RAREFIED GAS DYNAMICS

Volume II

RAREFIED GAS DYNAMICS

PROCEEDINGS OF THE THIRD INTERNATIONAL SYMPOSIUM ON RAREFIED GAS DYNAMICS, HELD AT THE PALAIS DE L'UNESCO, PARIS, IN 1962

Edited by

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LOCKHEED MISSILES AND SPACE COMPANY, PALO ALTO, CALIFORNIA

Volume II

NEW YORK LONDON

ACADEMIC PRESS

ACADEMIC PRESS INC.
111 Fifth Avenue, New York 3, New York

United Kingdom Edition published by
ACADEMIC PRESS INC. (LONDON) LTD.
Berkeley Square House, London W.1

LIBRARY OF CONGRESS CATALOG CARD NUMBER: 60-16990

PRINTED IN THE UNITED STATES OF AMERICA

Third International Symposium on

Rarefied Gas Dynamics

Presented at the

PALAIS DE L'UNESCO, PARIS, JUNE 1962

Under the sponsorship of

Air Force Office of Scientific Research
Office of Naval Research
National Aeronautics and Space Administration
International Union of Theoretical and Applied Mechanics
Délégation Générale à la Recherche Scientifique et Technique
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Preface

The Third International Rarefied Gas Dynamics Symposium was convened in the month of June 1962, at the Palais de l'UNESCO, Paris, under the aegis of the Air Force Office of Scientific Research, the Office of Naval Research, the International Union of Theoretical and Applied Mechanics, the National Aeronautics and Space Administration, the Délégation Générale à la Recherche Scientifique et Technique (France), and the Université de Paris. Most of the papers delivered at the conference are reproduced here as a second supplement to the Advances in Applied Mechanics Series, the first supplement containing the Proceedings of the Second Symposium held at Berkeley in 1960.

Since its inception and its first meeting at Nice in 1958, the Rarefied Gas Dynamics Symposia have been held at regular biennial intervals. An originally advanced and highly specialized field has grown progressively in popularity and in practical importance since then, as evinced by steadily increased participation in the symposia, the number of papers published in the Proceedings being 31 in the first volume, 41 in the second, and 55 in the current one. It is anticipated, therefore, that the content of the present work will be ample enough to provide useful information to both the expert in the field as a compendium of some of the latest ideas in his subject and to the applied scientist who may be concerned with the practical significance of the results of research in rarefied gas dynamics as applied to flight in the upper atmosphere or to experimentation with wind tunnels and molecular beams.

The subject of rarefied gas dynamics is conveniently defined as the study of the flow of gases in which the mean free path within the gas is not negligible in comparison with a length typical of the structure of the flow being considered, e.g., the channel width in Poiseuille flow, or the boundary layer thickness of an external flow. The field is thus seen to be one that intrinsically involves the statistical kinetic theory of gases, the most commonly used description being the integro-differential Boltzmann equation. Limiting cases of this equation yield the continuum (Navier-Stokes) flow equations at one extreme of mean free paths and free-molecule (collisionless) flow at the other. The majority of problems in rarefied gas dynamics involve the central region between these limits, commonly called the transition regime.

Recently, problems involving ionized gas flows have become of increasing interest and now form a recognized subcategory of rarefied gas dynamics. As is well known in studies on plasma dynamics the definition of mean free paths and corresponding regimes of a plasma is a difficult task and indeed, ionized rarefied gas flow forms an immense, essentially uncharted, field of investigation. These symposium Proceedings will be found to contain the description of a few attempts to delineate some problems in this subject area. Future symposia, it is anticipated, will include an increasing number of such contributions.

When dealing with specific problems of rarefied gas flows, in contrast to continuum flows, the characteristics of the gas-surface interaction at a boundary influence the nature of the flow. It is natural, therefore, that the general study of gas-surface interactions should have become an important item in the Rarefied Gas Dynamics Symposia. In fact, an appreciable part of these Proceedings is concerned with the subject, most particularly with the use of molecular beam techniques to investigate a variety of interaction phenomena.

The arrangement of the contents of the Proceedings follows approximately the subdivisions of the meeting itself, falling into the following categories: fundamental kinetic theory, molecular beams and surface interactions, ionized flows, transition flow, theory and experiment, and, finally, experimental methods.

Included in the first section are a number of papers discussing the validity of the Boltzmann equation as applied in rarefied gas dynamics (including the ionized case) and various techniques for solving it. Other papers in this section consider more specific problems; in particular, three deal with the perennially favorite subject of shock wave structure. Many of these particular topics were motivated, not as much by a need for solving the problem itself, but by a desire to study the validity or accuracy of particular approximate methods of handling the Boltzmann equation. Such questions remain as some of the most difficult and intriguing ones in kinetic theory.

Molecular beams and surface interaction effects have always constituted a large part of the field of interest of rarefied gas dynamics. That this is still the case is clear from the number of papers on the subject in Section II. Besides a substantial number dealing with standard molecular beam techniques, newer methods are also discussed, including the increasingly popular nozzle-source method for producing beams, first mentioned at the Nice Rarefied Gas Dynamics Symposium.

Starting in the second volume, Section III is a brief one concerned mostly with the application of free molecule flow theory to ionized gas flows. An exception is the invited paper by Dr. Cox that surveys the ionized gas flow problems that are to be expected in upper atmosphere flight.

Various attempts, theoretical and experimental, to bridge the gap in knowledge between continuum and free-molecule flows are covered in Sections IV and V. Most of the theoretical papers (contained in Section IV) deal with the difficult and slow advance into this transition regime from the continuum limit. A few papers do attempt to treat the complete transition region, usually, however, only for specific simple cases. It is of interest to note that little further success has been achieved in the last two years in tackling the problem from the free molecule end, so that the past high hopes for a meeting of the two techniques, approaching each other from the continuum and free-molecule limits, have not yet been realized. The experimental work, described in Section V, presents a brighter picture. It appears that here we are at last arriving at a point where detailed experimental results are obtainable for the complete range of Knudsen numbers in a variety of flow configurations. Thus there is now appreciable overlap in available experimental data, and attempts, similar to those developed by Professor Sherman in his survey article in this section, to correlate and compare the various results should bear fruit, assisting in the pursuit of an adequate theoretical description.

The last section contains a few papers in a number of topics involving experimental methods, related to testing with both ionized and un-ionized gases in low density wind tunnels.

In addition to the written manuscripts of the talks given at the symposium, which constitute the bulk of these volumes, there is also included a selected number of questions and answers that followed in the discussion period after each presentation. This innovation in the Proceedings publications can be described as a partial success only, since a complete record of the discussions was not obtained, and thus the printed commentaries are not in all cases representative of the original verbal exchange. It is thought, however, that the discussions reported are useful and will be of interest to many readers.

A great number of dedicated individuals were instrumental in realizing the success of the symposium and in compiling these Proceedings. Principal among these has been Professor E. A. Brun, who not only acted as chairman for the meeting, but handled most of the editorial work involved with the papers contributed from Europe. To him and to his co-chairman, Dr. I. Estermann, is owed the largest debt of gratitude of those participating in the symposium. A large editorial committee of eminent scientists, both in the United States and France, reviewed the papers submitted to the meeting, making easier the difficult task of the editor in selecting, out of nearly 100 submissions, those included in the symposium. Their names, as well as those of all the session chairmen and secretaries, are too numerous to list here. However, special mention should be made not only of Professor Brun, but also of the University of California group at Berkeley for special assistance in the editorial work and of Professor P. Germain of the University of Paris, who initiated the editorial committee work in Europe. The symposium as always, relied heavily on an adequate secretarial staff, and a personal vote of thanks is due to Mrs. Barbara Moore for bearing the brunt of the American portion of this load. In the implementation of the symposium program itself, Professor Brun was assisted effectively by l'Ingénieur en chef de l'Air Deschamps and by efficient secretarial aid, notably that of Mrs. Emoré and Mrs. Williamson. The sponsoring agencies formed the indispensable base for the functioning of the meeting; without them these symposia would not exist. The editor would also like to comment on the generosity of the Lockheed Research Laboratory of Palo Alto and its director Dr. W. C. Griffith for making available both its funds and facilities in the extensive editorial work that preceded the symposium and the production of these volumes of the Proceedings.

Of the reader, for whose education and enlightenment this publication is aimed, we would ask his tolerance for any defect in concept or oversight in production. The editor, with the diligent and patient cooperation of the personnel of the Academic Press, has striven for the earliest possible publication date, realizing that the rapid dissemination of new research results is essential in the scientific tempo of today. The consequent lack of perfection can perhaps be accepted as the price to be paid for a more timely appearance of these symposium Proceedings.

Palo Alto, California
February 1963

J. A. LAURMANN

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Section 3

IONIZED GAS FLOWS

Some Aspects of Ionospheric Aerodynamics

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This paper considers the aerodynamic flow regimes which can exist for the motion of rockets or satellites in the ionosphere. The flow variables which are likely to be of importance in the flow of a partially ionized plasma past a body are introduced, and criteria for rarefied or continuum flows are suggested. The paper then deals with plasma wave motions. It is shown that ion electroacoustic waves can be strongly excited by a typical body in the ionosphere, and some results are given for linearized supersonic flows over thin bodies. A discussion of ion shock wave formation follows, together with a description of experimental work at RARDE on plasma flows.

I. Introduction

It is the primary purpose of this paper to discuss, with reference to conditions prevailing for moving bodies such as satellites or rockets in the ionosphere, the aerodynamic flow regimes which can exist in a partially ionized gas, and, in particular, the differences which can be expected between continuum and rarefied gas flows. The most obvious difference between the flow of a neutral gas and that of a charged gas is the appearance of collective phenomena (plasma oscillations and wave motions) which arise because of the long-range electrostatic (or Coulomb) forces between charged particles. Thus, even in a rarefied plasma flow, acoustic wave propagation may take place, and, if the motion is supersonic, the possibility of shocks forming about bodies must be considered. The resulting redistribution of the electron and ion densities in the flow field can affect the drag and hence the orbit of a satellite, and may have a large effect on the transmission and reflection of electromagnetic waves from the neighborhood of a vehicle.

Some aspects of the interaction of a satellite with the ionosphere, such as ionization by emission of secondary particles from a vehicle surface, and the calculation of the particle orbits near a charged vehicle assuming free molecular flow, are dealt with in other papers in this Symposium. However these papers specifically omit the plasma wave phenomena with which this paper is concerned. As is well known for supersonic flow of a neutral gas, the radiation of accoustic waves by a body normally leads to a radical change in the nature of the flow about a body; this would tend to be obscured by a particle treatment of the problem which did not include the wave motions.

II. Characteristic Flow Variables

At low pressures the behavior of a neutral gas flow can be characterized by two main nondimensional parameters, the Knudsen number, obtained by dividing the mean free path λ by a typical body dimension D, and the Mach number M, which is the ratio of the free stream velocity to the appropriate acoustic speed a.

It is normal in treating the fluid dynamics of a neutral gas to consider the flow of the gas past a body of characteristic dimension D to be a continuum flow if $\lambda \ll D$, and to be a rarefied gas flow if $\lambda \gg D$. In rarefied gas flow the region near the body is governed only by the Knudsen number. However, as pointed out by Grad (1959) beyond a distance of order (λ/D) from the body the flow will depend asymptotically on the Mach number, and in supersonic flow shock waves will begin to form, although they will be heavily damped and their strength will small $[O(D/\lambda)^2]$.

We consider first the properties of the electrostatic field for a fully ionized gas. A primary characteristic of a plasma is the preservation, in the large, of electrical neutrality. Any change in the relative number densities of ions and electrons will produce large electrostatic forces which tend to restore neutrality; it is, of course, these forces which are responsible for plasma wave phenomena. Thus, for a macroscopically large volume of a plasma, we have $(n_e - n_i)/n \ll 1$. (Here n_e and n_i are the number densities of electrons and ions, and n the mean number density.) It is evident, however, that there must be a region close to each charged particle or to a charged body where the potential must differ from the mean plasma potential. For a single ion the distribution of potential follows the Coulomb relationship ϵ^2/r, (where ϵ is the unit of electronic charge and r the distance).

However the presence of other charged particles alters this simple picture. If we consider a coordinate system coinciding with a given ion, we see that other ions will tend to be repelled, and electrons attracted; a particle of given sign is said to be screened by particles of the opposite sign. The potential obtained by solving the Poisson equation

$$\nabla^2\phi = 4\pi\epsilon(n_e - n_i) \tag{1}$$

is then given by

$$\phi = \text{const} \cdot \frac{1}{r} \cdot \exp - \frac{r}{r_D} \tag{2}$$

Here the Debye radius

$$r_D{}^2 = \frac{k\,T_e}{4\pi n_e\,\epsilon^2} \tag{3}$$

(where k is the Boltzmann constant) is the distance at which the potential has become $1/e$ times the Coulomb value. Screening is essentially a collective effect, depending as it does on the presence of a large number of particles, and the Debye radius is a measure of the distance over which the potential resulting from local charge separation can vary significantly from the mean value in a plasma. For the ionosphere the Debye radius is about 0.25 cm. It is thus small compared with vehicle dimensions, and the collective properties of the surrounding medium will therefore be of importance. (It is only for a region small compared with a Debye radius that collective effects can be neglected).

The definition of an effective mean free path for binary collisions in a plasma presents considerable conceptual difficulties. If two particles approach each other to within a distance small compared with a Debye radius, the screening effect of other particles is no longer important, and there will be a simple Coulomb potential between the particles. Their distance of closest approach, P_0, may be found by equating their average thermal energy $3kT$ to the electrostatic potential energy E^2/P_0. This gives

$$P_0 = \frac{\epsilon^2}{3kT} \tag{4}$$

and is the distance of approach for two particles which are deflected through 90°. To obtain the corresponding mean free path, λ_{90}, we assume the collision cross section, σ, to be $\pi P_0{}^2$, whence

$$\lambda_{90} \doteq \frac{1}{n\sigma} = \frac{1}{\pi n}\left(\frac{3kT}{\epsilon^2}\right)^2 \tag{5}$$

(for the ionosphere $P_0 \simeq 10^{-6}$ cm and $\lambda_{90} \simeq 3$ km at 350 km altitude).

Such close binary collisions are however relatively rare events in a plasma (λ_{90} is normally large compared with the interparticle spacing, d, which is $\simeq 10^{-2}$ cm in the ionosphere), and account must be taken of the contributions of the more frequent collisions occurring at distances larger than P_0, but less than the Debye radius, r_D. This effect has been calculated by Chandrasekhar (1951) and Spitzer (1956) and, assuming that the integrations to obtain the relaxation frequencies are carried out between P_0 and r_D, an expression of the form (for ion-ion collisions)

$$\lambda_{ii} = \frac{(kT_i)^2}{n\epsilon^4 \ln (r_D/P_0)} \tag{6}$$

is obtained, with similar expressions for ion-electron and electron-electron encounters.[1] For the ionosphere λ_{ii} is about 300 meters (at 350 km altitude).

A similar calculation using the Fokker-Planck equation has also been made by Thompson and Hubbard (1960). This avoids the necessity of making an assumption about the outer cutoff limit (r_D), but to include the effect of the close binary collisions, an inner cutoff at P_0 has to be used, and the resulting value for λ_{ii} is similar to that obtained by Chandrasekhar.

Lighthill (1960) argues that although the value given by (6) is the mean free path for all the directed momentum of a particle to be lost, the logarithmic term in Eq. (6) means that an appreciable part of the momentum transfer takes place for collisions occurring over distances smaller than the Debye radius, and that the effective mean free path may be considerably less than that given by Eq. (6).

Because of the general uncertainty surrounding this question, we will simply define an effective plasma mean free path, λ_p. The determination of this is obviously one of the important goals for theory and experiment, since the transport properties of the plasma will depend primarily on the value of λ_p for the appropriate species.

In a magnetic field a number of other characteristic lengths assume importance. The electrons and ions tend, for instance, to spiral about the lines of force at the Larmor radius, which, for the ionosphere is about 1 cm for electrons and 2 meters for O^+ ions. Various other lengths have been introduced by Grad (1959) to describe the perturbation of the magnetic field by the body, and the effect of induced currents. However for the ionosphere above about 130 km, where the magnetic

[1] Chopra (1961) suggests that $\lambda_{ii} \ll \lambda_{ee}$ in the ionosphere. This is manifestly untrue since Eqs. (5) and (6) hold whether the particles are both ions or both electrons.