Volume III Supersymmetry

THE QUANTUM THEORY OF FIELDS

STEVEN WEINBERG

量子场论

第3卷

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The Quantum Theory of Fields

Volume III
Supersymmetry

Steven Weinberg
University of Texas at Austin

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Preface To Volume III

This volume deals with quantum field theories that are governed by supersymmetry, a symmetry that unites particles of integer and half-integer spin in common symmetry multiplets. These theories offer a possible way of solving the 'hierarchy problem,' the mystery of the enormous ratio of the Planck mass to the 300 GeV energy scale of electroweak symmetry breaking. Supersymmetry also has the quality of uniqueness that we search for in fundamental physical theories. There is an infinite number of Lie groups that can be used to combine particles of the same spin in ordinary symmetry multiplets, but there are only eight kinds of supersymmetry in four spacetime dimensions, of which only one, the simplest, could be directly relevant to observed particles.

These are reasons enough to devote this third volume of *The Quantum Theory of Fields* to supersymmetry. In addition, the quantum field theories based on supersymmetry have remarkable properties that are not found among other field theories: some supersymmetric theories have couplings that are not renormalized in any order of perturbation theory; other theories are finite; and some even allow exact solutions. Indeed, much of the most interesting work in quantum field theory over the past decade has been in the context of supersymmetry.

Unfortunately, after a quarter century there is no direct evidence for supersymmetry, as no pair of particles related by a supersymmetry transformation has yet been discovered. There is just one significant piece of indirect evidence for supersymmetry: the high-energy unification of the $SU(3)$, $SU(2)$, and $U(1)$ gauge couplings works better with the extra particles called for by supersymmetry than without them.

Nevertheless, because of the intrinsic attractiveness of supersymmetry and the possibility it offers of resolving the hierarchy problem, I and many other physicists are reasonably confident that supersymmetry will be found to be relevant to the real world, and perhaps soon. Supersymmetry is a primary target of experiments at high energy planned at existing accelerators, and at the Large Hadron Collider under construction at the CERN laboratory.

After a historical introduction in Chapter 24, Chapters 25–27 present the essential machinery of supersymmetric field theories: the structure of the supersymmetry algebra and supersymmetry multiplets and the construction of supersymmetric Lagrangians in general, and in particular for theories of chiral and gauge superfields. Chapter 28 then uses this machinery to incorporate supersymmetry in the standard model of electroweak and strong interactions, and reviews experimental difficulties and opportunities. Chapters 29–32 deal with topics that are mathematically more advanced: non-perturbative results, supergraphs, supergravity, and supersymmetry in higher dimensions.

I have made the treatment of supersymmetry here as clear and self-contained as I could. Wherever possible I take the reader through calculations, rather than just reporting results from the literature. Where calculations are too lengthy or complicated to be included in a book of this sort, especially in Chapter 28, I have tried to present simpler versions that give the reader an idea of the physical issues involved.

I have made a point of including topics here that have generally not been covered in earlier books, some because they are too new. These include: the use of holomorphy to study perturbative and non-perturbative radiative corrections; the calculation of central charges; gauge-mediated and anomaly-mediated supersymmetry breaking; the Witten index; duality; the Seiberg–Witten calculation of the effective Lagrangian in $N = 2$ supersymmetric gauge theories; supersymmetry breaking by modular fields; and a first look at the rapidly developing topic of supersymmetry in higher dimensions, including theories with p-branes.

On the other hand, I have shortened the usual treatment of two topics that seemed to me to have been well covered in earlier books. One of these is the use of supergraphs. Many of the previous applications of the supergraph formalism in studying the general structure of radiative corrections can now be handled more easily by using the arguments of holomorphy described in Sections 27.6 and 29.3. The other is supergravity. In Sections 31.1–31.5 I have given a detailed and self-contained treatment of supergravity in the weak-field limit, which makes it clear why the ingredients of supergravity theories — the graviton, gravitino, and auxiliary fields — are what they are, and which allows us to derive some of the most important results of supergravity theory, including the formula for the gravitino mass and for the gaugino masses produced by anomaly-mediated supersymmetry breaking. In Section 31.6 I have outlined the calculations that generalize supergravity theory to gravitational fields of arbitrary strength, but these calculations are so lengthy and unlovely that I was content to quote other sources for the results. However, in Section 31.7 I have given a fuller than usual treatment of gravitationally mediated supersymmetry breaking. I regret that I have not been able to include exciting work of the past decade on supersymmetry related to string theory, but string theory is beyond the scope of this book, and I did not want to report results for which I had not provided a basis of explanation.

I have given citations both to the classic papers on supersymmetry and to useful references on topics that are mentioned but not presented in detail in this book. I did not always know who was responsible for material presented here, and the mere absence of a citation should not be taken as a claim that the material presented here is original, but some of it is. I hope that I have improved on the original literature or standard textbook treatments in several places, as for instance in the proof of the Coleman–Mandula theorem; in the treatment of parity matrices in extended supersymmetry theories; in the inclusion of new soft supersymmetry-breaking terms in the minimum supersymmetric standard model; in the derivation of supercurrent sum rules; and in the proof of the uniqueness of the Seiberg–Witten solution.

I have also supplied problems for each chapter. Some of these problems aim simply at providing exercise in the use of techniques described in the chapter; others are intended to suggest extensions of the results of the chapter to a wider class of theories.

In teaching a course on supersymmetry, I have found that this book provides enough material for a one-year course for graduate students. I intended that this book should be accessible to students who are familiar with quantum field theory at the level it is presented in the first two volumes of this treatise. It is not assumed that the reader has gone through Volumes I and II, but for the convenience of those fortunate readers who have done so I use the same notation here, and give cross-references to material in Volumes I and II wherever appropriate.

* * *

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ready this book for publication, and especially to my editor, Rufus Neal, for his continued friendly good advice.

STEVEN WEINBERG

Austin, Texas
May, 1999

Notation

The big issue in choosing notation for a book on supersymmetry is whether to use a two-component or a four-component notation for spinors. The standard texts on supersymmetry have opted for the two-component Weyl notation. I have chosen instead to use the four-component Dirac notation except in the first stages of constructing the supersymmetry algebra and multiplets, because I think this will make the book more accessible to those physicists who work on particle phenomenology and model building. It would be a pity to see the growth of a separate enclave of supersymmetry specialists, who communicate well with each other but are cut off by their notation from the larger community of particle theorists.

There is no great difficulty anyway in converting expressions in four-component form into the two-component formalism. In the representation of the Dirac matrices used throughout this book, in which γ_5 is the diagonal matrix with elements +1, +1, −1, and −1 on the main diagonal, any four-component Majorana spinor ψ_α (such as the supersymmetry generator Q_α, the superspace coordinate θ_α, or the superderivative $\mathscr{D}_\alpha$) may be written in terms of a two-component spinor χ_a as

$$\psi = \begin{pmatrix} e\chi^* \\ \chi \end{pmatrix} ,$$

where e is the 2×2 antisymmetric matrix with $e_{12} = +1$. The two-component spinor χ_a is what in other books is often called $\psi^*_a = \bar{\psi}_{\dot{a}}$, while $(e\chi^*)_a$ would be called ψ^a. A summary of useful properties of four-component Majorana spinors is given in the appendix to Chapter 26.

Here are some other features of the notation used in this book:

Latin indices i, j, k, and so on generally run over the three spatial coordinate labels, usually taken as 1, 2, 3. Where specifically indicated, they run over values 1, 2, 3, 4, with $x^4 = it$.

Greek indices μ, ν, etc. from the middle of the Greek alphabet generally run over the four spacetime coordinate labels 1, 2, 3, 0, with x^0 the time coordinate. Where it is necessary to distinguish between spacetime coordinates in a general coordinate system and in a locally inertial system, indices μ, ν, etc. are used for the former and a, b, etc. for the latter.

Greek indices α, β, etc. from the beginning of the Greek alphabet generally (except in Chapter 24) run over the components of four-component spinors. To avoid confusion, I depart here from the conventions of Volume II, and use upper-case letters A, B, etc. to label the generators of a symmetry algebra. Components of two-component spinors are labelled with indices a, b, etc. In particular, four-component supersymmetry generators are denoted Q_α, while two-component generators (the bottom two components of Q_α) are called Q_a.

Repeated indices are generally summed, unless otherwise indicated.

The spacetime metric $\eta_{\mu\nu}$ is diagonal, with elements $\eta_{11} = \eta_{22} = \eta_{33} = 1$, $\eta_{00} = -1$.

The d'Alembertian is defined as $\Box \equiv \eta^{\mu\nu}\partial^2/\partial x^\mu \partial x^\nu = \nabla^2 - \partial^2/\partial t^2$, where ∇^2 is the Laplacian $\partial^2/\partial x^i \partial x^i$.

The 'Levi–Civita tensor' $\epsilon^{\mu\nu\rho\sigma}$ is defined as the totally antisymmetric quantity with $\epsilon^{0123} = +1$.

Dirac matrices γ_μ are defined so that $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\eta_{\mu\nu}$. Also, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, and $\beta = i\gamma^0 = \gamma_4$. Where explicit matrices are needed, they are given by the block matrices

$$\gamma^0 = -i\begin{bmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{bmatrix}, \qquad \boldsymbol{\gamma} = -i\begin{bmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{bmatrix},$$

where $\mathbf{1}$ is the unit 2×2 matrix, $\mathbf{0}$ is the 2×2 matrix with elements zero, and the components of $\boldsymbol{\sigma}$ are the usual Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

We also frequently make use of the 4×4 block matrices

$$\gamma_5 = \begin{bmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{bmatrix}, \qquad \epsilon = \begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix},$$

where e is again the antisymmetric 2×2 matrix $i\sigma_2$. For instance, our phase convention for four-component Majorana spinors s may be expressed as $s^* = -\beta\,\gamma_5\,\epsilon\, s$.

The step function $\theta(s)$ has the value $+1$ for $s > 0$ and 0 for $s < 0$.

The complex conjugate, transpose, and Hermitian adjoint of a matrix or vector A are denoted A^*, A^{T}, and $A^\dagger = A^{*\mathrm{T}}$, respectively. We use an asterisk * for the Hermitian adjoint of an operator or the complex conjugate of a number, except where a dagger † is used for the transpose of the matrix formed from the Hermitian adjoints of operators or complex conjugates of numbers. +H.c. or +c.c. at the end of an expression indicates the addition of the Hermitian adjoint or complex conjugate of the foregoing terms. A bar on a four-component spinor u is defined by $\bar{u} = u^\dagger \beta$.

Units are used with $\hbar$ and the speed of light taken to be unity. Throughout $-e$ is the rationalized charge of the electron, so that the fine structure constant is $\alpha = e^2/4\pi \simeq 1/137$. Temperatures are in energy units, with the Boltzmann constant taken equal to unity.

Numbers in parenthesis at the end of quoted numerical data give the uncertainty in the last digits of the quoted figure. Where not otherwise indicated, experimental data are taken from 'Review of Particle Physics,' The Particle Data Group, *European Physics Journal C* **3**, 1 (1998).

Contents

Sections marked with an asterisk are somewhat out of the book's main line of development and may be omitted in a first reading.