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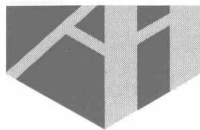
**PAUL D. GROVES**

**PRINCIPLES OF**

**GNSS, INERTIAL, AND MULTISENSOR  
INTEGRATED NAVIGATION  
SYSTEMS**

# Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems

Paul D. Groves



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# Preface

This book has four main aims:

- To provide an introduction to navigation systems suitable for those with no prior knowledge;
- To describe the principles of operation of satellite, inertial, and many other navigation technologies, both qualitatively and mathematically;
- To review the state of the art in navigation technology;
- To provide a detailed treatment of integrated navigation.

It is aimed at professional scientists and engineers in industry, academia, and government, and at students at final year undergraduate, master's, and Ph.D. levels.

The book begins with a basic introduction to the main principles of navigation and a summary of the different technologies. The different coordinate frames, attitude representations, multiframe kinematics, Earth models, and gravity are then carefully explained, while the basic principles of each topic in the body of the book are explained before going into the details.

To cover the state of the art in navigation technology, the book goes beyond global navigation satellite systems (GNSS) and inertial navigation to describe terrestrial radio navigation, dead reckoning, and feature matching techniques. Topics covered include Loran, wireless local area network (WLAN) and ultrawideband (UWB) positioning, magnetometers, attitude and heading reference systems (AHRS), altimeters, odometers, pedestrian dead reckoning, Doppler radar and sonar, terrain-referenced navigation, image matching, and map matching.

The GNSS chapters describe the legacy and new Global Positioning System (GPS), Global Navigation Satellite System (GLONASS), and Galileo signals together and cover a range of advanced topics, including differential and carrier-phase positioning, GNSS attitude, multipath mitigation, and operation in poor signal-to-noise environments. Inertial navigation coverage includes accelerometer and gyroscope technology, navigation equations, initialization, alignment, and zero velocity updates.

Integrated navigation is served by a navigation-focused chapter on Kalman filtering, together with chapters on inertial navigation system (INS)/GNSS integration, including deep, and on multisensor integration. To support these, the chapters on the navigation sensors include comprehensive descriptions of the processing chains and error sources. The book concludes with a chapter on integrity monitoring, showing how a navigation system can detect and recover from faults.

The emphasis throughout is on providing an understanding of how navigation systems work, rather than on engineering details. The book focuses on the physical principles on which navigation systems are based, how they generate a navigation solution, how they may be combined, the origins of the error sources, and their mitigation. Later chapters build on material covered in earlier chapters, with comprehensive cross-referencing.

The book is divided into four parts. Part I comprises a nonmathematical introduction, while Part II provides the mathematical grounding to describe navigation systems and their integration. Part III describes the navigation systems, starting with inertial navigation, moving on to satellite navigation, and finishing with the other technologies. Part IV describes integrated navigation, including fault detection and integrity monitoring. Appendixes on vectors, matrices, and statistics, as well as full lists of symbols and acronyms complete the book.

Like many fields, navigation does not always adopt consistent notation and terminology. Here, a consistent notation has been adopted throughout the book, with common alternatives indicated where appropriate. The most commonly used conventions have generally been adopted, with some departures to avoid clashes and aid clarity.

Scalars are italicized and may be either upper or lower case. Vectors are lower-case bold, and matrices are uppercase bold, with the corresponding scalar used to indicate their individual components. The vector (or cross) product is denoted by  $\wedge$ , while Dirac notation (i.e.,  $\dot{x}$ ,  $\ddot{x}$ , and so on) is generally used to indicate time derivatives. All equations presented assume base SI units, the meter, second, and radian. Other units used in the text include the degree (1 degree =  $\pi/180$  rad), the hour (1 hour = 3,600 seconds), and the g unit describing acceleration due to gravity ( $1g \approx 9.8 \text{ ms}^{-2}$ ).

Unless stated otherwise, all uncertainties and error bounds quoted are ensemble  $1\sigma$  standard deviations, which correspond to a 68 percent confidence level where a Gaussian (normal) distribution applies. This convention is adopted because integration and other estimation algorithms model the  $1\sigma$  error bounds.

References are denoted by square brackets, numbered sequentially, and listed at the end of each chapter. Many chapters also include a selected bibliography, listing further publications of interest.

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A list of updates and corrections and printable symbol and acronym lists may be found online. See the links page at <http://www.artechhouse.com>.

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# PART I

## Introduction



