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Pseudo-Complex General Relativity

FIAS Frankfurt Institute for Advanced Studies

Springer

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Preface

General relativity (GR), one of the most and best checked physical theories of our time, exhibits singularities: The theory predicts that when a sufficient large mass collapses, no known force is able to stop it until all mass is concentrated at a point. The theory also predicts so-called *coordinate singularities*. These are singularities in the metric which vanish after a transformation to different coordinates. For example, when an astronaut falls freely towards a black hole, he will not see anything special, except the gravitational force with its deadly tidal effect. However, a fixed observer at a safe distance will see at a certain distance from the center, the *Schwarzschild radius* for a nonrotating black hole, an event horizon. *No information can reach the observer from places at smaller radial distances*! This is a rather discomforting observation, telling that part of the space is excluded from the observation by a nearby observer. On the other hand: Why should GR still be valid in extremely strong gravitational fields, as one encounters near the Schwarzschild radius?

This was the reason why two of the authors of this book (P.O. Hess and W. Greiner) started to discuss this point several years ago. We believe that *no acceptable physical theory should have a singularity (!)*, not even a coordinate singularity of the type discussed above! The appearance of a singularity shows the limitations of the theory. In GR this limitation is the strong gravitational force acting near and at a supermassive concentration of a central mass. There are other very successful theories, like the Quantum Electrodynamics (QED), which exhibits singularities, infinities, due to taking into account the very large momenta corresponding to very small distances in space-time. Most of the physicists would agree that any field theory should not apply at very small distances. Methods of regularizing field theories have been developed, giving a recipe how to remove the infinite contribution. But that is what they are: *Recipes*! In 2007 the authors of this book published a new field theory, called *pseudo-complex Field Theory*, where they introduced pseudo-complex variables, which will play an important role in this book. Owing to the extension to pseudo-complex fields and operators, it is shown that the theory is automatically regularized. This is due to the appearance of a minimal length as a *parameter*. Because it appears only as a parameter, Lorentz transformation does not affect it, thus, *all continuous and discrete symmetries of nature are maintained*! However, due to the extremely small effects and the minimal length, there is no hope to measure the deviations in near future.

This was the reason why we started to look for extreme physical situations, such as strong gravitational fields near a large mass. The first question was: Is there a possibility to avoid the formation of the event horizon? This would mean that the large mass concentrations, for example at the center of galaxies, are still there but these objects are *no black holes*! It will be shown that there is one natural algebraic extension of GR, namely to pseudo-complex (pc) coordinates. We developed the *pseudo-complex General Relativity* (pc-GR) and found several observational effects which can be measured in near future (See Chap. 5 of this book). The *very long baseline interferometry* (VLBI), to which ALMA, the European observatory in the Atacama dessert in Chile belongs, will be able to resolve the central massive objects at the centers of our galaxy and in M87. Thus, as GR, also pc-GR is a testable theory!

This book contains several exercises with explicit and detailed solutions. It is therefore also of interest for students working in GR. Many of the exercises correspond to considerations not published in text books or at least not in detailed form. We therefore are convinced that this book is helpful also for students only starting to work in General Relativity.

The book is divided into seven chapters. In Chap. 1, the necessary basis is led to deal with pc-variables. This chapter is necessary to understand the content from the second chapter and further on. The noninterested reader can skip it but surely he will have to return soon to the first chapter.

Chapter 2 is a central piece of this book, where the pc-GR is introduced and the basic philosophy is discussed. First, a historical overview is given on former attempts to extend GR (which includes Einstein himself), all with distinct motivations. It will be shown that the only possible algebraic extension is to introduce pc-coordinates, otherwise for weak gravitational fields, nonphysical ghost solutions appear. *Thus, the need to use pc-variables.* We will see that the theory contains a minimal length with important consequences. After that, the pc-GR is formulated and compared to the former attempts. A new variational principle is introduced, which requires in the Einstein equations an additional contribution. Alternatively, the standard variational principle can be applied, but one has to introduce a constraint with the same former results. The additional contribution will be associated to vacuum fluctuation, whose dependence on the radial distance can be approximately obtained, using semiclassical quantum mechanics. *The main point is that pc-GR predicts that mass not only curves the space but also changes the vacuum structure of the space itself.* In the following chapters, the minimal length will be set to zero, due to its smallness. Nevertheless, the pc-GR will keep a remnant of the pc-description, namely that the appearance of a term, which we may call "dark energy," is inevitable.

The first application will be discussed in Chap. 3, namely solutions of central mass distributions. For a nonrotating massive object, it is the pc-Schwarzschild solution; for a rotating massive object, the pc-Kerr solution; and for a charged massive object, it will be the Reissner–Nordström solution. This chapter serves to become familiar on how to resolve problems in pc-GR and on how to interpret the results. One of the main consequences is that we can eliminate the event horizon and thus, *there will be no black holes*! The huge massive objects in the center of nearly any galaxy and the so-called galactic black holes are within pc-GR still there, but with the absence of an event horizon!

Chapter 4 gives another application of the theory, namely the Robertson–Walker solution, which we use to model different outcomes of the evolution of the universe. New solutions will appear as the limit of constant acceleration, the limit of zero acceleration after a period of a nonzero acceleration. We also discuss the possibility of an oscillating universe, with repeated big bangs, with no need to explain the smoothness of the universe.

The success of a theory depends on the capability to predict new phenomena. Chapter 5 is just dedicated to this purpose. We will see that at a large distance from a large massive object, GR and pc-GR will show no differences. However, near the Schwarzschild radius significant deviations of pc-GR from GR are predicted. The orbital frequency of a particle in a circular orbit and stable orbits in general will be calculated. As a distinct feature, in pc-GR there will be a maximal orbital frequency. We show that above a given spin of the star, there will be no *innermost stable circular orbit* (ISCO) and an accretion disk will reach the surface of the star. This has important consequences for the physics of the accretion disk: It will appear brighter (emit more light) and due to the maximum in the orbital frequency, a dark ring is predicted by pc-GR. Also the redshift will be calculated. This is of great importance: One observes so-called *quasi-periodic oscillations* (QPO) and the redshift of Fe Kα lines. Knowing the orbital frequency of a QPO and the redshift, GR and pc-GR get for each observable a radius for the position of the QPO. Both radii, obtained from both observables, should coincide. They do not in GR, but they do in pc-GR! Of course, this depends still on the interpretation of the nature of the QPO and the discussion is still on.

In Chap. 6, neutron stars are discussed and a primitive model for the coupling of mass to the dark energy is proposed. This chapter is of conceptional nature and is meant to show that large masses for neutron stars can be obtained. The jewel of this chapter is the discussion of the so-called *energy conditions*. They are used to see if an ansatz for an energy–momentum tensor, treating for example ideal fluids, makes sense. We found no book or article in the literature where these conditions are treated as extensively as here with detailed solutions. Thus, this chapter serves also for people interested only in the standard theory of GR.

Finally, in Chap. 7, the geometric differential structure of pc-GR is investigated. The motivation for this chapter is to complete the presentation of pc-GR in a rigorous manner. For a noninterested reader of differential geometry, this chapter can be skipped. However, he may find it to be useful, to learn more of this topic. No explicit knowledge of differential geometry is required, because all necessary definitions will be given. This makes this chapter especially useful.

The book appears within the series of the *FIAS Lecture Notes*, which is meant to publish on topics of interdisciplinary interest and new developments. We think that this is an ideal place for resuming all results obtained within pc-GR.

Finally, we would like to express our sincere thanks to all the people who contributed with their help to the realization of this book. We thank Gunther Caspar and Thomas Schönenbach for their contribution to Chaps. 3 and 5, Thomas Boller and Andreas Müller for their contribution to Chap. 5 and Isaac Rodríguez for his contribution to Chap. 6. The Chap. 3 is based mainly on the master theses of Thomas Schönenbach and Gunther Casper, Chap. 5 is on the Ph.D. thesis of Thomas Schönenbach, and Chap. 6 is based on the Ph.D. thesis of Isaac Rodríguez. We acknowledge useful comments by J. Kirsch. We also thank Laura Quist for their patience and logistic help. P.O.H. wants also to acknowledge financial help from DGAPA-PAPIIT (IN100315). M.S. acknowledges the support from Stiftung Polytechnische Gesellschaft.

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