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Nonlinear Control Systems with Discontinuity: Theory and Practice

Tsinghua University Press

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Selected papers from

the second China-Japan Joint Workshop

on Control Theory and Technology

Harbin,China, August 6, 2006

Tsinghua University Press

图书在版编目（CIP）数据

非连续非线性系统的控制理论与应用/申铁龙等编. —北京：清华大学出版社, 2007.10

ISBN 978-7-302-16144-8

Ⅰ.非… Ⅱ.申… Ⅲ.非线性控制系统–文集 Ⅳ. TP273-53

中国版本图书馆 CIP 数据核字（2007）第 144893 号

责任编辑： 张占奎
责任校对： 刘玉霞
责任印制： 李红英

出版发行： 清华大学出版社　　**地　　址：** 北京清华大学学研大厦 A 座
http://www.tup.com.cn　　**邮　　编：** 100084
c-service@tup.tsinghua.edu.cn
社 总 机： 010-62770175　　**邮购热线：** 010-62786544
投稿咨询： 010-62772015　　**客户服务：** 010-62776969
印 装 者： 三河市春园印刷有限公司
经　　销： 全国新华书店
开　　本： 165×235　**印张：** 15　**字数：** 373 千字
版　　次： 2007 年 10 月第 1 版　**印次：** 2007 年 10 月第 1 次印刷
印　　数： 1~500
定　　价： 78.00 元

本书如存在文字不清、漏印、缺页、倒页、脱页等印装质量问题，请与清华大学出版社出版部联系调换。联系电话：010-62770177 转 3103　产品编号：027094-01

Preface

Discontinuity in dynamic systems is a tough property for analysis and control of the system. However, in practical engineering, we are often faced with the discontinuity in control system design. In dynamical systems, the discontinuity is usually caused by natural phenomenon such as static friction, stroke changes etc., in mechanical systems or control actions engineered by controller design such as variable structure control, switching control, etc. For this kind of systems, the conventional framework for analysis and synthesis is not sufficient to provide a solution in exact theoretical sense. The field of control theory for dynamical systems with discontinuity is widely open for future research and development.

The aim of the second China-Japan joint workshop on control theory and technology is to provide a forum for scientists on automatic control from both China and Japan to exchange contemporary research results on this issue and to promote the applications of advanced control theory to practical engineering problems, and as a result to enhance the development and spread new results of control theory on the dynamical systems with discontinuities. The papers included in this book are selected from the workshop, and this selection is focused on the papers dealing with the closely related topics of *nonsmooth, switching and hybrid systems* and its applications in self driven hybrid systems such as the power systems, aircraft, land vehicles, mechanical and electrical systems.

This book is organized as three parts: the first part contains 7 papers that addressed theoretical problems in design of nonlinear control systems with discontinuity and time-delay. The second part contains 6 papers with physical application of mechanical and electrical system, and the last part collects 4 papers that focus on the control problem in vehicular systems including railway, automotive engine, etc.

The editors appreciate the Control and Simulation Center, Harbin Institute of Technology. The second China-Japan joint workshop on control theory and technology was hosted by the center as oneday event in Victoria Hotel, Harbin, at 6th of August, 2006. The workshop was sponsored technically by Technical Committee on Control Theory, CAA(China), Committee of Electric Mathematics, CALC(China), Committee on Adaptive Learning Control, SICE(Japan). Moreover, it should be noted that this book is the second book published by Tsinghua University Press for the China-Japan joint workshop on this topic. The

first one was published in May of 2005 that contains the selected papers from the first China-Japan joint workshop on control theory and applications held in Fragrant Hill Hotel Beijing, 22-26, Sep., 2004, under financial support from Natural Science Foundation of China (NSFC) and Japan Science Promotion Society (JSPS).

Finally, we would like to express our thanks to Dr. Fenghua He for helping in organizing the workshop, and the PhD Candidate, Mr. Kai Zheng for technical helping in editing the book.

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Beijing, 30 June, 2007

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Part I

Design Method of Nonlinear Systems

Design of Decentralized Robust Full-MRACS Based on Sliding Mode Control

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1 Introduction

Mathematical theory of networks and systems has been significantly enriched in the last few decades due partly to the injection of new blood from different backgrounds. The contributions of past and present investigators span a vast body of theory and applications. Of particular importance in this arena is the field of decentralized adaptive control, whose relevant results are teeming in the literature [1].

In the broad sense, the existing results on decentralized adaptive control can be categorized as in the following, where due to lack of space not are all of the pertinent works cited:

- Decentralized linear adaptive control [12~16];
- Decentralized intelligent adaptive control [17~20];
- Decentralized optimal adaptive control [21];
- Decentralized robust adaptive control [22~26];
- Decentralized nonlinear adaptive control [27~30].

It is noted that it is possible to have different categorizations (e.g. vs. linear/nonlinear systems, or delay-free/delay systems) or to further ramify each category and/or to have their integrations. Actually, many of the existing results present an integrated approach and do not fully belong to one of the above categories. Here, in particular, (in some sense) from the integration of the last two we single out the sequel branch,

- *Decentralized adaptive variable-structure (or sliding-mode) control (VSC or SMC)* [31~42][3], since it will be conceptually used in this work.

To the best of our knowledge, and broadly speaking, all the existing decentralized MRAC methods suffer from two major drawbacks, see [1] for details:

[3] It should be noted that the results of [34] and [37] are erroneous, as reported in [42].

- accommodation of reference models for subsystems only, i.e., not for interconnections;
- necessity for the so-called matching conditions.

In the spirit of a recursive nonlinear design methodology - the so-called integrator back stepping - developed in the early 1990s [43], which alleviated the necessity for matching conditions in centralized adaptive control, the second drawback was partially solved in the mid 1990s [44]. More precisely, for the special class of large-scale systems transformable to decentralized strict feedback form, the aforementioned necessity was relaxed in [44]. Later, it has been tackled by a few other works as well, see [1] for details.

As for the first problem, the situation is worse. In fact, it seems to be folk knowledge that it is irrelevant or rather impossible to consider a full reference model whose interconnection parts are not identical with those of the system, since the system has no adjustable parameters in those parts. Nevertheless, this argument, though appealing, is no mathematical proof.

On the other hand, incorporation of a desirable reference model for interconnections, i.e., a full model reference whose interconnection parts are not identical with those of the system, will result in performance enhancement, since the overall system will behave like a reference model whose interconnections, as well as subsystems, are desirable. Inspired by this, the aforementioned folk knowledge is examined in this work and this long-standing stereotype is shown to be wrong. More precisely, for systems satisfying the matching conditions, a *full*-MRAC with non-identical interconnections is proposed in this paper.

Before embarking on the main result, a general picture of the status quo of MRAC is drawn in the following.

In both centralized and decentralized MRAC, the subsequent assumptions are standard [43,45~52], where in the decentralized case the plant refers to each of the isolated subsystems:

- The plant is single-input single output (SISO);
- The plant is minimum phase;
- The relative degree of the plant is known, usually one or two;
- The sign of the high-frequency gain is known;
- There is no nonlinearity, disturbance, perturbation;
- There is no delay in interconnections.

It is of course well known that in the realm of centralized MRAC there are *many* results alleviating any one or more of the above assumptions or rather shortcomings [43,45~52], but, to the best of our knowledge, not all of them together. In the realm of decentralized MRAC the situation is poorer: there are only *a few* works addressing some of them. In particular, to our best knowledge, only some VSC-based schemes have been able to relax the first and/or second of the aforementioned defects. Also, the relative degree of the plant can be larger than two, but an upper bound of it should be known a priori [11], the sign of the high-frequency gain can be unknown [12], and perturbations, disturbances, nonlinearities, and delays can be handled, e.g., [22~26]. In addition, it should be mentioned that the case of unknown time-delay interconnections has been tackled only by few works like [25, 26, 40].

On the other hand, in both centralized and decentralized cases, adaptation and control laws depend on whether the plant (each isolated subsystem) is known or unknown. That is, to our best knowledge, there is no universal MRAC scheme treating known and unknown systems in a unified framework with the same adaptation and control laws.

What is presented in this work is a full-MRAC with non-identical interconnections that rectifies *all* the above-mentioned shortcomings as well. More precisely, without any of the aforementioned standard assumptions, we consider a class of *known* systems with unknown time-varying interconnection delays and unknown time-varying perturbations (model uncertainties, nonlinearities, or external disturbances), upper bounds of the latter being unknown. The accommodation of this kind of perturbations - along with a *reformulation* of the problem - enables us to consider *unknown* systems in the same framework, as will be explained in the next section. For this class a decentralized robust full-MRAC with non-identical interconnections is presented. To this end, conceptually similar to [31 $\sim$ 42] in particular [40][4], a stable decentralized full-model-reference adaptive sliding-mode control is proposed. The controller has two parts: the adaptive part which compensates for the perturbations whose upper bounds are unknown, and the linear part, together enforcing the sliding motion and global asymptotic exact tracking. Moreover, as will be seen, the proposed controller does not suffer from the inherent shortcomings of standard VSC, namely, need for a priori knowledge of the upper bound of perturbations (as mentioned above), and the notorious chattering phenomenon.

The organization of this work is as follows. In Section II the system is described and the problem is formulated. Controller design is presented in Section III, where the stability of the system is proven using an improved Lyapunov function. Finally, conclusions are drawn in Section IV. This work presents part of [1] in continuation of [53, 54].

2 System Description

Consider a large-scale system consisting of N interconnected subsystems whose ith one is represented by,

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + v_i + \sum_{j \neq i}^{N} A_{ij} x_j(t - \tau_{ij}) \\ x_i &= \varphi_i, t \in [-T, 0], \end{aligned} \tag{1}$$

[4] This work can also be seen as an extension of [40] to full-MRAC and also to the case of unknown systems, see Remark 3.4 and 3.5 for details.

in which $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$, and $v_i(x_i, u_i, x_j, t) \in \mathbb{R}^{n_i}$ are the state, input, and uncertain nonlinear time-varying parameter perturbation of the ith subsystem. More precisely, v_i denotes *any* unknown time-varying perturbation in the form of model uncertainty ($\Delta A_i x_i$, $\Delta B_i x_i$, $\sum\limits_{j \neq i}^{N} \Delta A_{ij} x_j$), external disturbance, or nonlinearity. The scalar function $0 \leqslant \tau_{ij}(t) \leqslant T < \infty$ represents the unknown nonnegative continuous bounded time-varying delay, where T is a known positive constant. The continuous function $\varphi_i(t) \in \mathbb{R}^{n_i}$ denotes the arbitrary initial condition. Moreover, all the matrices A_i, B_i, A_{ij} are known, of appropriate dimensions, with obvious meanings, and (A_i, B_i) is a controllable pair. The terms $\sum\limits_{j \neq i}^{N} \Delta A_{ij} x_j$ denote the interconnections with other subsystems. In case of unknown systems, the matrices A_i, B_i, A_{ij} are chosen arbitrarily up to the assumptions to follow shortly. The rest of the unknown parts are lumped and denoted by v_i. This reformulation enables us to treat both known and unknown systems in the same framework with the same adaptation and laws.

$$\dot{x}_{mi} = A_{mi} x_{mi} + B_{mi} r_i + \sum_{j \neq i}^{N} A_{mij} x_{mj} \tag{2}$$

in which the involved terms have obvious meanings. In addition, A_{mi} is chosen such that (A_{mi}, B_i) is a controllable pair. Defining the tracking error vector as,

$$e_i = x_i - x_{mi}, \quad i = 1, \cdots, N \tag{3}$$

the error dynamics is given by $i = 1, \cdots, N$,

$$\dot{e}_i = A_{mi} e_i + (A_i - A_{mi}) x_i + B_i u_i + \sum_{j \neq i}^{N} A_{ij} x_j (t - \tau_{ij}) - B_{mi} r_i - \sum_{j \neq i}^{N} A_{mij} x_{mj} + v_i \tag{4}$$

As stated before, as in the majority of the literature, we consider the class of systems satisfying the matching conditions. As such, there exist continuous matrix and vector functions M, H, J, G, and f (in general, non-unique) of appropriate dimensions such that,

$$\begin{aligned} A_i - A_{mi} &= B_i M_i \\ A_{ij} &= B_i H_{ij} \\ A_{mij} &= B_i J_{ij} \\ B_{mi} &= B_i G_i \\ v_i &= B_i f_i \end{aligned} \tag{5}$$

It should be noted that although this class does not cover all systems, it is still fairly large and embodies a huge body of real-world systems like many mechanical ones [1~52]. Besides, it is clear that the above conditions are satisfied, based on the theory of generalized inverse [55], iff $\text{rank}(X; B) = \text{rank}(B)$, where $X = BY$.

For this class the following assumptions are made, as in the literature, where the first two are clearly feasible and natural in practice:
A1) Local controllability and availability of all states for measurement in all time;
A2) Continuous differentiability and piecewise continuity in time;
A3) Existence of unknown positive scalar constants α_i and β_i such that [56],

$$||f_i|| \leqslant \alpha_i + \beta_i||x_i|| \tag{6}$$

for all time. This assumption will be used in the stability proof.

Remark 2.1 *For the aforementioned large class of real-world systems we are concerned with actual signals, systems, and quantities. The assumption A3 is thus natural and does not impose any restriction on the class of systems considered, since it is actually equivalent to the Lipschitz boundedness of the perturbations. For the sake of completeness, on the other hand, the following mathematical argument can be made. If an upper bound on $||f_i||$ is given in terms of higher orders of $||x_i||$, e.g., a polynomial of high order, still the first order given bound is valid, because we are concerned with actual signals and quantities, which are all bounded, not with abstract mathematical quantities. However, in this case, due to conservatism, the control effort will be in general larger.*

Under the aforementioned assumptions A1~A3 and (5), the error dynamics is reduced to,

$$\dot{e}_i = A_{mi}e_i + B_iM_ix_i + B_iu_i + \sum_{j\neq i}^{N} B_iH_{ij}x_j(t-\tau_{ij}) - B_iG_ir_i - \sum_{j\neq i}^{N} B_iJ_{ij}x_{mj} + B_if_i \tag{7}$$

In the proceeding section a decentralized full-model-reference adaptive sliding-mode controller will be proposed by which the global stability of the error dynamics is guaranteed. Prior to this, one of the merits of the advocated methodology is restated.

Remark 2.2 *The above unified analysis has a nice interpretation and is worth paraphrasing. For both known and unknown systems, the system can be perturbed, disturbed, nonlinear, and/or time varying. All the designer needs to do is to choose the system parameters (A_i, B_i, A_{ij}) some constant matrices satisfying (5) with (A_i, B_i) controllable (and to pursue the proposed synthesis approach of the next section). Needless to say, in general, the closer these parameters to their true values, the smaller the v_i, and hence the smaller and smoother the control action and the transient error.*

3 Controller Design

The majority of the existing MRAC methods fall in three main categories: Lyapunov function method, hyperstability method, and SMC (or VSC) method. The advantages and disadvantages of each category have been extensively studied in the literature. The adaptive SMC strategy is adopted in this work.

3.1 Conventional SMC Schemes

The principle of the conventional SMC method is to employ a discontinuous control law to drive the system to the origin along a user-specified trajectory, the so-called sliding mode [57~62]. Broadly speaking, the design procedure can be divided into two steps. In the first step, some discontinuous surfaces (i.e., sliding hyperplanes) are designed in such a way that not only the sliding motion can take place on them, but also some desired performance is achieved. In the second step, a discontinuous control law is derived by which the sliding motion is guaranteed.

It is well-known that VSC exhibits well transient response and is a powerful tool for handling uncertain perturbations, nonlinearities, and disturbances. Two major disadvantages, nevertheless, are associated with it. The first one is the notorious chattering phenomenon, which is not only detrimental to the control object, but also excites the high frequency unmodeled plant dynamics. The second one is that, in general, upper bounds of perturbations (model uncertainties, nonlinearities, or disturbances) are needed to be known a priori, whilst in practice it is often difficult or impossible to have a (non-conservative) measure of them.

These shortcomings are rectified in the ensuing development.

3.2 The Proposed Controller

To compensate for the inherent, undesirable chattering problem, apart from quick fixes like the variable sliding surface [31], boundary-layer technique [59], and filtering and variable step-size implementation [60], there has been a fundamental modification in the traditional SMC scheme in that the discontinuous control law is replaced by a continuous one [62], while preserving all the other features of the approach. As will be seen, our proposed controller is also continuous and thus does not suffer from the chattering problem (see also Remarks 3.4 and 3.5). The sliding surface is defined as,

$$s_i = C_i e_i - \int_0^t C_i (A_{mi} + B_i K_i) e_i \, \mathrm{d}\tau \qquad i = 1, \cdots, N \tag{8}$$

where $C_i \in \mathbb{R}^{n_i \times n_i}$ is of full rank and chosen such that $C_i B_i$ is invertible. The matrix $K_i \in \mathbb{R}^{m_i \times n_i}$ is designed such that $A_{mi} + B_i K_i$ is Hurwitz, see Remark 3.1.

As for the unknown upper bounds of perturbations, similar to the method of [56], later used in [40], the following strategy is adopted. It is assumed that the earlier-mentioned assumption A3, namely, $||f_i|| \leqslant \alpha_i + \beta_i ||x_i||$ holds. Then, some adaptation laws are proposed for the unknown upper bounds, and an adaptive sliding-mode controller is designed accordingly. More precisely, the scalars $\hat{\alpha}_i$ and $\widehat{\beta}_i$ are used to adapt the unknown constants α_i and β_i, respectively. The adaptation errors are defined by $\Delta\alpha_i \stackrel{\text{def}}{=} \hat{\alpha}_i - \alpha_i$ and $\Delta\beta_i \stackrel{\text{def}}{=} \widehat{\beta}_i - \beta_i$. The controller is proposed as,

$$u_i = u_i^{lin} + u_i^{adp} \tag{9}$$

where the linear and adaptive parts are respectively given by,

$$\begin{aligned} u_i^{lin} &= -M_i x_i + G_i r_i + K_i e_i - (C_i B_i)^{-1}\lambda_i s_i \\ &\quad -(C_i B_i)^{-1} s_i ||s_i||^{-1}(\sum_{j\neq i}^{N} ||C_j B_j|| ||H_{ji}|| x_i^{\sup}(t)) \\ &\quad -(C_i B_i)^{-1} s_i ||s_i||^{-1}(\sum_{j\neq i}^{N} ||C_j B_j|| ||J_{ji}|| ||x_{mi}||) \\ u_i^{adp} &= -B_i^{\mathrm{T}} C_i^{\mathrm{T}} s_i ||s_i^{\mathrm{T}} C_i B_i||^{-1}(\widehat{\alpha}_i + \widehat{\beta}_i ||x_i||) \end{aligned} \tag{10}$$

in which λ_i is a user-specified positive constant and $x_i^{\sup}(t) \stackrel{\text{def}}{=} \sup_{t-T\leqslant h\leqslant t} ||x_i(h)||$. The adaptation parameters are updated by,

$$\begin{aligned} \dot{\widehat{\alpha}}_i &= \rho_i \left(\frac{||s_i^{\mathrm{T}} C_i B_i||}{||s_i||}\right) \\ \dot{\widehat{\beta}}_i &= \mu_i \left(\frac{||s_i^{\mathrm{T}} C_i B_i|| ||x_i||}{||s_i||}\right) \end{aligned} \tag{11}$$

in which the positive scalars ρ_i, μ_i denote the adaptation gains. In theory, the higher the adaptation gains, the higher the adaptation rates. In practice, this of course has an upper bound due to actuator limitations and other practical considerations including the safe operation region of the plant.

The main result of this work is presented in the sequel theorem.

Theorem 3.1 *Given the large-scale system (1), the full reference model (2), and all the earlier-mentioned assumptions. Using the sliding hyperplanes (8), if the sub-controllers $u_i (i = 1, \cdots, N)$ are designed by (9),(10) and adapted by (11), then the error dynamics (7) is globally asymptotically stable.*

Proof: Consider the improved Lyapunov function,

$$V = \sum_{i=1}^{N} ||s_i|| + \sum_{i=1}^{N} \frac{1}{2}\left(\rho_i^{-1}\tilde{\alpha}_i^2 + \mu_i^{-1}\tilde{\beta}_i^2\right) \tag{12}$$

The derivative of V is,

$$\dot{V} = \sum_{i=1}^{N} \frac{s_i^{\mathrm{T}} \dot{s}_i}{||s_i||} + \sum_{i=1}^{N} \left(\rho_i^{-1}\tilde{\alpha}_i\dot{\tilde{\alpha}}_i + \mu_i^{-1}\tilde{\beta}_i\dot{\tilde{\beta}}_i\right) \tag{13}$$

Similar to [40], it is easy to show that $\dot{V} \leqslant -\sum_{i=1}^{N} \lambda_i ||s_i||$ and conclude that $\lim_{t\to\infty} s_i \to 0$. In other words, the sliding motion will eventually be obtained. When the system is in the sliding mode $s_i = 0$, $\dot{s}_i = 0$, where the latter is obtained by the equivalent control,

$$u_i^{equ} = -M_i x_i - \sum_{j\neq i}^{N} H_{ij} x_j(t-\tau_{ij}) - f_i + G_i r_i + \sum_{j\neq i}^{N} J_{ij} x_{mj} + K_i e_i \tag{14}$$

yielding $\dot{e}_i = (A_{mi} + B_i K_i)e_i$, guaranteeing the global asymptotic stability of the error dynamics. □

Some remarks are in order.

Remark 3.1 *The above theorem implies that the system state and the adaptive parameters are all bounded.*

Remark 3.2 *Among* [53, 54], *eigenstructure assignment can be employed as a powerful technique to design K_i so that some desired error dynamics is achieved. Due to lack of space, and because it is not the objective of this paper, it is not further followed inhere. However, the following trade-off is noted as a rule of thumb: the larger K_i, the faster the error convergence to zero, but the larger the control effort in the transient phase.*

Remark 3.3 *The difference between this work and that of* [40] *is remarkable: a) incorporation of a full reference model with non-identical interconnections, which is novel, b) applying the same method in the same framework without any modifications in adaptation and control laws to unknown systems (the above-cited paper considers only known systems) by the way of reformulation of the problem, which is a remarkable distinction of this work, not existing even in centralized adaptive control results* [43,45-52]*, and c) giving more insight to the details of the approach. On the other hand, formulation wise, the differences are not considerable: the presence of the reference model for interconnections, A_{mij}, resulting in the modifications of the matching assumptions (5) and u_i^{lin}.*

Remark 3.4 *A mistake in* [40, page 1217] *is pointed out, or rather, the following explanation is added: Contrary to what they say, their controller - similar to ours - is continuous and thus does not suffer from the chattering problem, for multi-input multi-output (MIMO) subsystems. However, it is clear that if the ith subsystem is SISO, then s_i and C_iB_i will be scalars, and thus $s_i||s_i||^{-1}$ and $B_i^{\mathrm{T}}C_i^{\mathrm{T}}s_i||s_i^{\mathrm{T}}C_iB_i||^{-1}$ will be sign functions (of their arguments), making the ith control input discontinuous. In this case, any of the previously-mentioned quick fixes can be used for that subsystem.*

Remark 3.5 *Another mistake in* [40, page 1215] *is pointed out: They argue that since in finite time s_i will not be exactly equal to zero, in practice the adaptive parameters (11) will increase until $s_i = 0$, and thus they propose the dead-zone technique as a remedy. In their statement the conclusion (i.e., unbounded growth of the adaptive parameters, and the use of the dead-zone technique as*