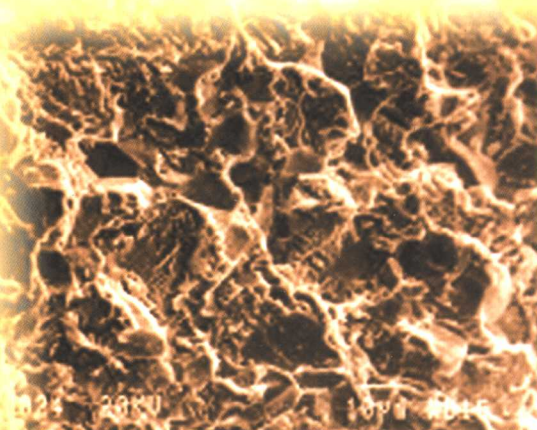
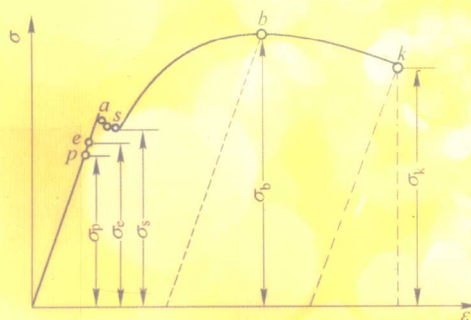
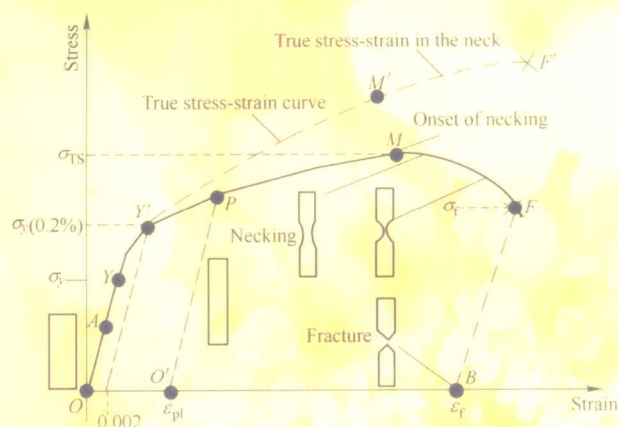


# Mechanical Properties of Materials

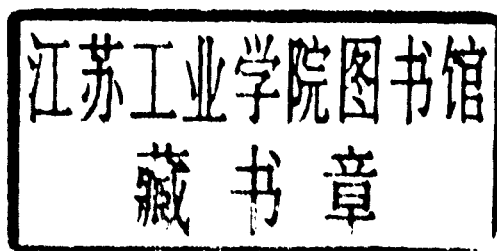
**Yu Haisheng    Sergiy N. Shukayev    Mamoun Medraj**



**Metallurgical Industry Press**

# Mechanical Properties of Materials

*Yu Haisheng*  
*Sergiy N. Shukayev*  
*Mamoun Medraj*



Beijing  
Metallurgical Industry Press

2005

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Published and distributed by

Metallurgical Industry Press

39 Songzhuyuan Beixiang, Beiheyuan Dajie

Beijing 100009, P. R. China

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### 图书在版编目(CIP)数据

材料的力学性能/于海生等著. —北京: 冶金工业出版社, 2005. 7

ISBN 7-5024-3748-7

I. 材… II. 于… III. 材料力学性质—教材—英文 IV. TB303

中国版本图书馆 CIP 数据核字(2005)第 051577 号

出版人 曹胜利 (北京沙滩嵩祝院北巷 39 号, 邮编 100009)

责任编辑 宋 良 方茹娟 美术编辑 李 心

责任校对 侯 瑁 李文彦 责任印制 牛晓波

北京百善印刷厂印刷; 冶金工业出版社发行; 各地新华书店经销

2005 年 7 月第 1 版, 2005 年 7 月第 1 次印刷

787mm × 1092mm 1/16; 15.75 印张; 380 千字; 240 页; 1-1500 册

30.00 元

冶金工业出版社发行部 电话: (010)64044283 传真: (010)64027893

冶金书店 地址: 北京东四西大街 46 号(100711) 电话: (010)65289081

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# Preface

As we know, metals are the most widely used materials in industry. Their service performances, such as physical properties, technological properties and mechanical properties, vary from metal to metal. Among them the mechanical properties for structural materials are the most important.

What do mechanical properties mean? Mechanical properties mean the behaviors of materials under external loads or combined effects of external loads with environmental factors, such as temperature, medium, loading rate and so on. Mechanical properties are also called mechanical behaviors, for example, deformation and fracture of metals caused by external loads.

Mechanical properties of materials include strength, hardness, elasticity, toughness, wear ability, notch sensitivity and so on. Briefly speaking, mechanical properties of materials mean the abilities of materials to resist deformation and fracture caused by external loads. These abilities are indicated by some parameters of mechanical property, such as parameters of strength  $\sigma_b$ ,  $\sigma_{0.2}$  and  $\sigma_{-1}$ , parameters of elasticity  $\delta$ ,  $\psi$ , parameters of toughness  $A_K$ ,  $K_{IC}$  and so on. These parameters are the main criteria for proper material selecting and strength calculating in engineering design.

What are the major factors that affect mechanical properties of materials?

Often materials are subject to forces when they are used. Materials deform or break as a function of applied load, time, temperature, and other conditions. The major factors that affect mechanical properties of materials include both intrinsic and external factors. Among intrinsic factor are chemical composition, structural configuration, metallurgy quality, residual stress, surface and internal defects and so on; Among external factors are types of loading (static load and dynamic load), loading spectrum, state of stress (tension, compression, bending, torsion, combined stress and so on). There are two examples to illustrate the effect of both intrinsic and external factors on mechanical properties of materials.

First example: The stress-strain curve of a type of low carbon steel under normal static tensile test condition is illustrated in Figure p-1a. From the picture we can tell that, the low carbon steel is a metal with a great toughness and ductility in nature. But if we do the same test only under lower temperature, in this case it appears to be

a brittle metal other than ductile one. This can be seen by examination of the curves in Figure p-1b. Another example: Instead of static load we impose a cyclic load on the steel specimen. It can be observed that some tiny cracks have occurred over time on the surface of the specimen, which will lead to a final fracture of the specimen. This type of failure is called fatigue fracture. In this way you can imagine the changes in mechanical properties caused by internal factor and external factors.

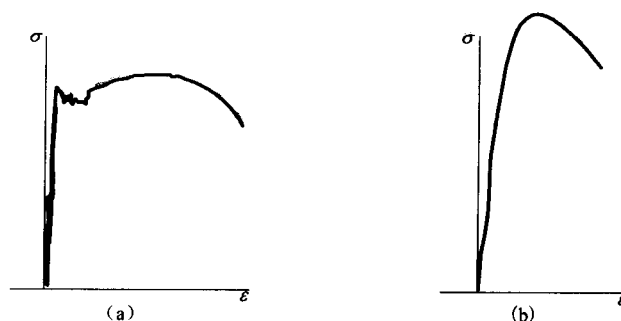


Figure p-1 Engineering stress-strain curve of low carbon steel

*Mechanical Properties of Materials* is categorized under experimental science. Different mechanical properties of materials are obtained through different experimentation. In microscopic terms, mechanical properties of materials are related to interactions among dislocations and interactions between dislocations and point defects. Therefore, we should study mechanical properties of materials from both macroscopic and microscopic considerations. Also we should follow some basic guidelines to study mechanical properties of materials:

- 1) To master the testing procedures and techniques for studying mechanical properties of materials;
- 2) To study the deformation and fracture phenomena and microscopic mechanism of materials under different conditions;
- 3) To study the essence of mechanical properties of materials, physical conceptions, actual meanings and relationships among different kinds of mechanical properties;
- 4) To study various affecting factors to different kinds of mechanical properties of materials;
- 5) To provide proper methods to improve mechanical properties of materials in the engineering practice.

Based on the fundamental theory of mechanical properties of materials, we should master the methods of proper material selection and proper design of components in order to improve the service performances of materials.

## LIST OF SYMBOLS

$a$ : distance between slip planes  
 $a_c$ : critical crack length  
 $A$ : amplitude ratio  
 $A_0$ : original cross section area of a specimen  
 $A_f$ : cross section area of a specimen at fracture  
 $A_{ku}$  ( $A_{kv}$ ): Charpy U (V) impact energy of the notched specimen  
 $b$ : Burger's vector  
COD : crack open displacement  
 $da/dN$ : fatigue crack propagation rate  
 $E_c$ : compressive modulus of elasticity  
 $E$ : modulus of elasticity in tension  
 $E_b$ : modulus of elasticity in bending (flexural modulus of elasticity)  
 $E_c$ : Young's modulus in compression  
 $e$ : true strain  
 $e_B$ : true strain at the stage of uniform deformation  
 $e_f$ : true strain at fracture  
 $F$ : testing force  
 $F_{bb}$ : maximum bending force  
 $F_{bc}$ : maximum compressive force  
FATT: fracture appearance transition temperature  
 $f$ : bending deflection  
 $G$ : torsional modulus of elasticity (shear modulus)  
 $G_{IC}$ : critical strain energy release rate  
HB: Brinell hardness  
HK: Knoop hardness  
HRA, HRB, HRC...: Rockwell hardness  
HS: Shore Hardness  
HV: Vickers Hardness  
IG : intergranular fracture  
 $J$ ,  $J_1$ : J integral  
 $J_{Ic}$ : critical J integral for mode I crack  
 $K_f$ : fatigue notch factor  
 $K_t$ : theoretical stress concentration factor  
 $K_I$ : stress intensity factor of mode I crack

- 
- $K_{IC}$ : critical stress intensity factor (plane strain fracture toughness)  
 $K_c$ : plain stress fracture toughness  
 $\Delta K_{th}$ : threshold of fatigue crack propagation  
 $\Delta K_I$ : stress intensity factor range  
 $m'$ : stress sensitivity exponent of dislocation motion velocity  
 $n$ : strain hardening exponent  
 $NDT$ : nil ductility transition  
 $NSR$ : notch sensitivity ratio  
 $N$ : number of fatigue cycles to failure  
 $R$ : stress ratio  
 $S$ : true stress  
 $t_k$ : ductile-brittle transition temperature  
 $TG$ : transgranular fracture  
 $U_e$ : elastic strain energy  
 $U$ : fracture energy  
 $Y$ : dimensionless geometry factor  
 $\alpha$ : coefficient of state of stress  
 $\gamma_s$ : effective surface energy  
 $\gamma$ : shear strain  
 $\delta$ : percentage elongation after fracture  
 $\Delta T$ : torque increments  
 $\Delta \varphi$ : torsion angle increments  
 $\varepsilon$ : strain  
 $\dot{\varepsilon}$ : strain rate  
 $\varepsilon_e$ : elastic strain  
 $\varepsilon_p$ : plastic strain  
 $\varepsilon_{pb}$ : given amount of non-proportional bending strain  
 $\nu$ : Poisson's ratio  
 $\rho$ : glissile dislocation density  
 $\bar{v}$ : average glissile dislocation velocity  
 $\sigma_b$ : tensile strength  
 $\sigma_{bn}$ : tensile strength for notched specimen  
 $\sigma_m$ : mean stress  
 $\sigma_a$ : stress amplitude in fatigue  
 $\sigma_{max}$ : maximum stress  
 $\sigma_{min}$ : minimum stress  
 $\sigma_p$ : stress for a given amount of non-proportional elongation  
 $\sigma_r$ : stress for a given amount of residual elongation  
 $\sigma_t$ : stress for a given amount of total elongation

- 
- $\sigma_{pc}$ : compressive stress for a given amount of non-proportional compression  
 $\sigma_{bc}$ : compressive strength  
 $\sigma_c$ : cleavage fracture stress  
 $\sigma_{sc}$ : compressive yield point  
 $\sigma_{pb}$ : bending stress for a given amount of non-proportional strain  
 $\sigma_{bb}$ : flexural strength  
 $\sigma_i$ : total resistance to dislocation motion  
 $\sigma_u$ : upper yield point  
 $\sigma_{sl}$ : lower yield point  
 $\sigma_{FL}$ : fatigue limit.  
 $\sigma_s, \sigma_{0.2}$ : yield strength (offset yield strength)  
 $\sigma_{-1}$ : bending fatigue limit of the symmetric stress cycle  
 $\sigma_{-1n}$ : bending fatigue limit of the symmetric stress cycle for notch specimen  
 $\tau_s$ : torsional yield point  
 $\tau_p$ : torsion stress for a give amount of non-proportional shear strain  
 $\tau_f$ : frictional resistance to slip.  
 $\tau_b$ : torsion strength  
 $\psi$ : Percent reduction in area  
 $\omega$ : dislocation width



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# Concepts of Stress and Strain

## 1. 1 Introduction

Engineers study the mechanics of materials mainly in order to have a means of analyzing and designing various machines and structures. Both the analysis and the design of a given structure involve determining of stresses and strains.

Stress and strain are important mechanical engineering and materials science concepts. Materials are often chosen on the basis of their response to applied force (stress). Strain is a measure of the temporary deformation such stresses can cause.

When choosing a material, there is a need to determine what properties are most important, so stress and strain that the material can undergo before permanent deformation are common factors in these decisions. As a result of action of external forces in a body there are additional (internal) forces of interaction between its separate particles. *The method of sections* is applied to reveal internal interactions between various parts of a body. In this method the body is mentally dissected by a surface (commonly, a plane) into two parts, one of which is mentally rejected, and its action is replaced with forces allocated on a surface of section.

If the body as a whole is in equilibrium, any part of it also must be in equilibrium. This consideration leads to the following fundamental conclusion: the externally applied forces to one side of an arbitrary cut must be balanced by the internal forces developed at the cut, or, briefly, the external forces are balanced by the internal forces. As a result of the equilibrium condition we may determine the resultant vector and resultant moment of internal forces that act upon the section surface. This is the essence of the method of sections.

This chapter concentrates on the fundamental concepts of stress and strain, and the relationship between stress and strain, to prepare the reader for further study of mechanical properties of metals.

## 1. 2 Stress

Stress is defined as a force acting upon some area. Since forces are vectors, a force inclined to a plane can always be described as a combination of normal and shear forces (Figure 1-1).

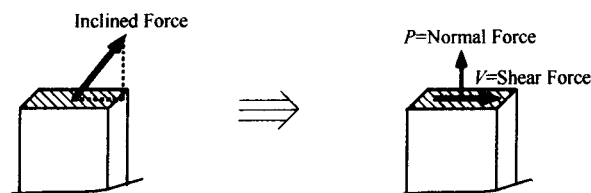


Figure 1-1 Transformation of an inclined force ( $F$ ) acting upon some area ( $A$ )

If you take the normal force  $P$  on Figure 1-1 above and divide it by a cross-sectional area  $A$ , you would get the normal stress. It results in an equation

$$\sigma = \frac{P}{A} \quad (1-1)$$

where, the Greek letter  $\sigma$  (sigma) is used to represent normal stress that acts normally upon the reference plane of the element (Figure 1-2). The *shear stress* can be defined in a similar manner. If you take the shear force  $V$  and divide it by a cross-sectional area  $A$ , you will get the shear stress,

$$\tau = \frac{V}{A} \quad (1-2)$$

where,  $\tau$  is shear stress acting in - plane of the element (Figure 1-2).

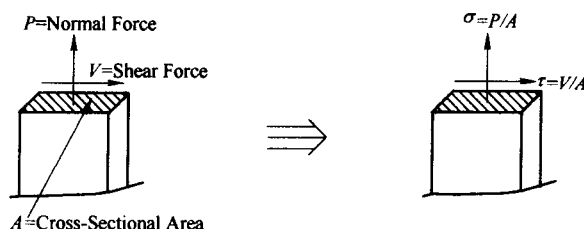


Figure 1-2 Definition of the normal and shear stresses

Stress has units of force over area. In the SI system it is newtons per square meter, and is defined as a Pascal (abbreviated Pa). In general, the internal forces acting upon infinitesimal areas of a cut are of varying magnitudes and directions. The stress upon the arbitrary infinitesimal area  $\Delta A$  with normal  $\nu$  (Figure 1-3) can be expressed as

$$P_\nu = \lim_{\Delta A \rightarrow 0} \left( \frac{\Delta F}{\Delta A} \right) \quad (1-3)$$

where,  $\Delta F$  is a vector sum of all of the internal forces acting upon a given infinitesimal area  $\Delta A$ . In other words, stress is the intensity of internal force at a point.

You can draw infinitely many planes at different orientation in space, and through a point. Stress will be different on each plane. Under static equilibrium, only nine stress components from three planes are needed to describe the stress state at a point. Figure 1-4 illustrates a general

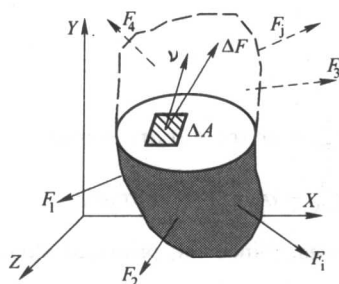


Figure 1-3 Representation of cross-section of a general body

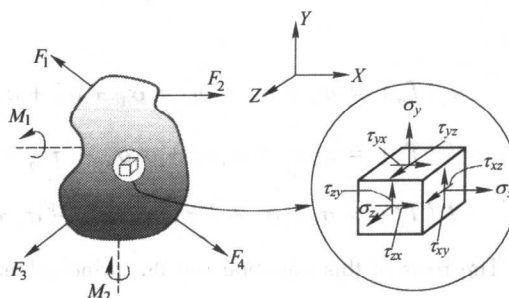


Figure 1-4 A general three-dimensional stress element

three-dimensional stress element, showing three normal stresses  $\sigma_x$ ,  $\sigma_y$  and  $\sigma_z$ , all positive; and six shear stresses  $\tau_{xy}$ ,  $\tau_{yx}$ ,  $\tau_{yz}$ ,  $\tau_{zy}$ ,  $\tau_{zx}$ , and  $\tau_{xz}$ , also all positive. The element is in static equilibrium and hence

$$\tau_{xy} = \tau_{yx} \quad \tau_{yz} = \tau_{zy} \quad \tau_{zx} = \tau_{xz} \quad (1-4)$$

Outwardly directed normal stresses are considered as tension and are positive. Shear stresses are positive if they act in the direction of a reference axis. The first subscript of a shear stress component specifies the normal to the element face. The shear stress component is parallel to the axis of the second subscript. The negative faces of the element will have shear stresses acting in the opposite direction; these are also considered as positive. These nine components can be organized into the matrix:

$$T_\sigma = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix} \quad (1-5)$$

This grouping of the nine stress components is known as the *stress tensor*.  $T_\sigma$  is symmetric and has only six independent components. We can always find some change of basis that transforms  $T_\sigma$  into a nice diagonal form

$$T_\sigma = \begin{vmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{vmatrix} \quad (1-6)$$

$\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  are called the principal stresses. The principal stresses acting in three mutually perpendicular planes, which pass through the point considered (Figure 1-5).

The relation between the principal stresses at a point and the stresses on an arbitrary oriented plane passing through this point is described by the following cubic equation:

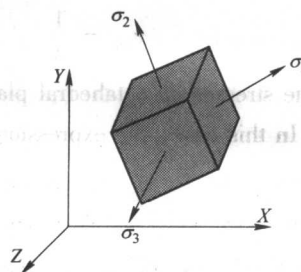


Figure 1-5 A principal stress element in rectangular system

$$\sigma^3 - I_1(T_\sigma)\sigma^2 + I_2(T_\sigma)\sigma - I_3(T_\sigma) = 0 \quad (1-7)$$

where,

$$I_1(T_\sigma) = \sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2(T_\sigma) = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \quad (1-8)$$

$$I_3(T_\sigma) = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 = \sigma_1\sigma_2\sigma_3$$

The roots of this equation are the principal normal stresses. Since the principal stresses do not vary with change of the coordinate axes, i. e., they are independent of the method of their determination, the coefficients  $I_1(T_\sigma)$ ,  $I_2(T_\sigma)$ , and  $I_3(T_\sigma)$  from Eq. (1.7) are also independent of the choice of a coordinate system; in other words, they are stress tensor invariants with respect to choice of coordinates. In addition to expressions (1.8), there are other functions of stress tensor components that are invariant with respect to the coordinate axes. Yet all of them can be represented as functions of the three invariants presented above.

The state of stress is said to be triaxial when none of the three principal stresses is zero. For plane, or biaxial stress state one of the principal stresses equals zero. When only one principal stress is nonzero, the state of stress is called *uniaxial*. It can be shown that there are three pairs of planes where the shear stresses reach extrema. These extreme values of the shear stresses are called the principal shear stresses:

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}, \quad \tau_{23} = \frac{\sigma_2 - \sigma_3}{2}, \quad \text{and} \quad \tau_{31} = \frac{\sigma_3 - \sigma_1}{2} \quad (1-9)$$

The plane inclined equally to the principal axes is called octahedral. The normal stress on this plane is equal to the arithmetic mean of the three principal normal stresses

$$\sigma_{\text{oct}} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \sigma_o \quad (1-10)$$

whereas the shear stress on the same plane is equal to the root mean square of the three principal shear stresses

$$\tau_{\text{oct}} = \frac{2}{3} \sqrt{\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2} \quad (1-11)$$

This formula can also be presented in the form

$$\tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (1-12)$$

The stresses on octahedral planes can also be expressed in terms of six stress tensor components. In this case, the expressions for  $\sigma_{\text{oct}}$  and  $\tau_{\text{oct}}$  have the form

$$\sigma_{\text{oct}} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} \quad (1-13)$$

$$\tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} \quad (1-14)$$

The stress tensor can be presented as the sum of the *hydrostatic stress tensor* ( $T_\sigma^\circ$ ) and of the stress deviator ( $D_\sigma$ ):

$$T_\sigma = T_\sigma^\circ + D_\sigma \quad (1-15)$$

where,  $T_\sigma^\circ$  is composed of the components which predetermines a change in the volume of body (e. g. hydrostatic pressure):

$$T_\sigma^\circ = \begin{vmatrix} \sigma_o & 0 & 0 \\ 0 & \sigma_o & 0 \\ 0 & 0 & \sigma_o \end{vmatrix} \quad (1-16)$$

$D_\sigma$  is composed of the components which are related only to distortion of the body:

$$D_\sigma = \begin{vmatrix} \sigma_x - \sigma_o & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma_o & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma_o \end{vmatrix} \quad (1-17)$$

and  $\sigma_o$  is the arithmetic mean of the normal stress:

$$\sigma_o = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{I_1}{3} \quad (1-18)$$

Since the first invariant of the hydrostatic stress tensor coincides with the first invariant of the stress tensor

$$I_1(T_\sigma^\circ) = \sigma_o + \sigma_o + \sigma_o = 3\sigma_o = \sigma_x + \sigma_y + \sigma_z = I_1 \quad (1-19)$$

the first invariant of the stress deviator is equal to zero. The second one is given by

$$I_2(D_\sigma) = \frac{1}{6}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)] \quad (1-20)$$

$$\text{or} \quad I_2(D_\sigma) = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2] \quad (1-21)$$

In the solution of a number of problems of mechanics of materials it is convenient to use generalized stresses  $\sigma_i$ , which are proportional to the second invariant of the stress deviator or the octahedral shear stresses

$$\sigma_i = \sqrt{3} \cdot \sqrt{I_2(D_\sigma)} = \frac{3}{\sqrt{2}} \tau_{\text{oct}} \quad (1-22)$$

The generalized stress  $\delta_i$  is called *the stress intensity*.

## 1.3 Strain

All structures and mechanical components deform under load. The collective displacements of



points in a body relative to an external reference frame is known as deformation. Deformation describes the transformations from some initial to some final geometry. Strain is a measure of the deformation of a solid body. In strain theory are entered the normal (or axial) strain  $\varepsilon$  and shear strain  $\gamma$ . The normal strain is defined as the fractional change in length, or the deformation of the original length divided by the original length, and is denoted by the Greek symbol epsilon

$$\varepsilon = \frac{\Delta(dl)}{dl} \quad (1-23)$$

where,  $dl$  is the original element length, and  $\Delta(dl)$  is the change in length  $dl$ , i. e. deformation. Normal strain may be compressive or tensile. Tensile strain is positive and the compressive strain is negative. Note that strain has no units, it is dimensionless. This is because we have divided one length by another. Sometimes you may see strain represented in percentage. Shear strain  $\gamma$  (gamma) is the change in the initial right angle between any two imaginary lines in a body. These angles are measured in radians. An example of the strains is shown in Figure 1-6. Let us consider two elementary lengths  $ba$  and  $bc$  starting from the point  $b$  and which are along coordinate axes  $x$  and  $y$  (Figure 1-6). As a result of deformation the lengths and original right angle between them is changed. The axial strains are equal to  $\varepsilon_x = \Delta(dl_x)/dl_x$ ,  $\varepsilon_y = \Delta(dl_y)/dl_y$ , and the shear strain is  $\gamma_{xy} = \Delta(\angle abc)$ . In general an arbitrary point of a body under deformation has three axial ( $\varepsilon_x, \varepsilon_y, \varepsilon_z$ ) and six shear ( $\gamma_{xy}, \gamma_{yx}, \gamma_{yz}, \gamma_{zy}, \gamma_{zx}, \gamma_{xz}$ ) strains. They characterize the strain state at a point.

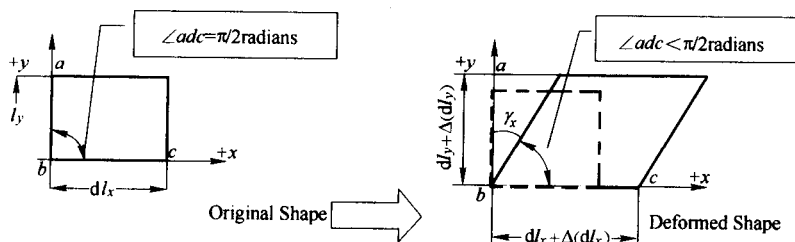


Figure1-6 Two-dimensional strained element in initial and final positions

The normal and the shear strains together define strain tensor, which is analogous to the stress tensor. The strain tensor measures the change of distances between close points in the deformed state with respect to the distances in the undeformed state. The strain tensor is symmetric and can be written as

$$T_\varepsilon = \begin{vmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \varepsilon_z \end{vmatrix} \quad (1-24)$$