

The theory of linear induction machinery

MICHEL POLOUJADOFF

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Foreword by Professor Louis Néel

Although one can date the concept, albeit in embryo form, back to the end of the last century, linear induction motors have not really been studied until the last thirty years. Since then researches in this domain have continued to gain momentum, encouraged by the importance of applications in the domain of transport, notably for aerodynamically, magnetically or electrodynamically suspended vehicles. Thus Professor Poloujadoff's monograph 'The theory of linear induction machinery' is timely.

The author has for twenty years taught the different branches of electrical engineering in engineering schools and notably at the Institut National Polytechnique de Grenoble, where he is Professor. He is therefore well qualified to compile this work. We have already had two textbooks from him: 'Treatise on electrical energy' and 'Electromechanical conversion', remarkable for their clarity of exposition. But Professor Poloujadoff is not only a teacher: he has also made important personal contributions to the theory of all kinds of electrical machinery, particularly asynchronous motors, printed circuit motors, and hysteresis motors. Since 1963 he has been interested in linear induction motors and has developed an analytical theory of 'return paths' in which he assumes that in a well designed motor the current lines become practically rectangular. After this he has developed numerical methods to calculate the real shape of the current sections.

The work presented here expounds and compares the analytical methods recommended by different investigators and among them those of the author. These methods are distinguished by three main features: the number of spatial components (one, two, or three) taken account of for each vector, the eventual replacement of the real structure of the motor by a theoretical equivalent structure, and finally the mathematical methods of solving the equations. By the

different combinations of these three features, one can arrive at the original methods that have been developed here. It is interesting to note besides that theories as different as the one-dimensional theory with return paths, the two-dimensional theory in the x - y plane, or in the x - z plane, lead to practically the same characteristics for the entry effect.

The author, very at ease in handling approximations, has also taken care to simplify the results of complete analysis in order to compare them with the results of simplified analysis. It is thus easier to compare the merits of three-dimensional with those of two-dimensional analysis. This work is, furthermore, rich in personal comments which give it a certain individuality. One will naturally find a treatment of the effects of braking as well as the application, under certain conditions, of the methods that have been explained to problems of magneto-hydrodynamics and the functioning of liquid metal pumps.

The work concludes with an excellent chapter on the present state of research and investigations on linear motors, followed by a very full bibliography comprising over 200 references.

I believe that this monograph will interest all specialists in electrical engineering and give great service to students, researchers, and engineers.

Professor Louis Néel,
Membre de l'Institut de France,
Foreign Member Royal Society,
Nobel Laureate,
Honorary President of the Institut National Polytechnique
de Grenoble.

Preface and Acknowledgements

The first nine chapters of this book were originally written during the academic year 1975-76. It is intended to present, in a logical order, most of the theories which have been published in the literature, and to comment on their similarities and differences. However, many material incidents delayed the preparation of the text which was ready only in 1978. Moreover, the publisher asked for the addition of chapter 10, which is intended to give an idea of the main projects, completed or nearly completed, at the time of writing; the writing of this chapter took very long and took place mainly in the academic year 1977-78.

I wish to thank all the authors which have contributed to this very interesting subject; I am very happy to have met most of them.

Indeed, it is a great pleasure to acknowledge more particularly the friendship or help of the following people:

Y. Pelenc and E. Remy are the two people who asked me to begin my first investigations in this domain.

Ph. Reyx, J.C. Sabonnadiere, M. Ivanès, E. Pillet, and B. De Fleury, have written several papers with me; J.L. Fanjeau has worked with me on the subject.

Y. Baudon, H. Bolton, M. Iwamoto, M. Jufer, M. Kant, S. Nonaka, K. Oberretl, and H. Weh have kindly given the authorization to reproduce some of their own results.

R. Pacaut provided the results of the experiments that he made under the direction of M. Ivanès.

E.R. Laithwaite and S. Yamamura invited me to visit their laboratories, and helped me to contact many people in their countries.

A special note is to be made of the various people who helped me to write chapter 10, and particularly Mr. Y. Kyotani (Japanese National Railways), MM. J.L. Giovachinni and J.P. Pascal (Institut de recherche des transports, Arceuil, France), M. Guarino (Department of transportation, Washington, U.S.A.), Y. Hosoda (Sumitomo Electric Industry Ltd, Osaka, Japan), and Professors G. Slemon (Toronto University, Canada), H. Weh (University of Braunschweig, Fed. Rep. of Germany), and S. Nonaka (Kyushy University, Japan).

M. Poloujadoff
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List of Symbols

B_0	$\mu_0 J_m / kg$ maximum primary field for an infinitely long motor (teslas)
\bar{B}_0	$-iB_0$
c	secondary thickness
F	thrust
F_0	$0.5 B_0 J_m l p \lambda$ (see F_m) (newtons)
F_m	$0.5 F_0$ maximum thrust of a constant current-fed, infinitely long motor, for p pole pairs
f	supply frequency (Hz)
G	goodness factor
$G_0(\xi)$	Fourier transform of $\bar{J}_0(x)$
g	air gap length
i	$\sqrt{-1}$
J_m	maximum primary current density (A/m)
$\bar{J}_m(x)$	actual primary current density of the limited length motor, expressed in terms of a complex function of x
\bar{k}^2	$= k^2 + i s w \mu \mu_0 / \rho_c$: defined pp. 157 and 168 for the study in the x - z plane
\bar{k}	separation constant used in section 6.2 for the study in the x - y plane (see p.119)
k	$= 2\pi / \lambda$ wave number
l	primary stack length
l'	secondary width
L	motor length (often equal to $p\lambda$)
p	number of pole pairs (or number of wavelengths)
R_1	primary resistance
R_m, R'_m, R''_m, R'''_m	magnetic Reynold's numbers
s	$= 1 - V/V_s$ slip

- V actual speed of the motor (m/s)
- $V_s = f\lambda$ synchronous speed (m/s)
- X_m magnetizing reactance of the primary; its value is given by equations (9.15a) and (9.15b) where it has been called either X_{ms} (approximation for a small air gap) or X_{m1} (exact X_{ms} value for a large air gap)
- X_{m1}, X_{ms} , see X_m
- x_f primary leakage reactance
- x, y, z coordinates (see Reference frame in the subject index)
- $\alpha = \mu_0 c / \rho g$: a constant used in the one-dimensional and two-dimensional ($x-y$) theories
- α_c same as α , if ρ is replaced by ρ_c
- $\bar{\gamma}^2 = k^2 + is\omega\alpha$ a constant defined for the study in the $x-y$ plane (p.85)
- γ^2, γ'^2 complex constants defined for the three-dimensional theory
- $\epsilon = g/2$: half the air gap; this symbol is used only for the study in the $x-z$ plane
- γ_0^2 defined on p.17
- ψ an angle defined by $\tan \psi = s\omega\alpha/k^2$ in Chapter 5 (see p. 89), or by $\tan \psi = s\omega\alpha/(k^2 + v_0^2)$ in Chapters 2 and 4 (see p. 20)
- ρ secondary resistivity
- ρ_c see corrected resistivity in the subject index
- λ wavelength
- μ_0 vacuum permittivity
- μ relative permittivity of the secondary
- ξ Fourier variable
- $\omega = 2\pi f$ supply angular frequency

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1. Introduction

1.1. LINEAR INDUCTION MACHINERY

Linear travelling-field induction machines have been the object of intensive study for the last thirty years; some other linear induction devices have also been studied, but less extensively.

Linear induction motors are quite easy to build and to study. Their history, basic principles, and main applications are assumed to be familiar to the reader of the present book; many references related to these topics may be found in a journal article by the author (Poloujadoff 1971); however some ideas which have not been quoted in the more easily available literature are described, or at least given references in section 1.5. Linear induction motors can have various configurations: the air gap can be flat or cylindrical, and the flux can be longitudinal or transverse; most of these possibilities are described in detail by Eastham and Laithwaite (1973) and by Laithwaite (1975).

Magnetohydrodynamic generators are linear induction generators with a gaseous or liquid moving part; a huge mass of information can be found in the proceedings of the meetings organized by the International Atomic Energy Agency, and in the proceedings of several symposia on the engineering aspects of magnetohydrodynamics (abbreviated to MHD).

The scope of this book is primarily the theoretical study of the flat air-gap travelling-field induction motor or generator, such as is described in section 1.2. This covers the case of the liquid-stream MHD generator if the fluid velocity is the same for all points of the channel, and if the Hall effect is negligible. It also covers the case of some liquid-metal pumps.

The case of some braking effects is also considered in sections 3.6 and 6.6 for two reasons. First, these studies shed light on the nature of extremity effects, which play an important role in travelling-field linear induction machines; and, secondly, their theory is easily obtained as a simpli-

fication of the theories which are the main subject of the book.

1.2. GENERAL DESCRIPTION OF THE LINEAR INDUCTION MOTOR AND GENERATOR. MAIN HYPOTHESES

The ordinary double-sided motor is shown roughly in Fig.1.1(a). The ordinary liquid-metal travelling-wave MHD

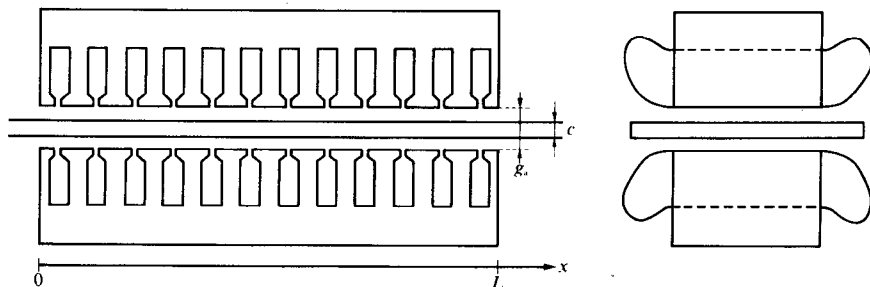


Fig.1.1. Front and left side view of a double-sided linear induction motor. In the front view, the windings are not represented.

generator is roughly represented in Fig.2.8 (see section 2.7); it is to be noted that a more complete drawing would show thermal insulation (often made of fibreglass) between the winding and the high-temperature channel.

Two slotted iron cores hold a set of windings similar to those of ordinary rotating induction motors, as explained below (Figs. 1.3 and 1.4). When fed by a system of polyphase balanced currents, these windings set up a primary gliding field in the air gap. Only the case of a long secondary is considered here. Indeed, the length of the secondary is assumed to be infinite.

The main hypotheses which are made for the study of the double-sided linear induction motor (LIM) are as follows:

- (i) The actual slotted air gap of length g_a is replaced by a fictitious unslotted one of length g ; the ratio g/g_a is the Carter coefficient, which is defined in any book on induction machines (e.g. see Alger (1965)).

- (ii) The windings on the top and bottom of the air gap are idealized into two plates of infinitely thin conductors laid along the surface of the fictitious air gap (see section 9.3).
- (iii) The iron is assumed to be laminated; therefore its conductivity is negligible. Its reluctance is also generally neglected, with the exception of section 2.8, where it is taken into account.

The single-sided linear induction motor is shown schematically in Fig.1.2. The motor of Fig.1.2(a) is just equivalent to one half of the motor of Fig.1.1, and this point is

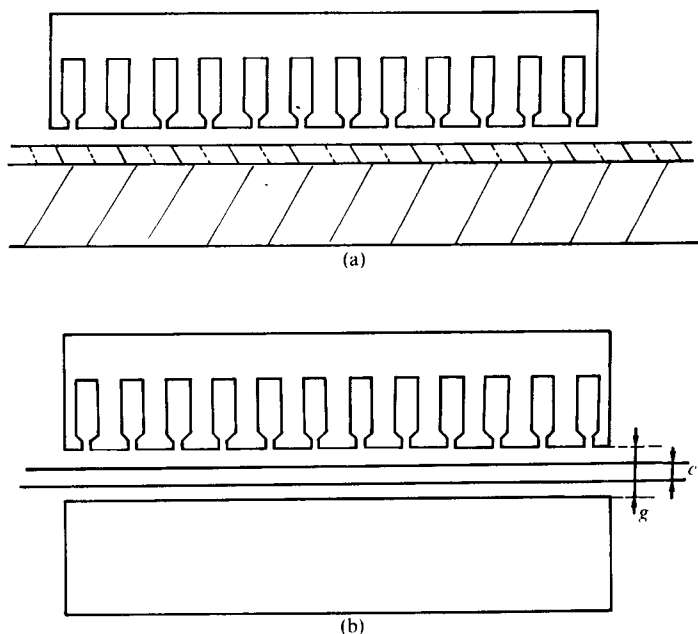


Fig.1.2. Single-sided motors.

used in many instances (for example section 7.3.1, Fig.7.17).

Therefore, it is not generally necessary to distinguish the theories of the double-sided and the single-sided motor, although their uses are different.

The *polyphase windings* in use in the linear induction machine are of two kinds: diametral single layer, or chorded double layer.

The diametral single-layer winding can be obtained by

the Gramme method, but this has the disadvantage of introducing a large primary leakage flux. It can also be obtained by the method outlined in Fig.1.3(a), which necessitates very bulky end turns, particularly in the very common case when the slots are very deep. This drawback can be reduced by the

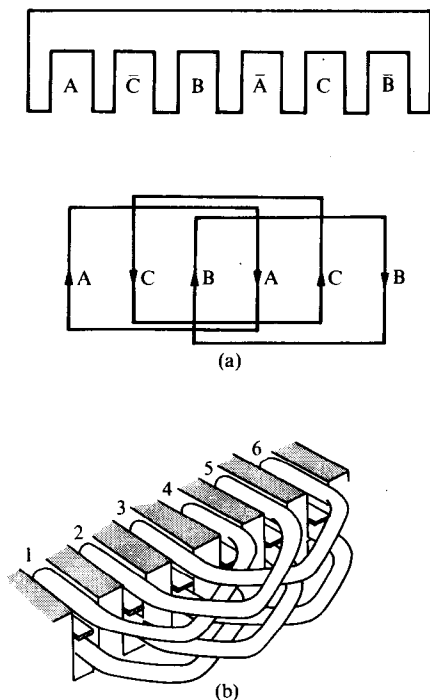


Fig.1.3.(a) Ordinary single-layer two-pole three-phase winding.(b) Superposition of two single-layer windings (From Payen 1971).

use of two single layers laid one over the other, as shown in Fig.1.3(b) (Payen 1971).

An example of a chorded double-layer winding is given in Fig.1.4. It is seen that, if the pitch is $2/3$, and if there are S slots per pole and phase, $2S$ slots have to be half filled at each extremity of the motor; an alternative is to suppress some slots at both ends (1 to 6 and 19 to 24, in the example of Fig.1.4) and let the corresponding coil sides be in the open air. The existence of half-filled slots (or of non-embedded conductors) is a drawback of this kind of winding. An example of a 60° phase belt, $2/3$ chorded, five

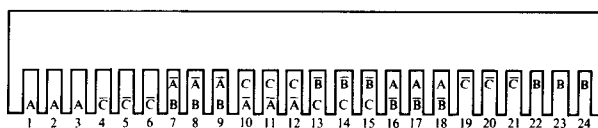


Fig.1.4. Chorded double-layer three-phase winding.

slots per pole and phase, is the five-pole pair Garret motor (Chirgwin 1973); in this case, there are 140 full slots, and 10 half-filled slots at each extremity of the motor. Other information on the corresponding experiments is to be found in Kalman (1974).

Other points of comparison between the single- and double-layer windings can be found in classical works on induction machines, for example in Chapter 3 of Alger (1965).

In the present book, it will be assumed that the winding is a single-layer one.

1.3. PURPOSE OF THE BOOK

The main purpose of this book is to present the most important methods which can be used to study the steady-state performance of the double-sided or single-sided linear induction motor or generator when they are fed by an a.c. sinusoidal supply.

Since we limit our investigations to the steady state, any component of an electric or magnetic field is specified by its phase and its amplitude, that is to say by a complex number which will be called its *value*.

Specifying the location of the points in the air gap with respect to a reference frame fixed to the primary as shown in Fig.1.5, the main theories of the linear motor can be classified as follows:

- (i) In one-dimensional theories, the values of the various electric or magnetic fields are assumed to be functions of the x -coordinate alone.
- (ii) In two-dimensional theories, the values of the various fields are assumed to be functions of only a pair of coordinates: x and y (if the analysis