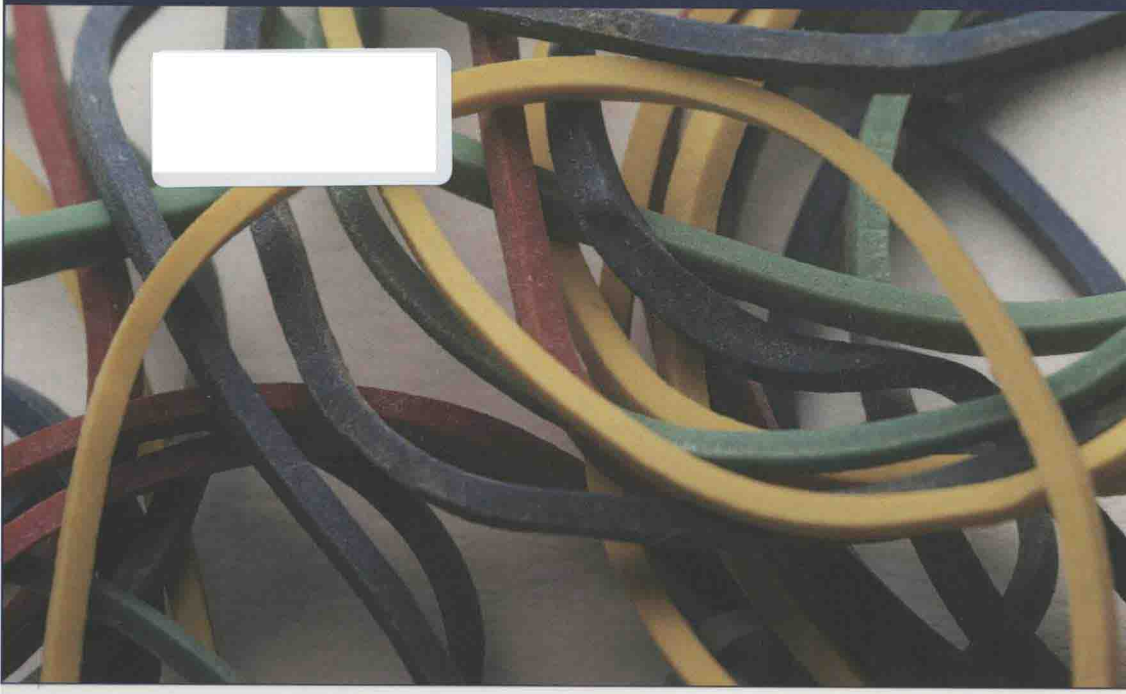

Mohsin Hamzah

# Mechanical Behaviour of Filled Elastomer

## Theoretical, Numerical & Experimental Investigations

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**Impressum / Imprint**
Bibliografische Information der Deutschen Nationalbibliothek: Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.d-nb.de abrufbar.

Bibliographic information published by the Deutsche Nationalbibliothek: The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.

Coverbild / Cover image: www.ingimage.com

Verlag / Publisher:
LAP LAMBERT Academic Publishing
ist ein Imprint der / is a trademark of
AV Akademikerverlag GmbH & Co. KG
Heinrich-Böcking-Str. 6-8, 66121 Saarbrücken, Deutschland / Germany
Email: info@lap-publishing.com

Herstellung: siehe letzte Seite /
Printed at: see last page
**ISBN: 978-3-659-32723-0**

Mohsin Hamzah

# Mechanical Behaviour of Filled Elastomer

## Dedication

*To*

*Nawar, Mustafa, and Sarah*

# Acknowledgments

First and foremost, I would like to express my gratitude and appreciation to the *Institute of International Education* and *Jefferson Scholars Foundation* for their support and assistance.

I would like to thank my family for their constant help, support, and encouragement. I am greatly indebted to my wife and my beloved children.

Most of the content of this book is based on the author experience during preparing his PhD thesis work, which entitled "***Large Strain and Time-Dependent Analysis of Elastomers under Monotonic and Cyclic Loading***", submitted to the Mechanical Engineering Department - University of Baghdad.

# Preface

The mechanical behaviour of elastomeric materials is known to be incompressible or nearly incompressible, hyperelastic with non-linear load-extension behaviour, rate-dependent and exhibit hysteresis upon cyclic loading. Prediction of such a complex behaviour of elastomers subjected to general strain histories is a task that requires extensive experimental and theoretical works.

Experimental investigations of the present work were performed on four types of rubbers that subjected to monotonic and cyclic loading. The carbon black was used as a filler material. The type of tests is standard compression test according to ASTM D575 for rubber testing. For each rubber type used different carbon black concentrations were used. It was found that the stiffness, strength, hardness, the amount of hysteresis and the time-dependency response of the material increase as the carbon black ratios increases. Also, after initial straining, the residual strains of the material increases as the carbon black ratios increase. In addition, the stress-strain response of the material while loading is different than the stress strain response of the material while decreasing strain, unloading.

Theoretical works are considered for both modeling and numerical investigations. In modeling, the mechanical behaviour is assumed to be decomposed into two networks: an equilibrium network corresponding to the state that is approached in long time stress relaxation tests; and a second

network capturing the non-linear rate-dependent deviation from the equilibrium state. The equilibrium response was obtained using multi-relaxation tests at different strain level, while the time-dependent parameters identification was obtained using Prony series.

New constitutive hyperelastic model has been proposed based on intensive study of the available hyperelastic constitutive models and the observations from experimental investigation of the present work. The validity, accuracy and robustness of the proposed model has been studied and showed that this model has the ability to capture the mechanical behaviour of elastomers for both small and large strain values as compared with the experimental results of the present work.

Numerical analyses were performed using the boundary elements method. The boundary elements are applied to elastomers by considering both small and large deformations. Under small deformations, the formulations are based on assuming that the elastomer is linear elastic isotropic incompressible solid. While for the large deformation, the formulation is based on decomposing the 1st Piola-Kirchhoff stresses into linear and nonlinear parts. Thereafter, the final derived equations are composed of both boundary integral and non-linear domain integrals. The non-linear analyses were performed using an incremental procedure with an iterative algorithm.

Solving numerical examples and comparing the results with that obtained from some available results and ANSYS 10.0 showed that the boundary elements method is a good numerical technique for solving incompressible elastomeric materials. And the formulation used for the boundary elements derivations for large strain analysis gave satisfactory results as compared with that of ANSYS ver. 10.0.

Druck: KN Digital Printforce GmbH · Schockenriedstraße 37 · 70565 Stuttgart

# Nomenclature

| | | |
|---|---|---|
| **B,** | Left Cauchy-Green strain tensor | |
| **C** | Right Cauchy-Green strain tensor | |
| $C_{ij}$ | Cauchy principal value | |
| **D** | Deformation rate tensor | |
| $E$ | Modulus of elasticity | N/m$^2$ |
| $E_{ij}$ | Lagrangian or Green's finite strain tensor | |
| $e_{ij}$ | Eulerian or Almansi's finite strain tensor | |
| $E\left(m, \frac{\pi}{2}\right)$ | Elliptical integral of the second kind | |
| **F** | Deformation gradient matrix | |
| **f** | Displacement derivative matrix | |
| $\mathbf{F}_i$ | Viscous part of the deformation gradient matrix | |
| $\mathbf{F}_e$ | Elastic part of the deformation gradient matrix | |
| $\mathcal{F}$ | Shape function matrix | |
| G | Shear modulus | N/m$^2$ |
| $G_{ij}$ | Euclidian metric tensors for the coordinate $X_i$ | |
| $g_{ij}$ | Euclidian metric tensors for the coordinate system $x_i$ | |
| $G_r$, $G_z$ | Galerkin's vectors in $r$ and $z$ direction | |
| $I_1$, $I_2$, $I_3$ | Stretch or strain invariants | |
| $\langle I_1 \rangle_m$ | Amplified stretch invariant | |
| $J$ | Determinant of deformation gradient | |
| $\mathbf{J}_\mathrm{D}$ | Jacobain matrix | |
| $k$ | Boltzmann's constant | |
| $K$ | Bulk modulus of compression | |
| $K\left(m, \frac{\pi}{2}\right)$ | Elliptical integral of the first kind | |
| **L** | Velocity gradient tensor | |
| $n$ | Chain density in Arruda-Boyce Model | |
| $N$ | Number of the statistical link | |
| $n_r$, $n_z$ | Direction cosines of the outward normal $\hat{n}$ | |
| $p$ | Hydrostatic pressure | |
| $Q_{\nu-1/2}^{-1}$ | Legendre function | |

| | | |
|---|---|---|
| $\mathbf{R}$ | Orthogonal rotation tensor | |
| $r_{chain}$ | Current chain length | |
| $S_{\mathrm{I}}$ | 1st Piola-Kirchhof stress | N/m$^2$ |
| $S_{\mathrm{II}}$ | 2nd Piola-Kirchhof stress | N/m$^2$ |
| $t$ | Traction components | N/m$^2$ |
| $T_{ij}$ | Traction kernel functions | |
| $\mathbf{V}$ | Left stretch tensor | m/m |
| $v_f$ | Volume fraction | m$^3$ |
| $u$ | Displacements components | m |
| $\mathbf{U}$ | Right stretch tensor | |
| $U_{ij}$ | Displacement kernel functions | |
| $\mathcal{W}$ | Strain energy function | |
| $\mathcal{W}^e$ | Strain energy associated with equilibrium part | |
| $\mathcal{W}^i$ | Strain energy associated with instantaneous part | |
| $\mathcal{W}_{vol}(J)$ | Volumetric strain energy function | |
| $\mathcal{W}_{dis}(\overline{\mathbf{C}})$ | Distortional strain energy function | |
| $X_i$ | Material coordinate description | |
| $x_i$ | Spatial coordinate description | |
| $\beta$ | Inverse Langevin function | |
| $\beta_r$ | Radial components of the body force | |
| $\beta_Z$ | Axial components of the body force | |
| $\delta_{ij}$ | Krönecker delta | |
| $\varepsilon_r,\ \varepsilon_\theta,\ \varepsilon_z$ | Strains in $r$, $\theta$, $z$ direction | |
| $\boldsymbol{\varepsilon}_G\ ,[\varepsilon_G]$ | Green's or Lagrangian strain tensor | m/m |
| $\boldsymbol{\varepsilon}_A\ \ [\varepsilon_A]$ | Almansi's or Eulerian strain tensor | m/m |
| $\varepsilon_L$ | Logarithmic strain tensor | |
| $\phi_i$ | Interpolation (shape) function | |
| $\boldsymbol{\Gamma}$ | Boundary of the problem | |
| $\boldsymbol{\Lambda}$ | Amplified strain | |
| $\lambda$ | Stretch or stretch ratio | m/m |
| $\lambda_1, \lambda_2, \lambda_3$ | Principal Stretches | m/m |
| $\upsilon$ | Poisson's ratio | |
| $\Theta$ | Temperature | $^o$C |
| $\sigma_{o,}\ \sigma_n$ | Engineering stress | N/m$^2$ |
| $\sigma$ | True or Cauchy stress | N/m$^2$ |
| $\xi$ , $\eta$ | Local coordinate system | m |

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