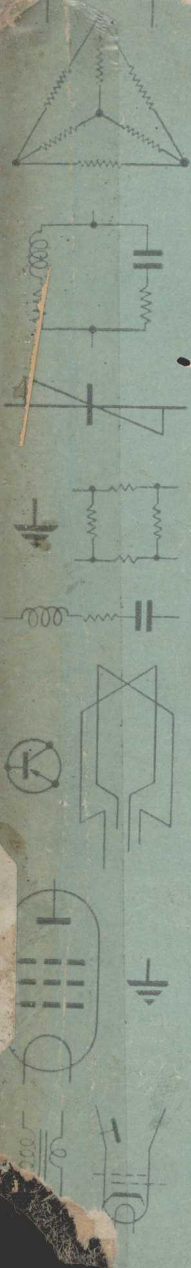

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RADIO ENGINEERING FORMULAE AND CALCULATIONS

Preface

PROBLEMS in radio engineering are usually either presented in the form of examination questions and answers, omitting many intermediate steps, or are framed for the instruction of the amateur. The examples given in this book have been selected from everyday practice in the design and operation of radio installations, rather than for their academic interest, and have been fully worked and generously illustrated by diagrams. The value of visual imagery as an aid in solving problems has been kept foremost in mind. There is no more useful device than a rough diagram to co-ordinate the known data and give a comprehensive grasp of the problem.

It too often happens that the mathematical tools of the practising engineer, who is chiefly occupied by administrative, constructional or maintenance work, become blunted with disuse, and he loses much of the dexterity in handling formulæ which he acquired in his student days. For him the book should be particularly useful in reviving his dormant knowledge. Although it is addressed primarily to him, it will be equally valuable to the technical assistant and student who is about to embark on an engineering career.

The introductory section shows how the common mathematical tools can be handled to the best advantage in making calculations, with simple illustrations of their application. For convenience of reference, formulæ and examples are classified under subject headings and followed by typical worked examples. The final section includes much useful data and the mathematical tables commonly required in practical work.

The information and examples in this book have been reprinted from the *Radio and Television Engineers' Reference Book*, supplemented by new sections and examples, and additional data.

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1. AIDS TO ENGINEERING CALCULATIONS

Accuracy

Nearly all engineering data is approximate to within the limits imposed by measuring instruments, and no useful purpose is served by carrying the result of a calculation to more than the number of significant figures given in the data. As an example, the diameter of a wire can be measured accurately with a micrometer to three significant figures. In calculating the resistance from the measured diameter and length, the accuracy of the result is not increased by working to more than three significant figures.

Again, suppose it is required to convert a wavelength of 16·4 metres to frequency in megacycles. The result would be correctly stated as 18·3 Mc/s. If the result were given as 18·29 Mc/s, no reliance could be placed on the second decimal, as the original figure is given to only three significant figures.

In most practical applications the results are seldom required to a greater accuracy than three significant figures, which is obtainable on a 10-in. slide rule. Two significant figures is often sufficient for rough mental computations.

A meter reading or a measurement given as 4·3 is understood to have limits of accuracy of $4·3 \pm 0·05$. If the reading were given as 4·30, the limits would be $4·30 \pm 0·005$, and so on. The procedure for adding, subtracting, multiplying and dividing data of equal accuracy when the limits are required is illustrated by the following examples :

ADDITION

$$\begin{array}{r} 4·3 \pm 0·05 \\ 3·2 \pm 0·05 \\ \hline 7·5 \pm 0·1 \end{array}$$

SUBTRACTION

$$\begin{array}{r} 4·3 \pm 0·05 \\ 3·2 \pm 0·05 \\ \hline 1·1 \pm 0 \end{array}$$

or $1·1 \pm 0·1$ if the errors are of the same sign.

MULTIPLICATION

$$(4·3 \pm 0·05)\,(3·2 \pm 0·05)$$
$$= 13·76 \pm 0·05\,(4·3 + 3·2) + 0·0025 \text{ neglecting the last term.}$$
$$= 13·76 \pm 0·37$$

DIVISION

$$\frac{4·3 \pm 0·05}{3·2 \pm 0·05} = \frac{4·35}{3·15} = 0·1381$$

$$\text{or } \frac{4·25}{3·25} = 0·1308$$

Find the mean of the two results to as many decimal places as the datum. Then

Mean quotient = 0·1344
Mean of difference = 0·0036
Limits of quotient = $0·1344 \pm 0·0036$

Approximations

An approximate calculation can be made to determine the position of the decimal point or to obtain a rough answer to an extended product or division by the following method. With practice the working can be performed mentally.

Write down each term of the expression as an approximate number of whole units to the left of the decimal point, multiplied or divided by the necessary number of tens written in index form, e.g.,

$$\begin{aligned} 0{\cdot}00365 &= 4 \,.\, 10^{-3} \\ 692 &= 7 \,.\, 10^{2} \end{aligned}$$

Examples

(1)

$$\frac{0{\cdot}00365 \times 692 \times 8{\cdot}2}{77{\cdot}4 \times 0{\cdot}01}$$

$$\simeq \frac{4 \times 7 \times 8}{8 \times 1} \times \frac{10^{-3} \times 10^{2}}{10^{1} \times 10^{-2}} = \underline{28 \text{ approx.}}$$

(2)

$$\sqrt{\frac{2{\cdot}1 \times 8{,}074}{39{\cdot}4 \times 0{\cdot}93}}$$

$$\simeq \sqrt{\frac{2 \times 8}{4 \times 1} \times \frac{10^{3}}{10^{1}}}$$

$$= 10\sqrt{4} = \underline{20 \text{ approx.}}$$

Short-Cuts in Calculations

Use of Indices. Numerical calculation is simplified by the use of indices or powers of ten. The index number, when positive, denotes the number of figures to the left of the decimal point; when negative to the right of the decimal point, e.g.,

$$\begin{aligned} 127{,}000 &= 1{\cdot}27 \times 10^{5} \\ 0{\cdot}0038 &= 3{\cdot}8 \times 10^{-3} \\ 0{\cdot}054 &= 5{\cdot}4 \times 10^{-2} \end{aligned}$$

The rules for operating on indices are

$$\begin{aligned} x^{m} \times x^{n} &= x^{m+n} & \qquad 10^{3} \times 10^{2} &= 10^{5} \\ x^{m}/x^{n} &= x^{m-n} & 10^{3}/10^{2} &= 10 \\ (x^{m})^{n} &= x^{mn} & (10^{3})^{2} &= 10^{6} \end{aligned}$$

Examples

(1)

$$\frac{3{\cdot}2 \times 0{\cdot}00625 \times 0{\cdot}73}{6{\cdot}29 \times 25{\cdot}64 \times 0{\cdot}01} = \frac{3{\cdot}2 \times 6{\cdot}25 \times 0{\cdot}73}{6{\cdot}29 \times 2{\cdot}564} \times \frac{10^{-3}}{10^{-1}}$$

$$= \underline{0{\cdot}00905}$$

(2) $\sqrt{0{\cdot}003 \times 0{\cdot}17} \quad = \sqrt{3 \times 1{\cdot}7 \times 10^{-4}}$

$= 10^{-2}\sqrt{5{\cdot}1}$

$= \underline{0{\cdot}0226}$

Squares and Square Roots

The calculation of square roots and cube roots is simplified by remembering that

$$\sqrt{x \times 10^n} = 10^{n/2}\sqrt{x}$$

$$\sqrt[3]{x \times 10^n} = 10^{n/3}\sqrt[3]{x}$$

Examples

(1) $\sqrt{0{\cdot}0009} = \sqrt{9 \times 10^{-4}} = 10^{-2}\sqrt{9} = \underline{0{\cdot}03}$

(2) $\sqrt[3]{27{,}000} = \sqrt[3]{27 \times 10^3} = 10\sqrt[3]{27} = \underline{30}$

In such expressions as a^2b^2, a^2/b^2, $\sqrt{a+b}/\sqrt{c+d}$ the operation of squaring or extracting the square root can be reduced to one operation.

Examples

(1) $\omega^2L^2 = (\omega L)^2$

(2) $a^2/n^2 = (a/n)^2$

(3) $\sqrt{R^2 + \omega^2L^2}/\sqrt{G^2 + 1/\omega^2C^2} = \sqrt{\{R^2 + (\omega L)^2\}/\{G^2 + 1/(\omega C)^2\}}$

Differences of Two Squares

The difference of two squares can be calculated by factorizing without having to square the two quantities.

$$a^2 - b^2 = (a+b)(a-b)$$

Other useful approximations obtained by squaring are :

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$= a^2 + 2ab \qquad \text{when } a \gg b$$

$$(a-b)^2 = a^2 - 2ab + b^2$$
$$= a^2 - 2ab \qquad \text{when } a \gg b$$

Examples

(1) $14^2 - 6{\cdot}5^2 = (14 + 6{\cdot}5)(14 - 6{\cdot}5) = \underline{153{\cdot}75 \text{ approx.}}$

(2) $42^2 = 40^2 + 2(40 \times 2) = \underline{1{,}760 \text{ approx.}}$

(3) $4{\cdot}8^2 = 5^2 - 2(5{\cdot}0 \times 0{\cdot}2) = \underline{23 \text{ approx.}}$

Products and Quotients

The following approximations for the products and powers of binominal factors can often be applied to shorten certain calculations.

$$(1 + x)(1 + y) \simeq 1 + x + y$$
$$(1 + x)(1 + y)(1 + z) \simeq 1 + x + y + z$$
$$(1 \pm x)^n \simeq 1 \pm nx$$
when x, y and $z \ll 1$

Examples

(1) $1{\cdot}003 \times 0{\cdot}995 = (1 + 0{\cdot}003)(1 - 0{\cdot}005)$
$= 1 + 0{\cdot}003 - 0{\cdot}005$ approx.
$= \underline{0{\cdot}998}$

(2) $\dfrac{1}{3{\cdot}006} = \frac{1}{3}(1 + 0{\cdot}002)^{-1}$
$= \frac{1}{3}(1 + (-1 \times 0{\cdot}002))$ approx.
$= \underline{0{\cdot}333}$

(3) $\dfrac{1}{\sqrt{0{\cdot}994}} = (1 - 0{\cdot}006)^{-\frac{1}{2}}$
$= 1 - (-\frac{1}{2})(0{\cdot}006)$ approx.
$= \underline{1{\cdot}003}$

Calculating Successive Values from a Formula

When an equation contains several variables, only one of which is varied, successive calculations for each different value of the variable factor are simplified by first working out the constant part of the expression, and then applying the appropriate law of proportions to the variable factor.

Example

The frequency in kc/s of a tuned circuit is given by the formula $f = 159/\sqrt{LC}$, where L is the inductance in microhenrys and C the capacitance in microfarads. Find f when $L = 50, 100, 150$ and $200\ \mu$H respectively and $C = 0{\cdot}0002\ \mu$F.

When $L = 50$, $f_1 = 159/\sqrt{LC}$
$$= \frac{159}{\sqrt{50 \times 2 \times 10^{-4}}} = \underline{1{,}590 \text{ kc/s}}$$

Note that, since $f \propto 1/\sqrt{LC}$

at any other frequency $f_2 = f_1/\sqrt{L_2/L_1}$
$= f_1 \sqrt{L_1/L_2}$

Hence

when $L = 100$ $\quad f_2 = f_1/\sqrt{2} = \underline{1{,}120 \text{ kc/s}}$

$L = 150$ $\quad f_3 = f_1/\sqrt{3} = \underline{920 \text{ kc/s}}$

$L = 200$ $\quad f_4 = f_1/2 = \underline{795 \text{ kc/s}}$

Expressions Containing ω or ω^2

Pulsatance ω ($= 2\pi \times$ frequency) can be evaluated quickly in many cases by remembering that at 100 c/s $\omega = 628$ approximately, and multiplying by 10^3 when the frequency is given in kc/s and 10^6 when in Mc/s.

Examples

(1) $f =$ 1,000 c/s. $\qquad \omega = \omega_{100} \times 10 = \underline{6{,}280}$

(2) $f =$ 5 .c/s. $\qquad \omega = \tfrac{1}{2}\omega_{100} \times 10^3 = \underline{3{\cdot}142 \times 10^5}$

(3) $f =$ 2·5 Mc/s. $\qquad \omega = \tfrac{1}{4}\omega_{100} \times 10^6 = \underline{1{\cdot}571 \times 10^8}$

(4) $f =$ 40 Mc/s. $\qquad \omega = \tfrac{1}{4}\omega_{100} \times 10^7 = \underline{2{\cdot}51 \times 10^{10}}$

ω^2 commonly occurs in such expressions as $\omega^2 LC$. Calculation can be shortened by considering that

$$\begin{aligned} w^2 &= 3{\cdot}94 f^2 \times 10^7 \text{ when } f \text{ is in kc/s} \\ &= 3{\cdot}94 f^2 \times 10^{13} \text{ when } f \text{ is in Mc/s} \end{aligned}$$

Examples

(1) $f =$ 5 kc/s. $\qquad \omega^2 = 3{\cdot}94 \times 5^2 \times 10^7 = \underline{9{\cdot}85 \times 10^7}$

(2) $f =$ 8 Mc/s. $\qquad \omega^2 = 3{\cdot}94 \times 8^2 \times 10^{13} = \underline{2{\cdot}52 \times 10^{15}}$

Conversion of Units

In radio technique quantities are frequently measured in submultiples of basic units, e.g., current in milliamperes, inductance in microhenrys and capacitance in microfarads; whereas fundamental formulæ are derived from basic units. The procedure in the following examples will often simplify the conversion of formulæ :

Examples

(1) Find the power in watts dissipated by a resistance of 2,000 ohms when carrying a current of 30 mA. Power in watts $P = I^2R$.

Instead of converting to amperes and squaring, it is simpler to divide mentally (milliamperes) by 1,000 and resistance by 1,000.

$$P = 0{\cdot}9 \times 2 = 1{\cdot}8 \text{ watts}$$

(2) The frequency in c/s is given by the formula $f = 1/2\pi\sqrt{LC}$, when L is in henrys and C in farads. What is the frequency in Mc/s when L is in microhenrys and C in microfarads ?

$$\begin{aligned} \sqrt{L_{\mu H} C_{\mu F}} &= \sqrt{L_H C_F \times 10^{12}} \\ &= 10^6 \sqrt{L_H C_F} \end{aligned}$$

Also $\qquad f_{\text{Mc/s}} = 10^6 f_{\text{c/s}}$

Hence the formula remains unchanged.

Where the conversion involves several factors, it is advisable to set down each factor step by step to reduce risk of errors.

Example

Find the number of kilogram-calories per hour equivalent of 1 kW, given that

1 kg.cal. = 3·968 B.Th.U.
1 B.Th.U./sec. = 1·058 kW

kW to B.Th.U./sec. $\div$ 1·058
B.Th.U./sec. to kg.cal./sec. $\div$ 3·968
kg.cal./sec. to kg.cal./hr. $\times$ 3,600

$$1 \text{ kW} = \frac{3{,}600}{1{\cdot}058 \times 3{\cdot}968}$$

$$= \underline{860 \text{ kg.cal./hr.}}$$

Use of Logarithms

The laws of indices expressed in logarithmic form are :

Product of two numbers . . . . $\log mn = \log m + \log n$
(Add the logs and take the antilog.)
Quotient of two numbers . . . $\log m/n = \log m - \log n$
(Subtract the logs and take the antilog.)
*n*th power of a number . . . . $\log m^n = n \log m$
(Multiply the log by the power and take the antilog.)
*n*th root of a number . . . . $\log \sqrt[n]{m} = (1/n) \log m$
(Divide the log by the power and take the antilog.)

In applying these principles to calculations, note that the decimal part of a logarithm is positive, but the whole number (the index) to the left of the decimal point may be either positive or negative, according as the number corresponding to the logarithm is greater or less than unity. When the index is negative, convert the logarithm to a positive quantity by the device of adding +10 −10, as shown in the following examples.

Examples

(1) Evaluate $\frac{29{\cdot}32 \times 0{\cdot}0076.}{3{\cdot}451}$

$$\log 29{\cdot}32 = 1{\cdot}4672$$
$$\log 0{\cdot}0076 = \bar{3}{\cdot}8808\ (+10 - 10) = 7{\cdot}8808 - 10$$
$$9{\cdot}3480 - 10$$
$$\log 3{\cdot}451 = 0{\cdot}5379$$
$$8{\cdot}8101 - 10$$
$$\text{antilog } 8{\cdot}8101 - 10 = \text{antilog } \bar{2}{\cdot}8101$$
$$= \underline{0{\cdot}06459}$$

(2) Evaluate $\sqrt[3]{\dfrac{2\cdot303 \times 69\cdot5}{26\cdot42 \times 0\cdot007 \times 0\cdot124}}$

Denominator

$$
\begin{aligned}
\log 26\cdot42 &= \bar{1}\cdot4219 \\
\log 0\cdot007 &= \bar{3}\cdot8451 \\
\log 0\cdot124 &= \bar{1}\cdot0934 \\
\hline
&\quad \bar{2}\cdot3604
\end{aligned}
$$

Numerator

$$
\begin{aligned}
\log 2\cdot303 &= 0\cdot3623 \\
\log 69\cdot05 &= 1\cdot8420 \\
\hline
&\quad 2\cdot2043 \\
&\quad \bar{2}\cdot3604 \\
\hline
\text{log quotient} &= 3\cdot8439
\end{aligned}
$$

$$
\begin{aligned}
\log \sqrt[3]{\ } &= 1/3\ (3\cdot8439) \\
&= 1\cdot2813 \\
\text{antilog} \sqrt[3]{\ } &= \underline{19\cdot11}
\end{aligned}
$$

Decibel Conversions

Tables 3 and 4 provide a quick means of converting any power ratio P_2/P_1 or voltage ratio E_2/E_1 into decibels N, and vice versa, without the use of logarithmic tables. These tables have been derived from the basic formulæ

$$N = 10 \log_{10} (P_2/P_1)$$

or

$$N = 20 \log_{10} (E_2/E_1)$$

and, conversely,

$$P_2/P_1 = \text{antilog } (N/10)$$

$$E_2/E_1 = \text{antilog } (N/20)$$

To convert a power ratio greater than the ratios given in the table, divide the ratio by 10 in succession, until the quotient falls within the ratio column. Read the corresponding number of decibels, and add +10 db for each division by 10.

Examples

(1) To express a power ratio of 580 in decibels.

$$\frac{580}{10 \times 10} = 5\cdot8$$

From the table

$$
\begin{aligned}
\text{A power ratio of } 5\cdot8 &= 7\cdot634 \text{ db} \\
\text{A power ratio of } 580 &= 7\cdot634 + 10 + 10 \text{ db} \\
&= \underline{27\cdot63 \text{ db}}
\end{aligned}
$$

To convert a power ratio smaller than the ratios given in the table. multiply by 10 in succession, until the product falls within the ratio column. Read the corresponding number of decibels, and add −10 db for each multiplication by 10.

(2) To express a power ratio of 0·036 in decibels.

$$0\cdot036 \times 10 \times 10 = 3\cdot6$$

$$
\begin{aligned}
\text{A power ratio of } 3\cdot6 &= 5\cdot563 \text{ db} \\
\text{A power ratio of } 0\cdot036 &= 5\cdot563 - 10 - 10 \text{ db} \\
&= \underline{-14\cdot44 \text{ db}}
\end{aligned}
$$

TABLE 3.—CONVERSION OF POWER, VOLTAGE OR CURRENT RATIOS TO DECIBELS

Power Ratio	*db*	*Power Ratio*	*db*	*Power Ratio*	*db*	*Power Ratio*	*db*
1·0	0·000	3·3	5·185	5·6	7·482	7·9	8·976
1·1	0·414	3·4	5·315	5·7	7·559	8·0	9·031
1·2	0·792	3·5	5·441	5·8	7·634	8·1	9·085
1·3	1·139	3·6	5·563	5·9	7·709	8·2	9·138
1·4	1·461	3·7	5·682	6·0	7·782	8·3	9·191
1·5	1·761	3·8	5·798	6·1	7·853	8·4	9·243
1·6	2·041	3·9	5·911	6·2	7·924	8·5	9·294
1·7	2·304	4·0	6·021	6·3	7·993	8·6	9·345
1·8	2·553	4·1	6·128	6·4	8·062	8·7	9·395
1·9	2·788	4·2	6·232	6·5	8·129	8·8	9·445
2·0	3·010	4·3	6·335	6·6	8·195	8·9	9·494
2·1	3·222	4·4	6·435	6·7	8·261	9·0	9·542
2·2	3·424	4·5	6·532	6·8	8·325	9·1	9·590
2·3	3·617	4·6	6·628	6·9	8·388	9·2	9·638
2·4	3·802	4·7	6·721	7·0	8·451	9·3	9·685
2·5	3·979	4·8	6·812	7·1	8·513	9·4	9·731
2·6	4·150	4·9	6·902	7·2	8·573	9·5	9·777
2·7	4·314	5·0	6·990	7·3	8·633	9·6	9·823
2·8	4·472	5·1	7·076	7·4	8·692	9·7	9·868
2·9	4·624	5·2	7·160	7·5	8·751	9·8	9·912
3·0	4·771	5·3	7·243	7·6	8·808	9·9	9·956
3·1	4·914	5·4	7·324	7·7	8·865	10·0	10·000
3·2	5·051	5·5	7·404	7·8	8·921		

To convert a voltage or current ratio, treat the value in the power ratio column as a voltage or current ratio, and double the corresponding number of decibels read in the decibel column. If the ratio is outside the range of the table, follow the procedure outlined above.

(3) To express a voltage ratio of 75 in decibels.

$$\frac{75}{10} = 7{\cdot}5$$

A power ratio of 7·5 = 8·751 db
A voltage ratio of 7·5 = 8·751 × 2 = 17·50 db
A voltage ratio of 75 = 17·5 + 10 = <u>27·5 db</u>

To convert values intermediate between 10 and 100 db, not included in the table, proceed as follows :

(1) Select from the decibel column the nearest value below the given decibels, and note the corresponding ratio.
(2) Subtract the selected decibels from the given decibels, and note the corresponding ratio.
(3) Multiply the two ratios thus found.

Table 4.—Conversion of Decibels to Power, Voltage or Current Ratios

Gain		+ *db*-	*Loss*	
Power Ratio	*Voltage or Current Ratio*		*Power Ratio*	*Voltage or Current Ratio*
1·000	1·000	0	1·0000	1·0000
1·259	1·122	1	0·7943	0·8193
1·585	1·259	2	0·6310	0·7943
1·995	1·413	3	0·5012	0·7079
2·512	1·585	4	0·3981	0·6310
3·162	1·778	5	0·3162	0·5623
3·981	1·995	6	0·2512	0·5012
5·012	2·239	7	0·1995	0·4467
6·310	2·512	8	0·1585	0·3981
7·943	2·818	9	0·1259	0·3548
10	3·162	10	10^{-1}	3·162 × 10
10^2	10	20	10^{-2}	10^{-1}
10^3	3·162 × 10	30	10^{-3}	3·162 × 10^{-1}
10^4	10^2	40	10^{-4}	10^{-2}
10^5	3·162 × 10^2	50	10^{-5}	3·162 × 10^{-2}
10^6	10^3	60	10^{-6}	10^{-3}
10^7	3·162 × 10^3	70	10^{-7}	3·162 × 10^{-3}
10^8	10^4	80	10^{-8}	10^{-4}
10^9	3·162 × 10^4	90	10^{-9}	3·162 × 10^{-4}
10^{10}	10^5	100	10^{-10}	10^{-5}

Examples

(1) To express a gain of 28 decibels as a power ratio.
From the table :

Power ratio for + 20 db = 100
Power ratio for + 8 db = 6·31
Power ratio for + 28 db = 100 × 6·31 = <u>631</u>

(2) To express a level of 15 decibels below zero level (1 mW) as power output in milliwatts.
From the table :

Power ratio for −10 db = 0·1
Power ratio for − 5 db = 0·3162
Power output (reference 1 mW) = 0·1 × 0·3162 × 1
= <u>0·0316 mW</u>

(3) To express a voltage gain of 44 decibels as output voltage when the input level is 0·5 volts. Input and output impedances are assumed to be equal.

From the table :

Voltage ratio for +40 db = 100
Voltage ratio for + 4 db = 1·585
Output voltage (reference 0·5 V) = 100 × 1·585 × 0·5
= 79·25 volts

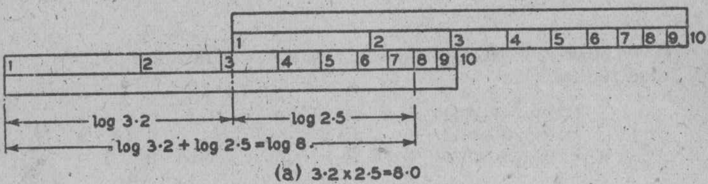
Slide-rule Calculations

The principal scales on the face of a slide rule are divided into lengths proportional to the logarithms of the numbers engraved opposite the lengths. On a 10-in. rule the full scale from 1 to 10 corresponds to $\log_{10} 10$, and the length in inches corresponding to any number between 1 and 10 is equal to the logarithm of that number multiplied by 10.

Multiplication of two numbers by addition of their logarithms is performed directly by adding the length representing one number on the sliding scale to the length representing the other on the fixed scale. Fig. 1 (*a*), with the aid of an example, will illustrate the principle of this operation. To multiply 3·2 × 2·5 set 1 on the sliding scale opposite 3·2 on the fixed scale. Opposite 2·5 on the sliding scale, read off the product 8·0 on the fixed scale.

Similarly, to divide two numbers by subtracting their logarithms, the lengths are subtracted, as in Fig. 1 (*b*). To divide 8·0 by 3·2 we set 3·2 on the fixed scale opposite 8·0 on the sliding scale. Opposite 1 on the fixed scale read off the quotient 2·5 on the sliding scale. The method of approximating to determine the position of the decimal point has been described in an earlier section.

The ordinary straight slide rule has four scales on the front face which will be referred to in the sections that follow as A, B, C, D. A and D are fixed to the body of the scale; B and C are on the slide. A and B are similar; C and D are also similar. On the reverse of the slide are three more scales for calculating sines, tangents and logarithms, which will be referred to as S, T and L respectively.



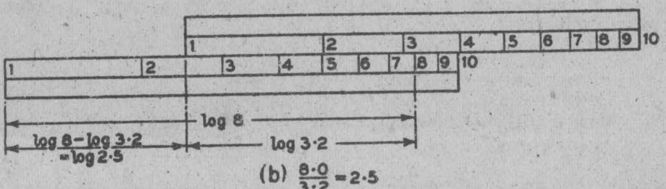


Fig. 1.—The Fixed and Sliding Scales of a Slide Rule, showing: (*a*) the Method of Multiplication, and (*b*) the Method of Division.

Multiplication and Division

When the numerator and denominator both contain two or more factors, the number of movements of the slide can be reduced by cross dividing and multiplying alternately.

Example

$$\frac{0{\cdot}038 \times 625}{89{\cdot}6 \times 0{\cdot}00011}$$

Approximation for the decimal point 2700.

(i) Set hair line on cursor over 3·8 on Scale D.
(ii) Slide Scale C until 8·96 appears under hair line.
(iii) Set hair line on cursor over 6·25 on Scale C.
(iv) Slide Scale C until 1·1 appears under hair line.
(v) Read off 241 under 1 on Scale D.

Answer to 3 significant figures = 2,410

Squares

Scales C and D are each twice the length of Scales A and B. Since $\log x^2 = 2 \log x$, numbers on Scale A are the squares of the numbers opposite them on Scale D.

Example

$$0{\cdot}0627^2$$

Approximation for the decimal point 0·0036.

(i) Set hair line on cursor over 6·27 on Scale D.
(ii) Read off 393 under hair line on Scale A.

Answer to 3 significant figures = 0·00393

Square Roots

Numbers on Scale D are the square roots of the numbers opposite them on Scale A. Use the left-hand half of Scale A if the number of figures to the left of the decimal point is odd, and the right-hand half if it is even. When the number is less than unity, use the left-hand half of Scale A if the number of 0s to the right of the decimal point is odd, and the right-hand half if it is even.

Examples

(1)

$$\sqrt{0{\cdot}00548}$$

Approximation for the decimal point 0·07.

(i) Set hair line on cursor over 5·48 on right-hand half of Scale A.
(ii) Read off 74 under hair line on Scale D.

Answer to 3 significant figures 0·074

(2)

$$\sqrt{39500}$$

Approximation for the decimal point 200.

(i) Set hair line on cursor over 395 on left-hand half of Scale A.
(ii) Read off 199 under hair line on Scale D.

Answer to 3 significant figures 199

Cubes

A cube is obtained on Scale A from the number on Scale D in the following manner:

Example

$$2{\cdot}56^3$$

(i) Move Scale C until 1 (or 10 if number >316 . . .) is opposite 2·56 on Scale D.
(ii) Set hair line on cursor over 2·56 on Scale B.
(iii) Read off answer on Scale A.

Answer 16·8

Cube Roots

The process is the reverse of cubing. The root is obtained on Scale D from the number of Scale A.

Example

$$\sqrt[3]{87}$$

(i) Set hair line on cursor over 87 on Scale A.
(ii) Move Scale C until 1 (or 10 if number >464 . . .) is opposite the same number on D as appears under cursor on B.
(iii) Read off answer on Scale D.

Answer 4·43

Sines

Use Scale S on back of slide for the angle, and read the sine on Scale B, and vice versa. Note that for angles from approximately $\frac{1}{2}°$ to $5\frac{1}{2}°$ the figures for the sine must be prefixed by 0·0, and for angles above about $5\frac{1}{2}°$ by 1.

Example

$$\sin 18° 30'$$

(i) Withdraw slide to right and set 18° 30′ on Scale S below the datum mark on the body of the scale.
(ii) Reverse rule and read off 3·17 on Scale B below 10 on Scale A at right-hand end.

Answer 0·317

Cosines

The cosine of an angle is equal to the sine of the complementary angle θ. Use the procedure for the sine and find $\sin(90° - \theta)$.

Example

$$\cos 62°$$

(i) $\cos 62° = \sin(90° - 62°) = \sin 28°$.
(ii) Withdraw slide to the right and set 28 on Scale S below the datum mark on the body of the rule.
(iii) Reverse rule and read off 4·7 on Scale B below 10 on Scale A at right-hand end.

Answer 0·47

Tangents

Use Scale T on back of slide for the angle, and read the tangent on Scale C above 1 on Scale D, and vice versa. Note that for angles between approximately $\frac{1}{2}°$ and $5\frac{1}{2}°$ the figures for the tangent must be prefixed by 0·0, and for angles between $5\frac{1}{2}°$ and 45° by 0. For angles between 45° and 90° the tangent varies between 1 and infinity. When the angle exceeds 45°, subtract the angle from 90° and read the tangent on Scale D below 10 on Scale C.

Example

$$\tan 63° \, 35'$$

(i) Since the angle is greater than 45°, use the complementary angle. $90° - 63° \, 35' = 26° \, 25'$.

(ii) Withdraw slide to left and set 26° 25′ above datum mark on body of rule.

(iii) Reverse the rule and read off 2·02 on Scale D below 10 at right-hand end of Scale C.

Answer 2·02

Cotangents

The contangent of an angle θ is equal to the tangent of its complementary angle. When the angle is less than 45°, find tan θ and read the tangent on Scale D below 10 on Scale C. When the angle is greater than 45°, find $\tan (90° - \theta)$ and read the tangent on Scale C above 1 on Scale D.

Example

$$\cot 21° \, 50'$$

(i) Withdraw slide to left and set 21° 50′ above datum mark on body on rule.

(ii) Reverse rule and read off 2·5 on Scale D below 1 on Scale C.

Answer 2·5

Common Logarithms

Use Scale D for the number and Scale L on the back of the slide for the mantissa of the corresponding logarithm, and vice versa. The index of the logarithm is determined in the manner previously described for logarithms.

Examples

(1) $\log_{10} 74·6$

(i) Since the number lies between 10 and 100, the index will be 1.

(ii) Set 1 on Scale C opposite 7·46 on Scale D.

(iii) Reverse rule and read off 8·73 on Scale L above datum mark on body of rule.

Answer 1·873

(2) $\log_{10} 0{\cdot}0382$

(i) Since the number lies between 0·01 and 0·1, the index will be $\bar{2}$.

(ii) Set 1 on Scale C opposite 3·82 on Scale D.

(iii) Reverse rule and read off 5·82 on Scale C above datum mark on body of rule.

Answer $\bar{2}{\cdot}582$

Complex Quantities

Complex quantities are readily resolved with the aid of the j operator. Just as the symbol $-$ prefixed to a quantity representing a vector denotes that the vector is displaced by an angle of 180°, so the symbol $\sqrt{-1}$, represented by the prefix j, denotes displacement through 90°. It follows that

$$\begin{array}{rcrl}
j & = & \sqrt{-1} & \text{rotation through } 90^\circ \\
j \times j = +j^2 & = & -1 & \text{rotation through } 180^\circ \\
-j \times j = -j^2 & = & +1 & \text{rotation through } 0^\circ \\
-j \times -j = +j^2 & = & -1 & \text{rotation through } 180^\circ \\
j \times j \times j = +j^3 & = & -1\sqrt{-1} & \text{rotation through } 270^\circ \\
j \times j \times j \times j = +j^4 & = & +1 & \text{rotation through } 360^\circ
\end{array}$$

A vector displaced by an angle intermediate between 0° and 90° can be regarded as the resultant of two component vectors a and b at 90° with respect to each other, and is designated by the complex quantity $a + jb$, of which a is termed the real part and jb the imaginary part. For example, the vector representing the impedance Z of a resistance R and reactance X in series is the resultant of two vectors R and jX, and forms the hypotenuse of a right-angled triangle, of which R and jX are the two sides (see Fig. 2). Hence:

$$Z = R + jX$$

$$|Z| = \sqrt{R^2 + X^2}$$

where the phase angle

$$\phi = \text{arc tan } (X/R)$$

Complex expressions must be added vectorially by treating the real and imaginary components separately. Addition, subtraction, multiplication and division are performed according to the ordinary rules of algebra. The scalar value of the resultant can then be calculated by taking the square root of the sum of the squares of the real and imaginary quantities.

ADDITION

$$Z_1 + Z_2 = (R_1 + jX_1) + (R_2 + jX_2) = (R_1 + R_2) + j(X_1 + X_2)$$

SUBTRACTION

$$Z_1 - Z_2 = (R_1 + jX_1) - (R_2 + jX_2) = (R_1 - R_2) + j(X_1 - X_2)$$

MULTIPLICATION

$$Z_1Z_2 = (R_1 + jX_1)(R_2 + jX_2) = (R_1R_2 - X_1X_2) + j(X_1R_2 + R_1X_2)$$

DIVISION

$$Z_1/Z_2 = (R_1 + jX_1)/(R_2 + jX_2) = (R_1R_2 + X_1X_2)/(R_2^2 + X_2^2) + j(X_1R_2 - R_1X_2)/(R_2^2 + X_2^2)$$