



Fudan Series in Graduate Textbooks



Advanced Statistical Physics

高等统计物理

戴显熹 编著

Xianxi Dai

复旦大学出版社

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0414. 2/Y39

2007.



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图书在版编目(CIP)数据

高等统计物理 = Advanced Statistical Physics/戴显熹编著. —上海:
复旦大学出版社, 2007. 11
(博学·研究生系列)
ISBN 978-7-309-05488-0

I. 高… II. 戴… III. 统计物理学-研究生-教材 IV. 0414. 2

中国版本图书馆 CIP 数据核字(2007)第 055399 号

Advanced Statistical Physics

高等统计物理

戴显熹 编著

出版发行 复旦大学出版社 上海市国权路 579 号 邮编 200433
86-21-65642857(门市零售)
86-21-65100562(团体订购) 86-21-65109143(外埠邮购)
fupnet@fudanpress.com <http://www.fudanpress.com>

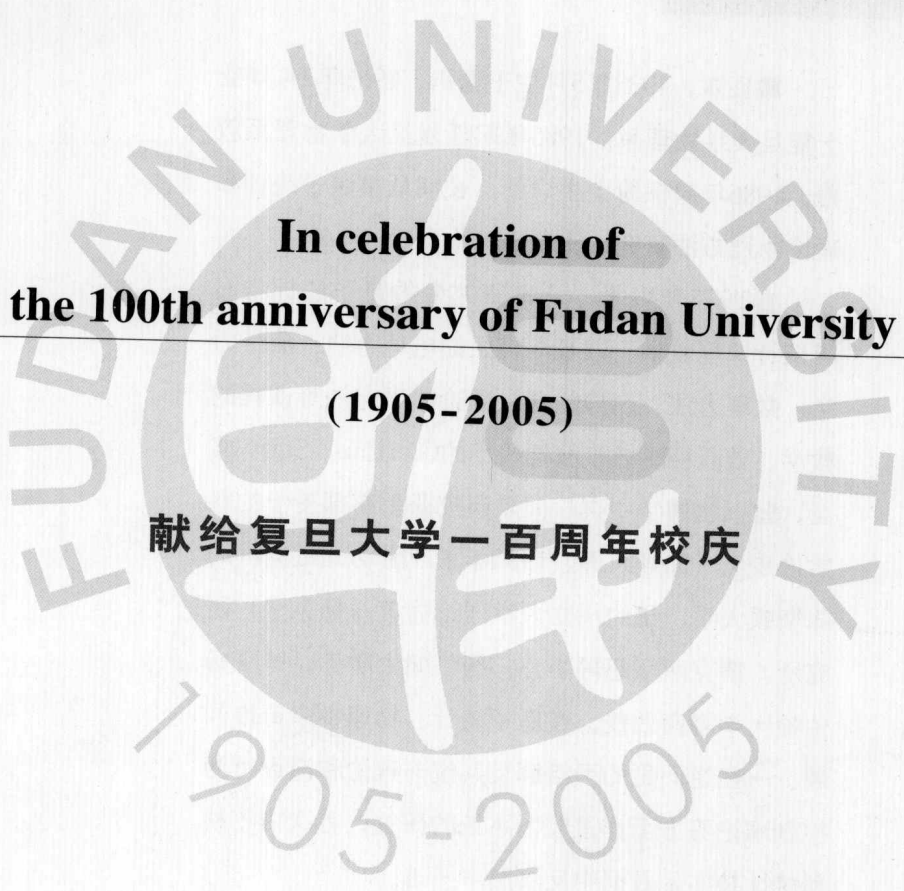
责任编辑 梁 玲
总 编 辑 高若海
出 品 人 贺圣遂

印 刷 上海华业装潢印刷厂有限公司
开 本 787 × 960 1/16
印 张 21.75
字 数 380 千
版 次 2007 年 11 月第一版第一次印刷
印 数 1—2 500

书 号 ISBN 978-7-309-05488-0/O · 395
定 价 38.00 元

如有印装质量问题, 请向复旦大学出版社发行部调换。

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**In celebration of
the 100th anniversary of Fudan University**

(1905-2005)

献给复旦大学一百周年校庆

作者简介

戴显熹，1938年5月生于温州。1961年7月毕业于复旦大学物理系。1985年起任复旦大学物理系教授，1986年起任博士生导师。长期从事量子统计和理论物理方法研究，发表学术论文100多篇。

自1978年以来，从事研究生的量子统计与高等统计课程教学，以及本科生的电动力学、量子力学、数理方法、超导物理、理论物理方法等课程的教学。曾获得杨振宁教授授予的Glorious Sun 奖金，曾以物理学中奇性问题研究获教育部授予的科技进步奖（二等）等。1980年来应邀访问过美国的休斯顿大学、纽约州立大学理论物理（杨振宁）研究所、德克萨斯超导中心、杨伯翰大学等，曾任杨伯翰大学客座教授。在量子统计、物理学中奇性问题、一些逆问题的严格解及其统一理论和渐近行为控制理论等方面作过较为系统的研究，首次由一材料的比热实际数据中反演出声子谱。

Abstract

Statistical physics establishes a bridge from the macroscopic world to study the microscopic world. This is a theory with the fewest assumptions and the broadest conclusions. Up to now there is no evidence to show that statistical physics itself is responsible for any mistakes.

Statistical physics has become an important branch of modern theoretical physics and this course has become one of the common fundamental courses of graduate students in different majors in physics departments.

Statistical physics is a branch of science engaged in studying the laws of thermal motion of macroscopic systems. Usually statistical physics as taught in fundamental courses for undergraduate students focuses mainly on classical statistics, while advanced statistics for graduate students mainly studies quantum statistics.

Chapter 1 of this book outlines the fundamental principles of statistical physics. Chapter 2, with simple applications of these principles, solves some typical problems in statistical physics, i.e. quantum perfect gases. Chapters 3 and 4 are devoted to the study of second quantization for many-particle systems and fields. Chapter 5 addresses Bose-Einstein condensation. Chapter 6 is devoted to the study of a class of inverse problems in quantum statistics, their Chen (or Möbius-Chen) exact solution formulas, Dai's exact solution formulas, asymptotic behavior control (ABC) theory, and concrete realizations of the inversion theories, especially obtaining the phonon spectrum from real specific heat data for high T_c superconductors. Chapter 7 is an introduction to the theory of Green's functions in quantum statistics. Chapter 8 presents the unified diagonalization theorem for Hamiltonians of quadratic form, for both Fermi and Bose systems. Chapter 9 is an introduction to the third formulation of quantum statistics and the functional integral approach. The first four chapters are fundamental, and should be well known. The last five chapters are recent developments.

This course was edited by revising the lecture notes of the author, from courses of quantum statistics and advanced statistics for graduate students in the Department of Physics, Fudan University, some institutes and universities, since 1978. At the same time, this work contains the research results of some related projects, supported by the National Natural Science Foundation of China.

PREFACE

Statistical physics establishes a bridge from the macroscopic world to study the microscopic world. All theories in which the Boltzmann constant appears involve statistical physics. This is a theory with the fewest assumptions and the broadest conclusions. Up to now there is no evidence to show that statistical physics itself is responsible for any mistakes, a reflection of the natural beauty of this science.

Statistical physics has become an important branch of modern theoretical physics. At the same time it has influenced many fields, so this course has become one of the common fundamental courses of graduate students in different majors in physics departments.

Statistical physics is a branch of science engaged in studying the laws of thermal motion of macroscopic systems. It has its own special laws, which cannot be derived from mechanical laws. However mechanical law is one of its foundations. Statistical physics when based on classical mechanics is called classical statistics, while statistical physics based on quantum mechanics is called quantum statistics. Usually statistical physics as taught in fundamental courses for undergraduate students focuses mainly on classical statistics, while advanced statistics for graduate students mainly studies quantum statistics.

Chapter 1 of this book outlines the fundamental principles of statistical physics. Chapter 2, with simple applications of these principles, solves some typical problems in statistical physics, i. e. quantum perfect gases. Chapters 3 and 4 are devoted to the study of second quantization for many-particle systems and fields. Chapter 5 addresses Bose-Einstein condensation. Chapter 6 is devoted to the study of a class of inverse

problems in quantum statistics, their Chen (or Möbius-Chen) exact solution formulas, Dai's exact solution formulas, asymptotic behavior control (ABC) theory, and concrete realizations of the inversion theories, especially obtaining the phonon spectrum from real specific heat data for high T_c superconductors. Chapter 7 is an introduction to the theory of Green's functions in quantum statistics (where double-time Green's functions are the main tool). Chapter 8 presents the unified diagonalization theorem for Hamiltonians of quadratic form, for both Fermi and Bose systems. Chapter 9 is an introduction to the third formulation of quantum statistics and the functional integral approach. Applying this formalism along with the diagonalization theorem established in chapter 8, an asymptotically exact solution is obtained in the thermodynamic limit for a model of superconductivity. The first four chapters are fundamental, and should be well known. The last five chapters are recent developments.

This course was edited by revising the lecture notes of the author, from courses of quantum statistics and advanced statistics for graduate students in the Department of Physics, Fudan University, since 1978. At the same time, this work contains the research results of some related projects, supported by the National Natural Science Foundation of China (NSFC: Nos. 19975009; 10174016; 19834010.) The author thanks the NSFC for its valued support over many years. I would like to thank my graduate students and students, especially including Dr. T. Wen, D.M. Ming, G. X. Hu, L. Sun, J. P. Ye, Mr. F. M. Ji, Y. He, Miss X. Xiang and etc, who studied these courses in a variety of majors in our and related departments, for their important support and discussions.

I also would like to sincerely thank Prof. C. N. Yang, my teacher, and Prof. Zhou Shixun, for their guidance and encouragement, Prof.

W. E. Evenson for his significant discussions and improving the English. I would like to express my special thanks to Prof. Wang Xun, for his important support and encouragement for this book.

I would like to thank all those who have helped me in writing and editing the book. Because of the limitations of the author, mistakes and errors are unavoidable, so all suggestions and comments which will help to improve the book are sincerely welcome.

Xianxi Dai

February, 2007

Fudan University

前 言

统计物理架起人们由宏观世界研究微观世界的桥梁。凡出现 Boltzmann 常数的理论就涉及统计物理。它是假设最少,结论众多的一门学科。至今还没有任何证据,确认某些错误必须由统计物理本身负责。这反映出该学科自然的美。

它已成为现代理论物理的一个重要部分。同时它又渗透到物理学的众多领域,因此自然成为物理系的多个专业的研究生基础课程。

统计物理是研究体系的热运动规律的学科。它有自己的特殊规律,不能从力学推导出来。但必须以力学为其基础之一。基于经典力学的,称经典统计,基于量子力学的,称量子统计。通常大学生基础课程中的统计物理以研究经典统计为主,而研究生的高等统计,则以量子统计为主。

本书的第一章是统计物理基本原理。第二章作为这些基本原理的简单应用,解决几个典型的统计物理问题:量子理想气体。第三章和第四章分别致力于多粒子体系和场的二次量子化。第五章研究玻色-爱因斯坦凝结。第六章研究一批量子统计中的反问题,它们的陈氏(或 Möbius-Chen)严格解公式,戴氏严格解公式及其渐近行为控制理论和反演理论的具体实现,特别是由高温超导体的实际比热数据,直接反演出声子谱。第七章是 Fermi 和 Bose 的二次型哈密顿量的统一的对角化定理。第八章是量子统计中的 Green 函数(以双时 Green 函数理论为主线)引论。第九章则介绍

量子统计第三种表述和泛函积分理论。并运用它和第七章的对角化定理,获得一个超导模型的热力学极限下的渐近严格解。本书的前四章是基础部分,是必须掌握的。后五章是最新发展。

本书是作者依据自 1978 年以来为复旦大学物理系研究生讲课所使用的量子统计课程和高等统计课程的教材修改而成的,同时也是国家自然科学基金的多个有关项目的研究结果。作者对国家自然科学基金委多年来的宝贵支持,对我所指导的多位研究生,以及多年来选读本课程的本系和外系的研究生和部分本科生,特别包括温涛、明灯明、胡光喜、孙磊、叶季平博士及季丰民、何源和相湘同学的讨论与支持,表示衷心的感谢。

作者对杨振宁教授,对我的老师周世勋教授多年来的关心、指导和鼓励表示衷心的感谢。对 Evenson 教授多年来的讨论和对本书的英语方面的修饰表示衷心感谢。作者还特别感谢王迅教授对本书的支持和鼓励。

作者还感谢所有支持和帮助过本书编写和出版的人们。限于作者的水平和时间,错误和不当之处在所难免,如蒙赐教,不胜感激。

戴显熹

2007 年 2 月于复旦大学

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