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Edited by **Roderick Melnik**

Mathematical and Computational Modeling

*With Applications in the Natural and Social
Sciences, Engineering, and the Arts*

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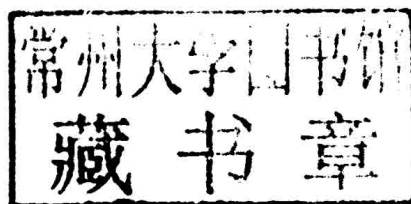
MATHEMATICAL AND COMPUTATIONAL MODELING

**With Applications in Natural and
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and the Arts**

Edited by

RODERICK MELNIK

Wilfrid Laurier University
Waterloo, Ontario, Canada



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**MATHEMATICAL AND COMPUTATIONAL
MODELING**

PURE AND APPLIED MATHEMATICS

A Wiley Series of Texts, Monographs, and Tracts

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Teachers who educate children deserve more honor than parents who merely gave birth; for bare life is furnished by the one, the other ensures a good life.
(Aristotle)

To my parents who were my first and most devoted teachers.

LIST OF CONTRIBUTORS

Julien Arino, Department of Mathematics, University of Manitoba, Winnipeg, Canada

Steven J. Brams, Department of Politics, New York University, New York, NY, USA

Emily A. Carter, Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ, USA; and Program in Applied and Computational Mathematics, and Andlinger Center for Energy and the Environment, Princeton University, Princeton, NJ, USA

Julia Chelen, Center for Advanced Modeling in the Social, Behavioral, and Health Sciences, Department of Emergency Medicine, Johns Hopkins University, Baltimore, MD, USA

Ronald R. Coifman, Mathematics Department, Yale University, New Haven, CT, USA

Johannes M. Dieterich, Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ, USA

Matan Gavish, Statistics Department, Stanford University, Palo Alto, CA, USA

Dimitrios Giannakis, Courant Institute of Mathematical Sciences, New York University, New York, NY, USA

Antonios Gonis, Physics and Life Sciences, Lawrence Livermore National Laboratory, Livermore, CA, USA

Ali Haddad, Mathematics Department, Yale University, New Haven, CT, USA

Bernd Hofmann, Faculty of Mathematics, Technische Universität Chemnitz, Chemnitz, Germany

D. Marc Kilgour, Department of Mathematics, Wilfrid Laurier University, Waterloo, Ontario, Canada

Eili Y. Klein, Center for Advanced Modeling in the Social, Behavioral, and Health Sciences, Department of Emergency Medicine, Johns Hopkins University, Baltimore, MD, USA; and Center for Disease Dynamics, Economics & Policy, Washington, DC, USA

David B. Krisiloff, Department of Chemistry, Princeton University, Princeton, NJ, USA

Mel Levy, Department of Chemistry, Duke University, Durham, NC, USA; Department of Physics, North Carolina A&T State University, Greensboro, NC, USA; and Department of Chemistry and Quantum Theory Group, School of Science and Engineering, Tulane University, New Orleans, LO, USA

Florian Libisch, Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ, USA

Andrew J. Majda, Courant Institute of Mathematical Sciences, New York University, New York, NY, USA

Michael D. Makowsky, Center for Advanced Modeling in the Social, Behavioral, and Health Sciences, Department of Emergency Medicine, Johns Hopkins University, Baltimore, MD, USA

Roderick Melnik, The MS2Discovery Interdisciplinary Research Institute, M²NeT Laboratory and Department of Mathematics, Wilfrid Laurier University, Waterloo, Ontario, Canada

Israel Michael Sigal, Department of Mathematics, University of Toronto, Ontario, Canada

Paul E. Smaldino, Center for Advanced Modeling in the Social, Behavioral, and Health Sciences, Department of Emergency Medicine, Johns Hopkins University, Baltimore, MD, USA

Iman A. Soliman, Department of Mathematics, Cairo University, Giza, Egypt

Ronen Talmon, Mathematics Department, Yale University, New Haven, CT, USA

Nicolae Tarfulea, Department of Mathematics, Computer Science, and Statistics, Purdue University Calumet, Hammond, IN, USA

Godfried T. Toussaint, Department of Computer Science, New York University Abu Dhabi, Abu Dhabi, United Arab Emirates

PREFACE

Mathematical and computational modeling has become a major driving force in scientific discovery and innovation, covering an increasing range of diverse application areas in the natural and social sciences, engineering, and the arts. Mathematical models, methods, and algorithms have been ubiquitous in human activities from the ancient times till now. The fundamental role they play in human knowledge, as well as in our well-being, is indisputable, and it continues to grow in its importance.

Significant sources of some of the most urgent and challenging problems the humanity faces today are coming not only from traditional areas of mathematics applications in natural and engineering sciences, but also from life, behavioral, and social sciences. We are witnessing an unprecedented growth of model-based approaches in practically every domain of human activities. This expands further interdisciplinary horizons of mathematical and computational modeling, providing new and strengthening existing links between different disciplines and human activities. Integrative, holistic approaches and systems–science methodologies are required in an increasing number of areas of human endeavor. In its turn, such approaches and methodologies require the development of new state-of-the-art mathematical models and methods.

Given this wide spectrum of applications of mathematical and computational modeling, we have selected five representative areas, grouped in this book into sections. These sections contain 12 selective chapters, written by 25 experts in their respective fields. They open to the reader a broad range of methods and tools important in many applications across different disciplines. The book provides details on state-of-the-art achievements in the development of these methods and tools, as well as their applications. Original results are presented on both fundamental theoretical and applied developments, with many examples emphasizing interdisciplinary nature of

mathematical and computational modeling and universality of models in our better understanding nature, society, and the man-made world.

Aimed at researchers in academia, practitioners, and graduate students, the book promotes interdisciplinary collaborations required to meet the challenges at the interface of different disciplines on the one hand and mathematical and computational modeling on the other. It can serve as a reference on theory and applications of mathematical and computational modeling in diverse areas within the natural and social sciences, engineering, and the arts.

I am thankful to many of my colleagues in North America, Europe, Asia, and Australia whose encouragements were vital for the completion of this project. Special thanks go to the referees of this volume. Their help and suggestions were invaluable. Finally, I am very grateful to the John Wiley & Sons editorial team, and in particular, Susanne Steitz-Filler and Sari Friedman for their highly professional support.

Waterloo, ON, Canada
August 2014–2015

RODERICK MELNIK

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