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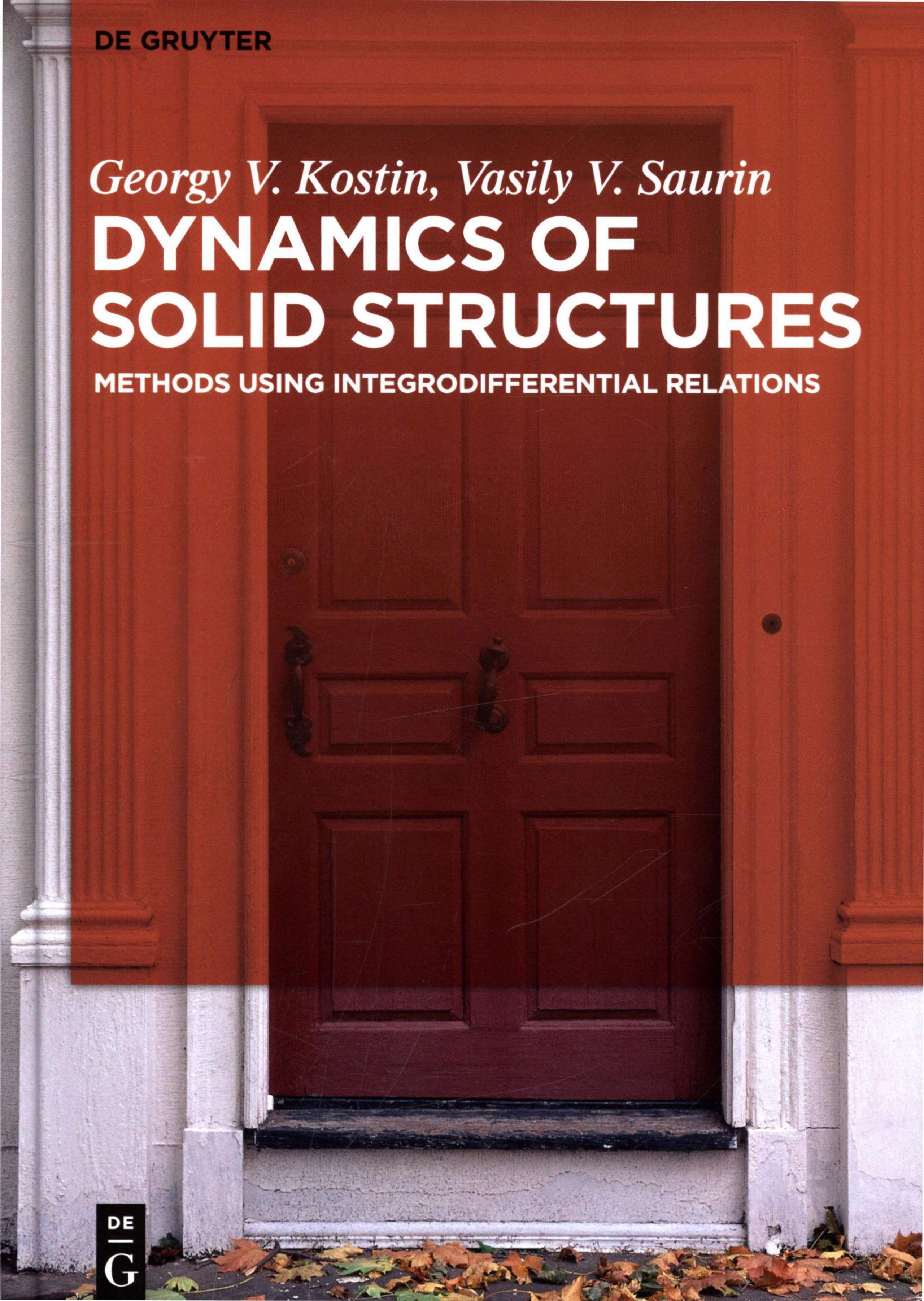
Georgy V. Kostin, Vasily V. Saurin

DYNAMICS OF SOLID STRUCTURES

METHODS USING INTEGRODIFFERENTIAL RELATIONS



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This monograph covers new variational and projection methods to study the dynamics within solid structures. To cope with the underlying initial-boundary value problems, the method of integrodifferential relations is employed. Applications and examples in physics, mechanics and control engineering range from natural vibrations or forced motions of elastic and viscoelastic bodies to heat and mass transfer processes.

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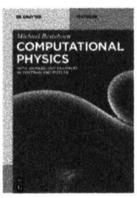

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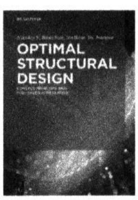

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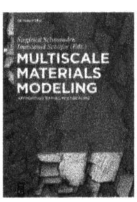

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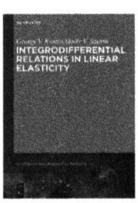

Integrodifferential Relations in Linear Elasticity
Georgy V. Kostin, Vasily V. Saurin, 2012
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To our families

Preface

The manuscript is a continuation of the monograph "Integrodifferential Approaches in Linear Elasticity" published by De Gryuter, the series "Studies in Mathematical Physics", Volume 10, 2012. The new book is a result of the authors' activity in solid dynamics and control theory in recent years at the Institute for Problems in Mechanics of the Russian Academy of Sciences.

Dynamics of solids has been actively studied in the scientific community over the past decades, and its applications are very actual and attractive nowadays. However, many fundamental questions in this area still remain open for discussion. Just take the conventional Hamilton principle in dynamics, for instance, that is formulated only for boundary value problems with respect to time which are not so substantive in practice. Although various numerical algorithms, among which Galerkin ones play a key role, have been intensively developed, a significant limiting factor in their evolution is the lack of convenient variational principles for initial-boundary value problems. Even more sophisticated barriers appear in formulating and solving inverse problems in dynamics, for which the very existence of a solution is often not grounded. All of this was the main reason for the authors to work out new variational and projection approaches to dynamic phenomena such as vibrations, forced motions of elastic or viscoelastic bodies and structures, heat and mass transfer processes in solids, and so on.

The key idea of the proposed approaches is that the state variables introduced can be always divided into two groups. The first group consists of the so-called measured quantities, e.g., displacements, strains, velocities, temperature. The second one includes unmeasured values: stresses, momenta, heat fluxes, etc. At the same time, governing equations can be split into three types: firstly, initial and boundary conditions, secondly, balance and continuity laws, and thirdly, constitutive relations. The first type of equations reflects the influence of the environment on the considered system. The second one describes the fundamental physical phenomena and hypotheses of continuum; all of these laws do not depend on media properties. In contrast, the constitutive relations connect measured and unmeasured unknowns and contain information about intrinsic properties of the object under study.

In physics, generalized statements suppose that some of the governing equations are weakened; these are typically balance equations. The essence of the method of integrodifferential relations (MIDR), which was grounded in the authors' book mentioned above, is that equations of the third type are represented in the integral form, whereas the other equations must be considered as strict constraints. The initial-boundary value problem modified in accordance with this scheme can be reduced to the minimization of a non-negative functional over all admissible variables. Such a reformulation became a starting point to develop advanced numerical techniques to state analysis, solution quality estimation, and optimization in dynamics of solids.

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Tentatively, the book can be divided into three principal parts. The first part deals with different variational formulations of initial-boundary value problems that provides the foundation for the other two parts.

In Chapter 1, the state of the art in the dynamics of solids is presented. A major emphasis in the survey is placed on a variety of direct as well as inverse dynamic problems and on the method of integrodifferential relations in statics and dynamics. The basic ideas of this approach are discussed with the example of deformations of elastic bodies.

In Chapter 2, generalized statements of conventional parabolic and hyperbolic problems are presented and their variational properties are analyzed. Methodological aspects of the MIDR are outlined by involving firstly the Cauchy problem for finite-dimension mechanical systems with elastic elements. Then the initial-boundary value problems describing vibrations of elastic structures and processes of heat transfer or diffusion in solids are considered.

The classical and novel, following the MIDR, variational principles in linear elasticity are introduced in the next chapter for dynamic and static cases. Special attention is paid to the relation between the novel formulations and the dual Hamilton principles.

Chapter 4 is devoted to variational statements in structural mechanics. In its beginning, the motions of an elastic beam and viscoelastic rod are discussed in detail. In the end of this chapter, variational properties of minimized functionals describing the behavior of mechanical structures with lumped as well as distributed parameters are discussed.

In the middle part of the book (Chapters 5–8), suitable variational and projection procedures based on the MIDR to solve initial-boundary and boundary value problems are described. Various ways to weaken the constitutive relations between balanced stresses and compatible strains as well as between momentum density and velocities of material points, which are given usually in the local form, are presented in this part.

In Chapter 5, usefulness of the Ritz method, conventionally applied in statics, in solving the initial-boundary value problems is shown. Specific features of this approach and its efficiency in dynamics are illustrated by numerical solutions on the basis of polynomial approximation and the finite element method.

Possible advantages and shortcomings of semi-discrete approximation to obtain numerical results for dynamic problems in mechanics of structures are discussed in Chapter 6. Productivity of such kind of approximations for variational and projection algorithms is analyzed with the example of elastic rod motions. After that, the comparison of the Ritz and Galerkin methods in beam dynamics is performed.

Chapter 7 is devoted to numerical analysis of eigenvalue problems based on variational as well as projection techniques and the MIDR. Firstly, eigenvalue properties are studied for elementary parabolic and hyperbolic differential equations. Then the semi-discretization developed in the previous chapter for initial-boundary value problems is extended to the case of natural in-plane vibrations of rectangular plates.

In Chapter 8, harmonic motions of elastic rectilinear beams with convex cross sections are investigated in the frame of the 3D model in the linear theory of elasticity. It is shown that the natural vibrations of the beams with cross sections having more than one axis of symmetry are decomposed into at least four independent motions, namely, longitudinal, torsional, and two types of lateral forms. Vice versa, the lack of symmetry leads to complex interdependent free and forced motions, as demonstrated for beams with asymmetric triangle cross sections.

The last part of the book deals with one type of inverse dynamic problems, namely, control problems. In accordance with the MIDR, a numerical procedure of double minimization is proposed in Chapter 9 to design optimal strategies of mechanical system motions. The key point of this approach is that a solution quality functional and an objective index are sequentially minimized over state and control parameters, respectively. Accordingly, finite element algorithms for the optimization of elastic rod or body motions controlled by boundary displacements and forces are described.

Questions concerning the incorporation of semi-discrete approximations and polynomial control laws into numerical schemes for inverse dynamic problems are considered in Chapter 10. Based on the MIDR and the variational formulation, a modification of objective functions aimed to regularize numerically ill-posed inverse problems is discussed. Additionally, Pontryagin's maximum principle combined with model reduction is applied to design a feedforward strategy for the optimal locomotion of a 3D beam.

Chapter 11 is devoted to various applications of the above-mentioned methods in mechatronics. The following optimal control problems are studied: rotations of an electromechanical manipulator with a flexible link, modeling and control of a high bay rack feeder with viscoelastic elements. It is shown that the proposed methodology is applicable to control of real-world technical systems and gives one the possibility to develop new efficient algorithms.

The Appendix contains the most important definitions and necessary information on the vector and tensor algebra as well as functional analysis.

Potential readers of this book will be mathematicians, engineers, as well as graduate and postgraduate students who are interested in learning the mathematical basis, modeling, and numerical technique in solid mechanics.

Basic notation

The following notation is used throughout the book unless otherwise specified.

Acronyms

BVP	boundary value problem
DAE	differential-algebraic equation
EVP	eigenvalue problem
FEM	finite element method
IBVP	initial-boundary value problem
LSM	least squares method
MIDR	method of integrodifferential relations
ODE	ordinary differential equation
PDE	partial differential equation

Functions and constants

$\mathbb{N} = \{0, 1, 2, \ldots\}$	set of natural numbers
$\mathbb{Z}_+ = \{1, 2, 3, \ldots\}$	set of positive integers
$\boldsymbol{a}(\boldsymbol{x}), \boldsymbol{b}(\boldsymbol{x}) \in \mathbb{R}^n$	basis vectors
$\mathbf{A}(t) \in \mathbb{R}^{n\times n}$	state matrix of a linear ODE system
$\mathbf{B}(t) \in \mathbb{R}^{n\times m}$	control matrix of a linear ODE system
$\mathbf{C}(\boldsymbol{x}) \in \mathbb{R}^{d^4}$	elastic modulus tensor
$d \in \{1, 2, 3\}$	space dimension
$\boldsymbol{f}(t, \boldsymbol{x}) \in \mathbb{R}^d$	(volume) force vector
$\{i, j, k, l, m, n\} \in \mathbb{Z}$	integer indices
$\boldsymbol{i}, \boldsymbol{j} \in \mathbb{Z}^n$	multi-indices
$J \in \mathbb{R}$	cost function in optimal control problems
$\mathbf{K} \in \mathbb{R}^{n\times n}$	stiffness matrix
$L > 0$	beam or rod length
$\mathbf{M} \in \mathbb{R}^{n\times n}$	mass matrix
$\boldsymbol{n}(\boldsymbol{x}) \in \mathbb{R}^d$	unit outward normal vector
$\boldsymbol{p}(t, \boldsymbol{x}) \in \mathbb{R}^d$	momentum density vector
$\boldsymbol{p}(t) \in \mathbb{R}^n$	generalized momenta of a mechanical system
$\boldsymbol{q}(t, \boldsymbol{x}) \in \mathbb{R}^d$	vector of surface stress/heat flux
$\boldsymbol{s}(t) \in \mathbb{R}^n$	generalized force vector
$t \in \mathbb{R}$	time
$T > 0$	terminal time instant
$\boldsymbol{u}(t) \in \mathbb{R}^m$	control vector

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$\boldsymbol{v}(t,\boldsymbol{x}) \in \mathbb{R}^d$	(residual) velocity vector
$V \subset \mathbb{R}^d$	spatial domain
$\boldsymbol{w}(t,\boldsymbol{x}) \in \mathbb{R}^d$	displacement vector
$\boldsymbol{w}(t) \in \mathbb{R}^n$	generalized coordinates of a mechanical system
$W(t) \in \mathbb{R}$	mechanical energy
$\boldsymbol{x} \in \mathbb{R}^d$	spatial coordinate vector
$\boldsymbol{x}(t) \in \mathbb{R}^n$	state vector of an ODE system
γ	weighting coefficient
Γ	boundary of a spatial domain
Δ	relative integral error
$\varepsilon(t,\boldsymbol{x}) \in \mathbb{R}^{d\times d}$	strain tensor
$\kappa(\boldsymbol{x}) > 0$	stiffness coefficient
$\lambda \in \mathbb{C}$	eigenvalue
$\nu(\boldsymbol{x}) \in \mathbb{R}$	Poisson's ratio
$\xi(t,\boldsymbol{x}) \in \mathbb{R}^{d\times d}$	residual strain tensor
$\rho(\boldsymbol{x}) > 0$	material volume density
$\sigma(t,\boldsymbol{x}) \in \mathbb{R}^{d\times d}$	stress tensor
$\Upsilon \in \mathbb{R}$	functional of action
$\Phi \geq 0$	constitutive functional
$\Psi \geq 0$	time integral of energy
Ω	domain of a function
$\omega \geq 0$	frequency

Contents