海外优秀理科类系列教材

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改编版

Linear Algebra

(Fourth Edition)

线性代数（第4版）

□ Stephen H. Friedberg
□ Arnold J. Insel　　原著
□ Lawrence E. Spence
□ 王殿军 改编

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出版者的话

为适应当前我国高校各类创新人才培养的需要，大力推进教育部倡导的双语教学，配合教育部实施的“高等学校教学质量与教学改革工程”和“精品课程”建设的需要，我社开始有计划、大规模地开展了海外优秀理科系列教材的影印及改编工作。海外优秀教材在立体化配套、多种教学资源的整合以及为课程提供整体教学解决方案等方面对我们有不少可资借鉴之处。但一个不容忽视的问题是，外版教材与我国现行的教学内容、教学体系、教学模式和习惯等存在着巨大的差异。譬如，重点课程的原版教材通常很厚，内容很多，容量是国内自编教材的好几倍，国外的情况是，老师未必会都讲，剩下大量的内容留给学生自学；而国内的情况则不尽相同。受国内教学学时所限，完全照搬是不合时宜的。教材的国际化必须与本民族的文化教育传统相融合，在原有的基础上吸收国外优秀教材的长处，这使得我们需要对外文原版教材进行适当的改编。改编不是简单地使内容增删，而是结合国内教学特点，引进国外先进的教学思想，在教学内容和方式上更中国化，使之更符合国内的课程设置及教学环境。

在引进改编海外优秀教材的过程中，我们坚持了两条原则：1. 精选版本，打造精品系列；2. 慎选改编者，保证品质。

首先，我们和 Pearson Education, John Wiley & Sons, McGraw-Hill 以及 Thomson Learning 等国外出版公司进行了广泛接触，经推荐并在国内专家的协助下，提交引进版权总数达 200 余种，学科专业领域涉及数学、物理、化学化工、地理、环境等。收到样书后，我们聘请了国内高校一线教师、专家学者参与这些原版教材的评介工作，从中遴选出了一批优秀教材进行改编，并组织出版。这批教材普遍具有以下特点：(1) 基本上是近几年出版的，在国际上被广泛使用，在同类教材中具有相当的权威性；(2) 高版次，历经多年教学实践检验，内容翔实准确，反映时代要求；(3) 各种教学资源配套整齐，为师生提供了极大的便利；(4) 插图精美、丰富，图文并茂，与正文相辅相成；(5) 语言简练、流畅，可读性强，比较适合非英语国家的学生阅读。

其次，慎选改编者。原版教材确定后，随之碰到的问题是寻找合适的

改编者。要改编一本教材，必须要从头到尾吃透它，有这样的精力自编一本教材都绰绰有余了。我们与国内众多高等院校的众多专家学者进行了广泛的接触和细致的协商，几经酝酿，最终确定下来改编者。大多数改编者都是有国外留学背景的中青年学者，他们既有相当高的学术水平，又热爱教学，活跃在教学第一线。他们能够承担此任，不单是因为他们了解引进版教材的知识结构、表达方式和写作方法，更重要的是他们有精力、有热情，愿意付出，有的甚至付出了比写一本新教材更多的劳动。我们向他们表示最真诚的谢意。

在努力降低引进教材售价方面，高等教育出版社做了大量和细致的工作，这套引进改编的教材体现了一定的权威性、系统性、先进性和经济性等特点。

这套教材出版后，我们将结合各高校的双语教学计划，开展大规模的宣传、培训工作，及时地将本套丛书推荐给高校使用。在使用过程中，我们衷心希望广大高校教师和同学提出宝贵的意见和建议。如有好的教材值得引进，也请与高等教育出版社高等理科分社联系。

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高等教育出版社

2005 年 5 月

To our families:
Ruth Ann, Rachel, Jessica, and Jeremy
Barbara, Thomas, and Sara
Linda, Stephen, and Alison

Preface

The language and concepts of matrix theory and, more generally, of linear algebra have come into widespread usage in the social and natural sciences, computer science, and statistics. In addition, linear algebra continues to be of great importance in modern treatments of geometry and analysis.

The primary purpose of this fourth edition of *Linear Algebra* is to present a careful treatment of the principal topics of linear algebra and to illustrate the power of the subject through a variety of applications. Our major thrust emphasizes the symbiotic relationship between linear transformations and matrices. However, where appropriate, theorems are stated in the more general infinite-dimensional case. For example, this theory is applied to finding solutions to a homogeneous linear differential equation and the best approximation by a trigonometric polynomial to a continuous function.

Although the only formal prerequisite for this book is a one-year course in calculus, it requires the mathematical sophistication of typical junior and senior mathematics majors. This book is especially suited for a second course in linear algebra that emphasizes abstract vector spaces, although it can be used in a first course with a strong theoretical emphasis.

The book is organized to permit a number of different courses (ranging from three to eight semester hours in length) to be taught from it. The core material (vector spaces, linear transformations and matrices, systems of linear equations, determinants, diagonalization, and inner product spaces) is found in Chapters 1 through 5 and Sections 6.1 through 6.5. Chapters 6 and 7, on inner product spaces and canonical forms, are completely independent and may be studied in either order. In addition, throughout the book are applications to such areas as differential equations, economics, geometry, and physics. These applications are not central to the mathematical development, however, and may be excluded at the discretion of the instructor.

We have attempted to make it possible for many of the important topics of linear algebra to be covered in a one-semester course. This goal has led us to develop the major topics with fewer preliminaries than in a traditional approach. (Our treatment of the Jordan canonical form, for instance, does not require any theory of polynomials.) The resulting economy permits us to cover the core material of the book (omitting many of the optional sections and a detailed discussion of determinants) in a one-semester four-hour course for students who have had some prior exposure to linear algebra.

Chapter 1 of the book presents the basic theory of vector spaces: subspaces, linear combinations, linear dependence and independence, bases, and dimension. The chapter concludes with an optional section in which we prove that every infinite-dimensional vector space has a basis.

Linear transformations and their relationship to matrices are the subject of Chapter 2. We discuss the null space and range of a linear transformation, matrix representations of a linear transformation, isomorphisms, and change of coordinates. Optional sections on dual spaces and homogeneous linear differential equations end the chapter.

The application of vector space theory and linear transformations to systems of linear equations is found in Chapter 3. We have chosen to defer this important subject so that it can be presented as a consequence of the preceding material. This approach allows the familiar topic of linear systems to illuminate the abstract theory and permits us to avoid messy matrix computations in the presentation of Chapters 1 and 2. There are occasional examples in these chapters, however, where we solve systems of linear equations. (Of course, these examples are not a part of the theoretical development.) The necessary background is contained in Section 1.4.

Determinants, the subject of Chapter 4, are of much less importance than they once were. In a short course (less than one year), we prefer to treat determinants lightly so that more time may be devoted to the material in Chapters 5 through 7. Consequently we have presented two alternatives in Chapter 4—a complete development of the theory (Sections 4.1 through 4.3) and a summary of important facts that are needed for the remaining chapters (Section 4.4). Optional Section 4.5 presents an axiomatic development of the determinant.

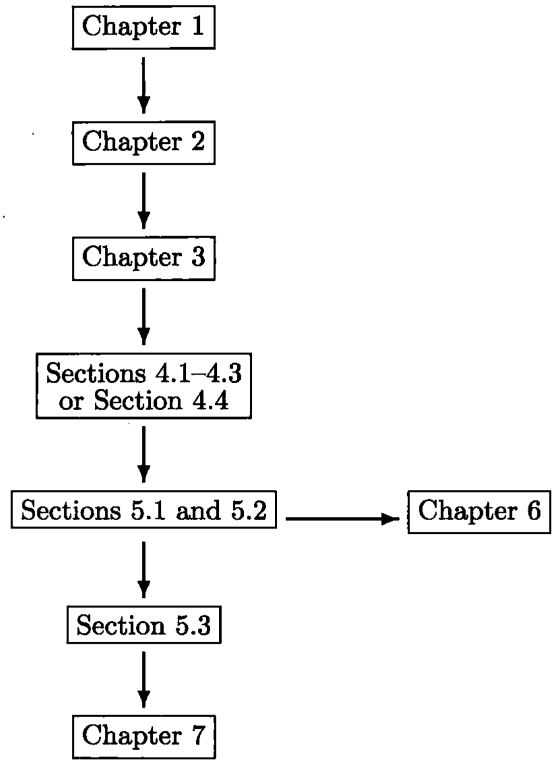
Chapter 5 discusses eigenvalues, eigenvectors, and diagonalization. One of the most important applications of this material occurs in computing matrix limits. We have therefore included an optional section on matrix limits and Markov chains in this chapter even though the most general statement of some of the results requires a knowledge of the Jordan canonical form. Section 5.3 contains material on invariant subspaces and the Cayley–Hamilton theorem.

Inner product spaces are the subject of Chapter 6. The basic mathematical theory (inner products; the Gram–Schmidt process; orthogonal complements; the adjoint of an operator; normal, self-adjoint, orthogonal and unitary operators; orthogonal projections; and the spectral theorem) is contained in Sections 6.1 through 6.6.

Canonical forms are treated in Chapter 7. Sections 7.1 and 7.2 develop the Jordan canonical form, Section 7.3 presents the minimal polynomial, and Section 7.4 discusses the rational canonical form.

There are five appendices. The first four, which discuss sets, functions, fields, and complex numbers, respectively, are intended to review basic ideas used throughout the book. Appendix C on polynomials is used primarily in Chapters 5 and 7, especially in Section 7.4. We prefer to cite particular results from the appendices as needed rather than to discuss the appendices independently.

The following diagram illustrates the dependencies among the various chapters.



One final word is required about our notation. Sections and subsections labeled with an asterisk (∗) are optional and may be omitted as the instructor sees fit. An exercise accompanied by the dagger symbol (†) is not optional, however—we use this symbol to identify an exercise that is cited in some later section that is not optional.

DIFFERENCES BETWEEN THE THIRD AND FOURTH EDITIONS

The organization of the text is essentially the same as in the third edition. Nevertheless, this edition contains many significant local changes that improve the book. Section 5.1 (Eigenvalues and Eigenvectors) has been streamlined, and some material previously in Section 5.1 has been moved to Section 2.5 (The Change of Coordinate Matrix). Further improvements include revised proofs of some theorems, additional examples, new exercises, and literally hundreds of minor editorial changes.

We are especially indebted to Jane M. Day (San Jose State University) for her extensive and detailed comments on the fourth edition manuscript.

Additional comments were provided by the following reviewers of the fourth edition manuscript: Thomas Banchoff (Brown University), Christopher Heil (Georgia Institute of Technology), and Thomas Shemanske (Dartmouth College).

To find the latest information about this book, consult our web site on the World Wide Web. We encourage comments, which can be sent to us by e-mail or ordinary post. Our web site and e-mail addresses are listed below.

web site: http://www.math.ilstu.edu/linalg

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K_ϕ	$\{x\colon (\phi(\mathsf{T}))^p(x) = 0$ for some positive integer $p\}$ $\{x\colon (\phi(\mathsf{T}))^p(x) = 0$对某个正整数$p\}$
L_A	left-multiplication transformation by matrix A 矩阵 A 的左乘变换
$\mathcal{L}(\mathsf{V})$	the space of linear transformations from V to V 从 V 到 V 的线性变换的空间
$\mathcal{L}(\mathsf{V},\mathsf{W})$	the space of linear transformations from V to W 从 V 到 W 的线性变换的空间
$\mathsf{M}_{m\times n}(F)$	the set of matrices with entries in F 元素在 F 上的 $m \times n$ 矩阵的集合
$\mathsf{N}(\mathsf{T})$	the null space of T T 的零空间
$nullity(\mathsf{T})$	the dimension of the null space of T T 的零空间的维数
O	the zero matrix 零矩阵
$\mathrm{per}(M)$	the permanent of the 2×2 matrix M 2×2 矩阵 M 的积和式
$\mathsf{P}(F)$	the space of polynomials with coefficients in F 系数在 F 上的多项式空间
$\mathsf{P}_n(F)$	the polynomials in $\mathsf{P}(F)$ of degree at most n $\mathsf{P}(F)$ 上次数至多为 n 的全体多项式
ϕ_β	standard representation with respect to basis β 关于基 β 的标准表示
R	the field of real numbers 实数域
$rank(A)$	the rank of the matrix A 矩阵 A 的秩
$rank(\mathsf{T})$	the rank of the linear transformation T 线性变换 T 的秩

$\mathsf{R}(\mathsf{T})$	the range of the linear transformation T 线性变换 T 的值域
$S_1 + S_2$	the sum of sets S_1 and S_2 集合 S_1 与 S_2 的和
$span(S)$	the span of the set S 集合 S 的生成集
$S^\perp$	the orthogonal complement of the set S 集合 S 的正交补
$[\mathsf{T}]_\beta$	the matrix representation of T in basis β T 在基 β 下的矩阵表示
$[\mathsf{T}]_\beta^\gamma$	the matrix representation of T in bases β and γ T 在基 β 和 γ 下的矩阵表示
T^{-1}	the inverse of the linear transformation T 线性变换 T 的逆变换
T^*	the adjoint of the linear operator T 线性算子 T 的伴随算子
T_0	the zero transformation 零变换
T_θ	the rotation transformation by θ 旋转 θ 角的旋转变换
T_W	the restriction of T to a subspace W T 在子空间 W 的限制
$tr(A)$	the trace of the matrix A 矩阵 A 的迹
V/W	the quotient space of V modulo W V 的模 W 的商空间
$\mathsf{W}_1 + \cdots + \mathsf{W}_k$	the sum of subspaces W_1 through W_k W_1 到 $\mathsf{W}_k k$ 个子空间的和
$\sum_{i=1}^k \mathsf{W}_i$	the sum of subspaces W_1 through W_k W_1 到 $\mathsf{W}_k k$ 个子空间的和

List of Symbols

A_{ij}	the ij-th entry of the matrix A 矩阵 A 的位于第 i 行第 j 列处的元素
A^{-1}	the inverse of the matrix A A 的逆矩阵
A^*	the adjoint of the matrix A A 的伴随矩阵
$\tilde{A}_{ij}$	the matrix A with row i and column j deleted 删去 A 的第 i 行第 j 列后得到的矩阵
A^t	the transpose of the matrix A A 的转置矩阵
$(A\|B)$	the matrix A augmented by the matrix B A 关于 B 的增广矩阵
$B_1 \oplus \cdots \oplus B_k$	the direct sum of matrices B_1 through B_k B_1 到 $B_k k$ 个矩阵的直和
$\mathcal{B}(\mathsf{V})$	the set of bilinear forms on V V 上双线性型的集合
β_x	the T-cyclic basis generated by x 由 x 生成的 T- 循环基
C	the field of complex numbers 复数域
$cond(A)$	the condition number of the matrix A 矩阵 A 的条件数
$\mathsf{C}^n(R)$	set of functions f on R with $f^{(n)}$ continuous R 上 n 阶可微函数 f 的集合
$\mathsf{C}(R)$	the vector space of continuous functions on R R 上连续函数的向量空间
$\mathsf{C}([0,1])$	the vector space of continuous functions on $[0,1]$ 闭区间 $[0,1]$ 上连续函数的向量空间

C_x	the T-cyclic subspace generated by x 由 x 生成的 T- 循环子空间
$\det(A)$	the determinant of the matrix A 矩阵 A 的行列式
δ_{ij}	the Kronecker delta 克罗内克 δ 符号
$\dim(\mathsf{V})$	the dimension of V V 的维数
e_i	the ith standard vector of F^n F^n 的第 i 个标准向量
E_λ	the eigenspace of T corresponding to λ T 的对应 λ 的特征空间
F	a field 域
$f(A)$	the polynomial $f(x)$ evaluated at the matrix A A 的矩阵多项式
F^n	the set of n-tuples with entries in a field F 域 F 上的 n 元数组的集合
$f(\mathsf{T})$	the polynomial $f(x)$ evaluated at the operator T 算子 T 的多项式
$\mathcal{F}(S, F)$	the set of functions from S to a field F S 到域 F 的函数的集合
H	space of continuous complex functions on $[0, 2\pi]$ $[0, 2\pi]$ 上的连续复函数的空间
I_n or I	the $n \times n$ identity matrix $n \times n$ 单位矩阵
$\mathsf{I_V}$ or I	the identity operator on V V 上的恒等算子
K_λ	generalized eigenspace of T corresponding to λ T 的关于 λ 的广义特征空间

$\mathsf{W}_1 \oplus \mathsf{W}_2$	the direct sum of subspaces W_1 and W_2
	子空间 W_1 与 W_2 的直和
$\mathsf{W}_1 \oplus \cdots \oplus \mathsf{W}_k$	the direct sum of subspaces W_1 through W_k
	W_1 到 $\mathsf{W}_k k$ 个子空间的直和
$\|x\|$	the norm of the vector x
	向量 x 的范数
$[x]_\beta$	the coordinate vector of x relative to β
	x 相对于 β 的坐标向量
$\langle x, y\rangle$	the inner product of x and y
	x 与 y 的内积
Z_2	the field consisting of 0 and 1
	由 0, 1 组成的域
$\overline{z}$	the complex conjugate of z
	z 的复共轭
0	the zero vector
	零向量

目　录