Community Experience Distilled

Advanced Quantitative Finance with C++

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Alonso Peña, Ph.D.

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He has publications in the fields of Quantitative Finance, applied mathematics, neuroscience, and the history of science. He has been awarded the Robert J. Melosh Medal—first prize for the best student paper on Finite Element Analysis, Duke University, USA; and the Rouse Ball Travelling Studentship in Mathematics, Trinity College, Cambridge. He has been to the Santa Fe Institute, USA, to study complex systems in social sciences.

His publications include the following:

- *The One Factor Libor Market Model Using Monte Carlo Simulation: An Empirical Investigation*
- *On the Role of Behavioral Finance in the Pricing of Financial Derivatives: The Case of the S&P 500*
- *Option Pricing with Radial Basis Functions: A Tutorial*
- *Application of extrapolation processes to the finite element method*
- *On the Role of Mathematical Biology in Contemporary Historiography*

He is currently working as a tutor for CQF (Fitch Learning) and a visiting faculty for the Indian Institute for Quantitative Finance, Mumbai.

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About the Reviewer

Marco Airoldi received his Ph.D. in Theoretical Condensed Matter Physics in 1995 from the International School for Advanced Studies (SISSA). He moved definitively to finance in 1999. Marco has been chosen as the head of financial engineering in one of the top financial institutions in Italy.

His expertise includes the Monte Carlo simulation for option pricing and pricing system architectures.

Table of Contents

Preface

Quantitative Finance is a highly complex interdisciplinary field, which covers mathematics, finance, and information technology. Navigating it successfully requires specialist knowledge from many sources, such as financial derivatives, stochastic calculus, and Monte Carlo simulation. Crucially, it also requires a hands-on ability to transform theory into practice effectively.

In *Advanced Quantitative Finance with C++*, we provide a guided tour through this exciting field. The key mathematical models used to price financial derivatives are explained as well as the main numerical models used to solve them. In particular, equity, currency, interest rates, and credit derivatives are discussed. The book also presents how to implement these models in C++ step by step. Several fully working, complete examples are given that can be immediately tested by the reader to support and complement their learning.

What this book covers

Chapter 1, What is Quantitative Finance?, gives a brief introduction to Quantitative Finance, delimits the subject to option pricing with C++, and describes the structure of the book.

Chapter 2, Mathematical Models, offers a summary of the fundamental models used to price derivatives in modern financial markets.

Chapter 3, Numerical Methods, reviews the three main families of numerical methods used to solve the mathematical models described in the *Chapter 2, Mathematical Models*.

Chapter 4, Equity Derivatives in C++, demonstrates the concrete pricing of equity derivatives using C++ in a basic contract (European Call/Put), and an advanced contract (multi-asset options).

Chapter 5, Foreign Exchange Derivatives with C++, illustrates the pricing of foreign exchange derivatives using C++ in a basic contract (continuous barrier) and an advanced contract (terminal barrier).

Chapter 6, Interest Rate Derivatives with C++, shows the pricing of interest rate derivatives using C++ in a basic contract and an advanced Interest Rate Swap (IRS).

Chapter 7, Credit Derivatives with C++, demonstrates the concrete pricing of credit derivatives using C++ in a basic contract (Merton model) and an advanced contract (**Credit Default Swap** (CDS)).

Appendix A, C++ Numerical Libraries for Option Pricing, gives a short guide to the various numerical libraries that can be used for option pricing.

Appendix B, References, lists all the bibliographic references used throughout the chapters of this book.

What you need for this book

In order to implement the pricing algorithms described in this book, you will need some basic knowledge of C++ and Integrated Development Environment (IDE) of your choice. I have used Code:Blocks, which is a free C, C++, and Fortran IDE, and is highly extensible and fully configurable. You can download it from `http://www.codeblocks.org/`. You will also need a C++ compiler. I have used MinGW, which is a part of the GNU Compiler Collection (GCC), including C, C++, ADA, and Fortran compilers. This compiler can be downloaded from `http://www.mingw.org/`.

Who this book is for

This book is ideal for quantitative analysts, risk managers, actuaries, and other professionals working in the field of Quantitative Finance who want a quick reference or a hands-on introduction to pricing of financial derivatives. Postgraduate, MSc, and MBA students following university courses on derivatives in corporate finance and/or risk management will also benefit from this book. It could be used effectively by advanced undergraduate students who are interested in understanding these fascinating financial instruments. A basic familiarity with programming concepts, C++ programming language, and undergraduate-level calculus is required.

Conventions

In this book, you will find a number of styles of text that distinguish among different kinds of information. Here are some examples of these styles, and an explanation of their meaning.

Code words in text, database table names, folder names, filenames, file extensions, pathnames, dummy URLs, user input, and Twitter handles are shown as follows: "An important feature of this algorithm is the function in code snippet 2 (`random.cpp`)."

A block of code is set as follows:

```
for (int i=0; i < N; i++)
{
  double epsilon = SampleBoxMuller();  // get Gaussian draw
  S[i+1] = S[i]*(1+r*dt+sigma*sqrt(dt)*epsilon);
}
```

New terms and **important words** are shown in bold. Words that you see on the screen, in menus or dialog boxes for example, appear in the text like this: "In this book, all the programs are implemented with the newest standard C++11 using **Code::Blocks** (`http://www.codeblocks.org`) and **MinGW** (`http://www.mingw.org`)".

> Warnings or important notes appear in a box like this.

> Tips and tricks appear like this.

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What is Quantitative Finance?

Quantitative Finance studies the application of quantitative techniques to the solution of problems in finance. It spans diverse areas such as the management of investment funds and insurance companies, the control of financial risks for manufacturing companies and banking industry, and the behavior of the financial markets. Quantitative Finance is eminently interdisciplinary building upon key expertise from the disciplines of finance, mathematics, and informatics.

In this book, we will focus on one aspect of Quantitative Finance—the pricing of financial derivatives using the programming language C++. In the following sections, we will describe the main features of the three key disciplines that constitute Quantitative Finance:

- Finance
- Mathematics
- Informatics

Discipline 1 – finance (financial derivatives)

In general, a financial derivative is a contract between two parties who agree to exchange one or more cash flows in the future. The value of these cash flows depends on some future event, for example, that the value of some stock index or interest rate being above or below some predefined level. The activation or triggering of this future event thus depends on the behavior of a variable quantity known as **the underlying**. Financial derivatives receive their name because they derive their value from the behavior of another financial instrument.

As such, financial derivatives do not have an intrinsic value in themselves (in contrast to bonds or stocks); their price depends entirely on the underlying.

A critical feature of derivative contracts is thus that their future cash flows are probabilistic and not deterministic. The future cash flows in a derivative contract are contingent on some future event. That is why derivatives are also known as **contingent claims**. This feature makes these types of contracts difficult to price.

The following are the most common types of financial derivatives:

- Futures
- Forwards
- Options
- Swaps

Futures and forwards are financial contracts between two parties. One party agrees to buy the underlying from the other party at some predetermined date (the maturity date) for some predetermined price (the delivery price). An example could be a one-month forward contract on one ounce of silver. The underlying is the price of one ounce of silver. No exchange of cash flows occur at inception (today, $t=0$), but it occurs only at maturity ($t=T$). Here t represents the variable time. Forwards are contracts negotiated privately between two parties (in other words, **Over The Counter** (**OTC**)), while futures are negotiated at an exchange.

Options are financial contracts between two parties. One party (called the **holder** of the option) pays a premium to the other party (called the **writer** of the option) in order to have the right, but not the obligation, to buy some particular asset (the underlying) for some particular price (the strike price) at some particular date in the future (the maturity date). This type of contract is called a **European Call** contract.

Example 1

Consider a one-month call contract on the S&P 500 index. The underlying in this case will be the value of the S&P 500 index. There are cash flows both at inception (today, $t=0$) and at maturity ($t=T$). At inception, ($t=0$) the premium is paid, while at maturity ($t=T$), the holder of the option will choose between the following two possible scenarios, depending on the value of the underlying at maturity $S(T)$:

- Scenario A: To exercise his/her right and buy the underlying asset for K
- Scenario B: To do nothing if the value of the underlying at maturity is below the value of the strike, that is, $S(T)<K$

The option holder will choose Scenario A if the value of the underlying at maturity is above the value of the strike, that is, $S(T)>K$. This will guarantee him/her a profit of $S(T)-K$. The option holder will choose Scenario B if the value of the underlying at maturity is below the value of the strike, that is, $S(T)<K$. This will guarantee him/her to limit his/her losses to zero.

Example 2

An **Interest Rate Swap** (**IRS**) is a financial contract between two parties A and B who agree to exchange cash flows at regular intervals during a given period of time (the life of a contract). Typically, the cash flows from A to B are indexed to a fixed rate of interest, while the cash flows from B to A are indexed to a floating interest rate. The set of fixed cash flows is known as the **fixed leg**, while the set of floating cash flows is known as the **floating leg**. The cash flows occur at regular intervals during the life of the contract between inception ($t=0$) and maturity ($t=T$). An example could be a fixed-for-floating IRS, who pays a rate of 5 percent on the agreed notional N every three months and receives EURIBOR3M on the agreed notional N every three months.

Example 3

A futures contract on a stock index also involves a single future cash flow (the delivery price) to be paid at the maturity of the contract. However, the payoff in this case is uncertain because how much profit I will get from this operation will depend on the value of the underlying at maturity.

If the price of the underlying is above the delivery price, then the payoff I get (denoted by function H) is positive (indicating a profit) and corresponds to the difference between the value of the underlying at maturity $S(T)$ and the delivery price K. If the price of the underlying is below the delivery price, then the payoff I get is negative (indicating a loss) and corresponds to the difference between the delivery price K and the value of the underlying at maturity $S(T)$. This characteristic can be summarized in the following payoff formula:

$$H(S(T)) = S(T) - K$$

Equation 1

Here, $H(S(T))$ is the payoff at maturity, which is a function of $S(T)$. Financial derivatives are very important to the modern financial markets. According to the **Bank of International Settlements** (**BIS**) as of December 2012, the amounts outstanding for OTC derivative contracts worldwide were Foreign exchange derivatives with 67,358 billion USD, Interest Rate Derivatives with 489,703 billion USD, Equity-linked derivatives with 6,251 billion USD, Commodity derivatives with 2,587 billion USD, and Credit default swaps with 25,069 billion USD. For more information, see `http://www.bis.org/statistics/dt1920a.pdf`.