl}{v(\underline{r})}, \tag{2.2}$$

where τ_m is the TOF along the path L_m.

In principle, in tomographic image reconstruction, at least as many TOF measurements are needed as pixels in the image. In practice, errors in the TOF and path-length measurement combine with inherent ill-conditioning in the tomographic inversion to require considerable measurement redundancy, in which case least-squares techniques could be employed to best estimate the temperature field. A priori information can be used both to reduce this dependency on redundant information and to mitigate the sensitivity of the inversion (ill-conditioning) to measurement errors.

The most important a priori constraint available to us is the assumption of symmetrical heat flow, which is often reasonable in bodies of simple geometric shape (e.g., of circular or rectangular cross-section). Knowledge that the temperature field is symmetrical drastically reduces the number of unknowns characterising the temperature field, and thus reduces the number of required measurements by a comparable amount. Furthermore, heat flow is well modeled by the thermal conductivity equation (a diffusion equation). Because temperature is a solution to this equation, it is, in effect, being subjected to a low-pass spatial filter whose spatial-frequency cutoff decreases in proportion to the square root of the cooling time. Stated another way, rapid spatial temperature fluctuations disappear with time due to thermal diffusion. This limit on the spatial frequency bandwidth (smoothness) of the temperature field implies the existence of a limit on the density of data sampling (and hence on the number of TOF

measurements) needed to characterise the temperature distribution. Compared to medical ultrasonic or x-ray tomography, effectively far coarser spatial resolution is sufficient to reconstruct the temperature field.

In practice, because of time constraints and experimental complications involved in coupling ultrasound in and out of a hot body, a relatively small number of TOF measurements are feasible. As a consequence, it is absolutely necessary to exploit object symmetries as well as the property that the temperature distribution rapidly assumes a smooth shape due to thermal diffusion. The possibility of reconstructing reasonably accurate temperature profiles with a small number of measurements relies crucially on the incorporation of such a priori information.

The constraint that the temperature field cannot be arbitrary, but must obey the thermal conductivity equation, suggests that we look for distributions in the general form of the solution to this equation. In an axially-symmetric object (in which the heat flow is assumed uniform or zero in the axial direction), the general solution reads

$$T(r,t) = T_{amb} + \sum_{n=1}^{\infty} c_n \, J_0(\alpha_n r) e^{-\kappa \alpha_n^2 t}, \tag{2.3}$$

where $J_0(\cdot)$ is the Bessel function of order zero. In this equation, T_{amb} is the ambient temperature (presumed known), c_n are unknown constants determined by some initial (and unknown) temperature state, α_n are unknown constants determined by the boundary conditions, and κ is the thermal diffusivity (presumed known). For a square geometry (again assuming constant heat flow in the z-direction), the general solution is

$$T(x,y,t) = T_{amb} + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_{nm} \cos(\alpha_n x) \cos(\alpha_m y) e^{-\kappa(\alpha_n^2 + \alpha_m^2)t}, \tag{2.4}$$

where, once again, the (unknown) constants c_{nm} and α_n are determined by the initial and boundary conditions. A reasonable approach would be to use the ultrasonic TOF measurements and the relationships (2.1) and (2.2) to fit the unknown parameters in the above temperature models, namely the c_n and α_n in equation (2.3) or the c_{nm} and α_n in equation (2.4), for a finite number of terms in the sum. Because we know that the lower-order terms dominate in a short time due to the exponential time dependence that increases rapidly with order, a reasonable first approximation would be to retain only the single lowest order terms in the sums (2.3) and (2.4). This approach has formed the basis of a practical inversion scheme that has been successfully checked against experiment [2]. When only the lowest-order terms are kept, the above temperature models simplify as follows.

For axial symmetry,

$$T(r,t) = T_{amb} + (T_c - T_{amb}) \, J_0(\alpha r) e^{-\kappa \alpha^2 t}, \tag{2.5}$$

where, for convenience, we have dropped the subscript one on α and renamed the first coefficient, c_1, by defining $c_1 = T_c - T_{amb}$. In the above temperature model note that $T_c = T(0,0)$ corresponds to the

axial temperature at an initial time when $t = 0$. In the above model there are only two undetermined parameters: T_c and α.

For a square cross-section, keeping the lowest-order term in equation (2.4) similarly yields

$$T(x, y, t) = T_{amb} + (T_c - T_{amb}) \cos(\alpha x) \cos(\alpha y) e^{-2\kappa\alpha^2 t}, \tag{2.6}$$

where, once again, T_c and α are the two undetermined parameters, and $T_c = T(0, 0, 0)$.

For purposes of illustration, consider the model (2.6) for a square cross-section. (The general procedure extends, of course, to the axially-symmetric case.) Suppose the TOF measurements are made through a square block along M parallel paths at heights y_m and at times t_m. Suppose further that the length of the side of the block is $2a$. Inserting equation (2.1) into (2.2) gives the "model-generated" measurements, $\tilde{\tau}$,

$$\tilde{\tau}(y_m, t_m) = \int_{-a}^{a} \frac{dx}{v_{ref} + \beta[T(x, y_m, t_m) - T_{ref}]}, \qquad m = 1, 2, \ldots, M, \tag{2.7}$$

where $T(x, y, t)$ is defined by equation (2.6) and M is the number of measurements. The parameters T_c and α are then chosen to minimize the mean-square error

$$E = \sum_{m=1}^{M} [\tau(y_m, t_m) - \tilde{\tau}(y_m, t_m)]^2, \tag{2.8}$$

where $\tau(y_m, t_m)$ is the *measured* TOF value at position y_m and time t_m, and $\tilde{\tau}(y_m, t_m)$ is the *computed* TOF value using equation (2.7). The numerical minimization of equation (2.8) with respect to T_c and α may be performed using well-known nonlinear least-squares algorithms.

Both the cylindrical and rectangular versions of the above reconstruction scheme were applied to TOF measurements made through, respectively, a 6 inch diameter cylinder and a 6×6 inch square block, both composed of 304 stainless steel. The TOF measurements were performed at temperatures ranging from 25° C to 750° C. Thermocouples embedded in the steel samples were used as an independent check of the temperature derived from the TOF measurements. Agreement between the thermocouple readings and the reconstructed temperature distribution was generally within 10° C, well within the experimental error expected from the estimated uncertainty in the TOF and path-length measurements. A detailed description of the experimental apparatus and the resulting temperature reconstructions are given in [2].

3. Determination of Electrical Conductivity Profiles from Frequency-Sweep Eddy Current Measurement

The problem of measuring a spatially-varying electrical conductivity profile in the interior of a conducting body has only recently been addressed in electromagnetic NDE, although this inverse problem has received some attention in geophysics. Several approaches to the conductivity inversion problem in the geophysical context were reported by Weidelt [7], Parker [8], and Parker and Whaler [9] in their work on depth-profiling the earth's conductivity from measurements of the time dependence of surface currents. The work reported below is an adaptation of Parker's [8] inversion scheme to problems in NDE [3].

The penetration of an ac magnetic field into a body of uniform conductivity is exponentially attenuated with a characteristic decay distance given by the well-known formula for skin depth,

$$\delta = \sqrt{2/\sigma\omega\mu_0}, \tag{3.1}$$

where σ is the conductivity, ω the angular frequency, and μ_0 the (free space) permeability. A measurement of impedance at the surface of the body will give a determination of the electrical conductivity. If the conductivity is allowed to vary with depth into the body, one could attempt to reconstruct the conductivity profile by performing surface impedance measurements at many frequencies. High frequency measurements would respond to the conductivity near the surface, whereas low frequency measurements would reflect conductivity values at greater depth. Thus, one would feel intuitively that an inversion procedure based on multi-frequency measurements could allow a reconstruction of an arbitrary conductivity profile without invoking specific a priori models (e.g., an assumed single surface layer).

The complexity of the general problem requires, however, that we limit the discussion to profiling planar stratified material, i.e., material in which the conductivity is a function only of the depth, z, below a planar surface. For computational convenience, we also assume that the material is terminated by a perfect conductor at a depth h. This assumption does no harm if h is much greater than the skin depth corresponding to the lowest measuring frequency available. This forces the electric field to vanish at the terminating conductor, allowing a solution in terms of discrete eigenfunctions. However, this requirement of an E-field node is automatically satisfied for all frequencies at the central plane of a symmetric plate, provided equal H-fields are applied to both sides. Also, an equivalent condition ($E = 0$ for $r = 0$) is automatically satisfied at all frequencies for a cylinder in a uniform H-field parallel to the cylindrical axis, provided the conductivity is a function of the radius only. Thus, the condition of a fixed E-field node can be assumed in many common NDE configurations.

The differential equation for the time-dependent electric field, $E(z,t)$, with the depth-dependent conductivity $\sigma(z)$, is

$$\frac{\partial^2 E}{\partial z^2} = \mu_0\sigma(z)\frac{\partial E}{\partial t}. \tag{3.2}$$

If we assume single-frequency excitation of the form $E(z,\omega)\exp(-i\omega t)$, we have

$$\frac{\partial^2 E}{\partial z^2} + i\omega\mu_0\sigma(z)E = 0, \tag{3.3}$$

and an equation for the magnetic field, H, of similar form. An implicit integral equation for E and its derivative, $E'(z)$, can be obtained by integrating equation (3.3) once, giving

$$E'(z) = E_0' + i\omega\mu_0\int_0^z E(z')\sigma(z')\,dz', \tag{3.4}$$

and integrating again to give

$$E(z) = E_0 + E_0{}'z + i\omega\mu_0 \int_0^z (z - z')E(z')\sigma(z')\,dz', \tag{3.5}$$

where E_0 and $E_0{}'$ are constants of integration and are fixed by the boundary condition at $z = 0$.

We now approximate the conductivity profile as a weighted set of N infinitesimally-thin, parallel conducting shells at depths z_i and conductivities τ_i, i.e., we let

$$\sigma(z) = \sum_{i=1}^{N} \tau_i \delta(z - z_i). \tag{3.6}$$

Between shells the magnetic field is constant, and thus the electric field varies uniformly with z. The current in each shell is proportional to the electric field at the shell and induces a jump in the magnetic field of an amount $\tau_i E(z_i)$. This causes a corresponding jump in the derivative of the electric field of an amount $i\omega\mu_0\tau_i E(z_i)$. That is, across the i-th shell, $E(z)$ is continuous, but

$$E'(z_i-) \to E'(z_i+) + i\omega\mu_o\tau_i E(z_i+), \tag{3.7}$$

as can be seen from equation (3.4).

A principal quantity of interest in the inversion problem is the so called admittance function, defined by

$$c(z,\omega) = \frac{E(z,\omega)}{E'(z,\omega)}. \tag{3.8}$$

This function can be measured at the surface $z = 0$ from measurements of the electric and magnetic fields $E(0,\omega)$ and $H(0,\omega)$ as a function of frequency. $c(\omega)$ can equivalently be derived from surface impedance measurements and knowledge of $H(0,\omega)$ [10]. The surface admittance is

$$c(\omega) \equiv c(0,\omega) = \frac{E(0,\omega)}{E'(0,\omega)} = \frac{E(0,\omega)}{i\omega\mu_0 H(0,\omega)}. \tag{3.9}$$

From equation (3.4) and (3.5), and in view of the definition (3.8), we see that in propagating from z_{i-1} to z_i between conducting shells, where no conductors are present, the admittance undergoes a change

$$c(z_{i-1}+,\omega) = -(z_i - z_{i-1}) + c(z_i-,\omega).$$

On the other hand, in propagation across the shell at z_i, it follows from equations (3.7) and (3.8) that the change is given by

$$c(z_i+,\omega) = \frac{1}{i\omega\mu_0\tau_i + \dfrac{1}{c(z_i-,\omega)}}.$$

Noting that $c(h,\omega) = 0$ (since the electric field vanishes at h), we can apply these rules successively to obtain a continued-fraction representation of the surface admittance:

$$c(\omega) = -h_0 + \cfrac{1}{i\omega\mu_0 r_1 + \cfrac{1}{-h_1 + \cfrac{1}{i\omega\mu_0 r_2 + \cfrac{1}{\ldots + \cfrac{1}{-h_N}}}}}, \tag{3.10}$$

where the $h_i = z_{i+1} - z_i$ are the spatial separations between the shells. When the above continued-fraction representation is rationalised it reduces to the ratio of two polynomials of degree N. This polynomial ratio can then be expanded in a sum of partial fractions, giving

$$c(\omega) = \sum_{n=1}^{N} \frac{A_n(\omega)}{\lambda_n - i\omega}, \tag{3.11}$$

where the λ_n are real and $A_n(\omega)$ are polynomial functions in ω. Thus, equation (3.10) has been cast in the form of a spectral density function. As written, equation (3.11) implies that $c(\omega)$ has N poles lying on the positive imaginary axis. This can be independently verified as follows. A set of real normal mode solutions to the eddy current equation (3.2) are the exponentially-damped functions $E(z,t) = u_n(z)\exp(-\lambda_n t)$. Inserting these modes into equation (3.2) results in

$$\frac{\partial^2 u_n}{\partial z^2} + \mu_0\sigma(z)\lambda_n u_n = 0, \tag{3.12}$$

where the eigen-solutions, u_n, are subject to the boundary conditions $u_n(h) = 0$ and $\partial u_n/\partial z = 0$ at $z = 0$. The boundary conditions generate a discrete set of normal modes, corresponding to the real eigenvalues $\lambda_n, n = 0, 1, \ldots$, which decay in time, each with its own time constant λ_n. Now, the Green's function for the eddy current equation (3.3) obeys the equation

$$\frac{\partial^2 G(z|z')}{\partial z^2} + i\omega\mu_0\sigma(z)G(z|z') = -\delta(z - z'), \tag{3.13}$$

subject to the boundary conditions $G(h, z') = 0$ and $\partial G(z|z')/\partial z = 0$ at $z = 0$. Performing the expansion of the Green's function in terms of the eigenfunctions of equation (3.12), $u_n(z)$, we have

$$G(z|z') = \sum_{n=1}^{\infty} \frac{u_n(z)u_n(z')}{\lambda_n - i\omega}. \tag{3.14}$$

With the help of equations (3.12) and (3.13) and Green's theorem, it is easy to verify that

$$G(0|0) = -\frac{E(0,\omega)}{E'(0,\omega)} = -c(\omega). \tag{3.15}$$

Comparing equations (3.14) and (3.15) to (3.11) shows that the finite shell problem corresponds exactly to the spectral expansion (3.14) truncated at N terms. This confirms that the λ_n in equation (3.11) are real and that the poles of $c(\omega)$ lie on the imaginary axis.

The proposed scheme of obtaining the conductivity profile is as follows:

1. From impedance measurements, obtain $c(\omega)$ at numerous values of the (real) frequency ω, in a range such that the skin depths span the dimensions of interest.
2. From the measurements of $c(\omega)$, obtain a best fit to a truncated expansion of the form of equation (3.11). The task of performing this fit with incomplete and imprecise data has been treated by Parker [8] and Parker and Whaler [9].
3. Transform the partial fraction form to the model of conductive shells by performing the expansion into the continued fraction form (3.10). The locations and strengths of the shells can, in principle, be picked off by inspection. Algorithms for this computation have also been implemented by Parker and Whaler [9]. This gives a profile in terms of δ-function shells.
4. Spread the conductances of the δ-function shells into the space between the shells. We arbitrarily bisect the regions between shells and spread the strength of each shell uniformly between the neighboring bisecting planes. This procedure gives the profile in the form of a series of flat steps. This last procedure is based on the concept that each δ-function shell obtained in the inversion process represents continuously distributed conductance.

The simplest realisable arrangement in which a uniform field may be applied to a sample is that of a long solenoid with a cylindrical core. If the conductivity depends only on the radial coordinate, and if we may neglect end effects, the problem may be transformed into the form of the previously treated case of the planar stratified medium. In the cylindrical case the admittance is defined by [10]

$$c(\omega) = \frac{E(R)}{i\omega\mu_0 H(R)} = \frac{E(R)}{\left.\frac{1}{r}\frac{\partial(rE)}{\partial r}\right|_R}, \tag{3.16}$$

where R is the radius of the sample. In addition, the inversion algorithm for a set of shells carries over from the planar case by the transformation:

$$c(\omega) \rightarrow Rc(\omega)$$
$$\tau_i \rightarrow \tau_i / R_i$$
$$h_i \rightarrow ({R_{i+1}}^2 - {R_i}^2)/2.$$

If these substitutions are made, the previous development applies.

Experimental tests were performed on several cylindrical samples, including a solid brass rod, a brass tube with a copper center, and a brass tube with a tungsten center. Impedance data were acquired after inserting the metal cylinders into a cylindrical coil. A detailed description of the experimental arrangement and the resulting conductivity profiles may be found in [3]. In the tests on metal cylinders, good qualitative experimental agreement was achieved; in particular, the locations of the discontinuities in conductivity at the interface between the different metals were accurately reproduced. Unfortunately, the quantitative agreement between the true and reconstructed conductivity values was quite inconsistent. The latter result may reflect limitations of the shell model [equation (3.6)] as well as the severe ill-conditioning inherent in the conductivity inversion problem. To improve the method, other geometric