

Textbooks for University

# **Fundamentals of Advanced Mathematics (I)**

Zhien Ma Miansen Wang

Fred Brauer



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# 高等数学基础( I )

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# **Fundamentals of Advanced Mathematics( I )**

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## About the book

The aim of this book is to meet the requirements of teaching Calculus in English or in bilingual education according to the customs of teaching and the present domestic conditions. It is divided into two volumes. The first volume contains Calculus of single variable, simple differential equations, infinite series, and the second volume contains the rest.

The selection of the contents is in accordance with the fundamental requirements of teaching issued by the Ministry of Education of China, and is based on the accomplishments of reform in teaching during the past ten years. The arrangement and explanation of the main contents in this book are approximately the same as the published Chinese version with the same title and edited in chief by the first two authors. It may help readers to understand the mathematics and to improve the level of their English by reading one of them and using the other one as a reference.

This book may be used as a textbook for undergraduate students in the science and engineering schools whose majors are not mathematics, and may also be suitable to the readers at the same level.

## Preface

In order to improve the English level of students in China and to make use of successful teaching experiences in Western countries, universities in China have begun to use bilingual teaching in classrooms. To accommodate this, English language textbooks are needed. However, course contents and teaching methods are quite different in China and North America. Although many good English language calculus textbooks are available, none seem to meet the teaching requirements set forth by the Ministry of Education of China. In order to meet the needs of Chinese students we have written this book.

The main subject of this book is calculus. In addition the book contains vector algebra and an introduction to matrices, spatial analytic geometry and ordinary differential equations. The book is divided into two volumes, the first of which contains calculus of functions of a single variable, elementary differential equations and infinite series. The remaining topics will be covered in the second volume.

The selection of the contents of the book is based on the fundamental requirements of teaching set forth by the Ministry of Education in China. In writing this book we have used the experience gained during the teaching reforms of the last decade. We hope that readers who study this book will acquire some mathematical knowledge and form a base for study of more mathematics in the future. In addition, we hope that readers will develop an appreciation for the applications of mathematics in other sciences.

We have attempted to give this book the following characteristics:

1. The book should have an appropriately theoretical basis. It should use the language, terminology and symbols of modern mathematics so that readers are able to gain the necessary nurturing and training in abstractions, logical and rigour, and to improve their preparation for the study of more advanced mathematics. A few portions indicated by the symbol “\*” are not part of the fundamental requirements and are

intended for readers who may be interested in learning some additional material.

2. Attention should be paid to the combination of Analysis, Algebra and Geometry. In this book, we apply vectors and matrices in multivariable calculus and differential equations, particularly in the study of line integrals of the second type and in the treatment of surface integrals combined with the theory of vector fields. In this way, we can better satisfy the requirements for the applications of mathematics in modern science and engineering. This will be beneficial in accustoming students to utilize vectors and matrices in later courses.
3. It is important to enhance training in the applications of mathematics. We do our best to bring to light the essentials of important mathematical concepts and methods, and to emphasize those ideas in mathematics which have important applications to problems in science and engineering. These include, for example, the idea of local linearization, the idea of approximation, the idea of optimization, the method of elements in integration, and transformations. Besides some practical examples in the fields of geometry and physics, we have added many other examples and exercises in the fields of engineering, life sciences, economics, medicine, and even daily life. We hope that this will enhance the attractiveness of the book to readers with particular interests in applied problems.
4. In order to keep the amount of material in the course reasonable it is necessary to decrease the coverage of some less important topics and to decrease the emphasis placed on some operational skills.

The arrangement and development of the main contents of this book are almost the same as the published Chinese textbook "Fundamentals of advanced mathematics" edited in chief by the first two authors. Reading one of them and using the other one as a reference may help readers to understand the mathematical contents and to improve the level of their English.

The initial manuscript of this book has been written by Zhien Ma and Miansen Wang, with some suggestions from Fred Brauer, and then revised by Fred Brauer with particular attention to English language.

The authors express their thanks to Xi'an Jiaotong University and the Higher Education Press for the foundation support, thanks to Dr. Juan Zhang



for help in putting the manuscript in order, and also thanks to the editor, Da-ying Dong, for touching up the book very meticulously.

The cooperation between Chinese and a foreigner to write a calculus book in English is our first attempt. In such a first attempt it is impossible to avoid some errors and unclear explanations. We would appreciate any constructive criticisms and corrections from readers.

Authors

## **Advice to students**

In order to learn calculus, it is not enough to read the textbook as if it were a newspaper. Learning requires careful reading, working through examples step by step, and solving many problems. Solving problems requires more than imitation of examples. It is necessary to think about what the problem really asks and to develop a method for that particular problem.

If something is still not clear after you have tried to understand it, you should ask a classmate, a more advanced student, or your teacher. If a classmate asks you a question, you may learn a great deal yourself from explaining the answer.

Authors

# Contents

<b>Introduction</b>	<b>1</b>
<b>Chapter 1 Theoretical Basis of Calculus</b>	<b>9</b>
1.1 Sets and Functions	9
1.1.1 Sets and their operations	9
1.1.2 Concepts of mappings and functions	13
1.1.3 Composition of mappings and composition of functions	18
1.1.4 Inverse mappings and inverse functions	20
1.1.5 Elementary functions and hyperbolic functions	22
1.1.6 Some examples for modelling of functions in practical problems	24
Exercises 1.1	27
1.2 Limit of Sequence	30
1.2.1 Concept of limit of a sequence	31
1.2.2 Conditions for convergence of a sequence	37
1.2.3 Rules of operations on convergent sequences	43
Exercises 1.2	50
1.3 Limit of Function	53
1.3.1 The concept of limit of a function	53
1.3.2 The properties and operation rules of functional limits	62
1.3.3 Two important limits	66
Exercises 1.3	70
1.4 Infinitesimal and Infinite Quantities	72
1.4.1 Infinitesimal quantities and their order	72
1.4.2 Equivalence transformations of infinitesimals	76
1.4.3 Infinite quantities	77

II	Contents
Exercises 1.4	78
1.5 Continuous Functions	80
1.5.1 The concept of continuous function and classification of discontinuous points	80
1.5.2 Operations on continuous functions and the continuity of elementary functions	85
1.5.3 Properties of continuous functions on a closed interval	88
Exercises 1.5	92
<b>Chapter 2 The Differential Calculus and Its Applications</b>	95
2.1 Concept of Derivatives	95
2.1.1 Definition of derivatives	95
2.1.2 Relationship between derivability and continuity	105
2.1.3 Some examples of derivative problems in science and technology	105
Exercises 2.1	109
2.2 Fundamental Derivation Rules	112
2.2.1 Derivation rules for sum, difference, product and quotient of functions	113
2.2.2 Derivation rule for composite functions	114
2.2.3 The derivative of an inverse function	117
2.2.4 Higher-order derivatives	119
Exercises 2.2	121
2.3 Derivation of Implicit Functions and Functions Defined by Parametric Equations	124
2.3.1 Method of derivation of implicit functions	124
2.3.2 Method of derivation of a function defined by parametric equations	125
2.3.3 Related rates of change	129
Exercises 2.3	133
2.4 The Differential	135
2.4.1 Concept of the differential	135
2.4.2 Geometric meaning of the differential	137
2.4.3 Rules of operations on differentials	138

2.4.4 Application of the differential in approximate computation	140
Exercises 2.4	141
2.5 The Mean Value Theorem in Differential Calculus and L'Hospital's Rules	143
2.5.1 Mean value theorems in differential calculus	144
2.5.2 L'Hospital's rules	152
Exercises 2.5	158
2.6 Taylor's Theorem and Its Applications	161
2.6.1 Taylor's theorem	162
2.6.2 Maclaurin formulae for some elementary functions	166
2.6.3 Some applications of Taylor's theorem	167
Exercises 2.6	170
2.7 Study of Properties of Functions	171
2.7.1 Monotonicity of functions	172
2.7.2 Extreme values of functions	174
2.7.3 Global maxima and minima	178
2.7.4 Convexity of functions	182
Exercises 2.7	187
Synthetic exercises	191
<b>Chapter 3 The Integral Calculus and Its Applications</b>	<b>193</b>
3.1 Concept and Properties of Definite Integrals	193
3.1.1 Examples of definite integral problems	193
3.1.2 The definition of definite integral	196
3.1.3 Properties of definite integrals	200
Exercises 3.1	203
3.2 The Newton-Leibniz Formula and the Fundamental Theorems of Calculus	206
3.2.1 Newton-Leibniz formula	206
3.2.2 Fundamental theorems of Calculus	208
Exercises 3.2	212
3.3 Indefinite Integrals and Integration	214

IV	Contents
3.3.1 Indefinite integrals	214
3.3.2 Integration by substitutions	217
3.3.3 Integration by parts	226
3.3.4 Quadrature problems for elementary fundamental functions	232
Exercises 3.3	233
3.4 Applications of Definite Integrals	236
3.4.1 Method of elements for setting up integral representations	236
3.4.2 Some examples on the applications of the definite integral in geometry	238
3.4.3 Some examples of applications of the definite integral in physics	243
Exercises 3.4	247
3.5 Some Types of Simple Differential Equations	251
3.5.1 Some fundamental concepts	251
3.5.2 First order differential equations with variables separable	256
3.5.3 Linear equations of first order	258
3.5.4 Equations of first order solvable by transformations of variables	262
3.5.5 Differential equations of second order solvable by reduced order methods	266
3.5.6 Some examples of application of differential equations	269
Exercises 3.5	277
3.6 Improper Integrals	280
3.6.1 Integration on an infinite interval	280
3.6.2 Integrals of unbounded functions	284
Exercises 3.6	288
<b>Chapter 4 Infinite Series</b>	291
4.1 Series of Constant Terms	291
4.1.1 Concepts and properties of series with constant terms	291
4.1.2 Convergence tests for series of positive terms	297
4.1.3 Series with variation of signs and tests for convergence	303
Exercises 4.1	308
4.2 Power Series	312



Contents	V
4.2.1 Concepts of series of functions	312
4.2.2 Convergence of power series and operations on power series	313
4.2.3 Expansion of functions in power series	320
4.2.4 Some examples of applications of power series	327
* 4.2.5 Uniform convergence of series of functions	330
Exercises 4.2	339
 4.3 Fourier Series	 343
4.3.1 Periodic functions and trigonometric series	343
4.3.2 Orthogonality of the system of trigonometric functions and Fourier series	344
4.3.3 Fourier expansions of periodic functions	346
4.3.4 Fourier expansion of functions defined on the interval $[0, l]$	353
4.3.5 Complex form of Fourier series	356
Exercises 4.3	358
Synthetic exercises	360
 <b>Appendix Answers and Hints for Exercises</b>	 361

# Introduction

“Fundamentals of advanced mathematics” is the first university mathematics course, and is also one of the most important courses for students in universities whose emphasis is on science and technology. Before we begin this course, let us recall very briefly the history of the evolution of mathematics.

The evolution of mathematics may be partitioned into three time periods:

The first period is from ancient Greek times (500 BC. to 300 BC) until the middle of the seventeenth century AD. During this long period, because of low social productivity, humans had limited understanding of nature and the universe. The objects of mathematical research were constants and simple regular geometric forms. This period may be described as the constant variable or elementary mathematics period. For example, the elementary algebra and geometry we learned in middle school were developed during this period.

The second period is from the year 1637, when the French mathematician Descartes invented analytic geometry, until the end of the nineteenth century AD. During the industrial revolution, human productivity improved significantly, and also stimulated development of many fields important for business and commerce, such as design of machinery, shipbuilding, mining, seafaring, and railway constructions. These new businesses drove the development of physics, mechanics, and astronomy, so that people achieved a deeper understanding of nature. There was a great demand for new mathematical tools to study the physical laws of moving objects. During this period Newton and Leibniz, working independently and from different perspectives, discovered calculus. Since then, the mathematical sciences have developed dramatically. Now we may speak of three main branches, namely ad-

vanced algebra, advanced geometry, and mathematical analysis. These main branches, together with some offshoots, are called advanced mathematics. During this period the objects studied in mathematics (including calculus) were variables and complex irregular geometric forms. In addition, because of the introduction of the rectangular coordinate system by Descartes, numbers and forms were closely linked together. Every point  $P$  in a plane can be represented by an ordered number pair  $(x, y)$ , and the curves in a plane can be represented by algebraic equations. Therefore, this time period may also be regarded as the variable or advanced mathematics period. Much of the material we learn in mathematics courses in college or university was developed during this period, although there have been great modifications during the modern period.

During the third period, from the end of the nineteenth century until now, mathematics has been developing continuously. This is regarded as the modern mathematics period. In the modern period the forms of expression of the objects being studied are more abstract. There has been a great leap in human understanding of nature and the universe, and this is a milestone in the development of the mathematical sciences. As the objects of study change, so do the study methodologies. Elementary mathematics uses mainly formal logic, and was developed in a static and solitary way, while advanced mathematics may use different methodologies.

A very important branch of advanced mathematics is calculus which is one of the greatest human achievements of the seventeenth century. Since then, calculus has shown its great power in the study of various fields such as mathematics, physics, engineering, life sciences, and even the social sciences. Calculus is the main content in this text.

But what is the main object of calculus? What are the main ideas and fundamental methods of calculus? What is the main difference between calculus and the elementary mathematics we learned in high school? We will uncover the veil of mystery of calculus to peep at its features through the following two classical problems.

**Problem 1** The problem of the instantaneous velocity of a motion with varying velocity along a straight line.

Given the variation law  $s = s(t) (a \leq t \leq b)$  where  $s$  is the displacement of a moving object on a straight line with varying velocity, with time  $t$ , the