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Glasshouse Crops Research Institute,
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Foreword

The application of mathematical modelling to plant physiology at the Glasshouse Crops Research Institute started in 1967–1968 with the appointment of Dr John Warren Wilson to the headship of the Plant Physiology Department. As the basis for a research programme he put forward an ambitious programme for the development of an 'idealized model of crop productivity'. The key position to implement this work was considered to be a biometrician, and it was to this post that Dr John Thornley was appointed in 1968. The modelling approach was quickly established, and in 1969, Acock, Thornley and Warren Wilson contributed a paper to a Symposium on 'Potential Crop Production' organized at the Welsh Plant Breeding Station. Their contribution was entitled 'Photosynthesis and Energy Conversion'. Another leading group of crop modellers from Wageningen, de Wit, Brouwer and de Vries, described a 'Dynamic Model of Plant and Crop Growth' at the same meeting, and both groups have provided a continuing stimulus to crop model building ever since.

It was Dr Thornley who first conceived the idea of holding regular meetings of ARC Crop Science Model Builders, and this led to the inaugural meeting in April 1971. The meetings continued annually at ARC headquarters until 1975 when they were transferred to The Royal Society. Many of those who were present at the earlier meetings have remained in the group, and I think it encouraging that crop model builders have established such strong bonds of collaborative research, which embrace scientists from research institutes and from university departments both in this country and overseas. The topics have been varied, although with an understandable emphasis on photosynthesis and productivity, but there have also been papers concerned with soil science, particularly the uptake of nutrients and fertilizer response, and with morphogenesis and developmental studies.

It is timely to recall the approach that Warren Wilson put forward in arguing the case for the development of a mathematical model that would describe quantitatively the processes involved in crop production. The model was to be based on traditional methods of growth analysis. These were to be progressively refined by incorporating quantitative treatments or 'sub-models' of the component processes of growth and development which could then be integrated into a general model. Once suitable data had been collected for one crop in a particular set of environmental conditions, the model was to be used to predict the course of crop production.

Five sub-models were originally proposed:

(1) *Light interception*—which was Warren Wilson's own particular interest;

(2) *Formation of assimilate*—Dry matter production and CO_2 uptake, the definition of which owed a great deal to the work of Monteith and his colleagues on crop photosynthesis;

(3) *Translocation of assimilate*—Source-sink relations and CO_2 enrichment. This sub-model was to be developed to examine translocation and partitioning, and to calculate the effects on productivity of changes in photosynthetic rate caused by CO_2 enrichment;

(4) *Utilization of assimilate*—A model relating carbohydrate levels to morphogenetic changes; this was to be of particular importance in the tomato crop since it could be expected to yield information on critical levels of carbon assimilation affecting truss development and flower abortion;

(5) The last sub-model was essentially a development of the fourth in that it sought to determine the optimal sizes of sources and sinks at various stages of growth and development. This sub-model was again of particular significance in the tomato, where fruit number and fruit size are affected by the amount of assimilate.

In the twelve years that have passed since Dr Warren Wilson's appointment a great deal has been achieved in crop modelling at the Glasshouse Crops Research Institute and at other research institutes and universities. Warren Wilson's confidence in his programme has been amply vindicated, as has the more cautious optimism of his supporters. Not only have a number of sub-models describing various aspects of photosynthesis and crop production been developed which are of fundamental interest, but other models with considerable practical importance in their application have also been devised.

Crop modelling is now established as a valuable research tool in delineating the constraints, and exploring the opportunities, of increased productivity in crop plants. To the researcher it has obvious attractions in enabling him to be more critical in the selection and design of his experimental programme—an aspect of vital importance at a time of increasing costs for experiments in glasshouses or controlled environment cabinets.

It was a special pleasure for me to welcome the ARC Crop Science Model Builders' Group to the 10th Annual Meeting because the Group was started by Dr Thornley from the Glasshouse Crops Research Institute. It was also the first time that the Group had met outside London at a residential meeting, and it is gratifying that to mark the occasion the proceedings have been published.

D. RUDD-JONES
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Glasshouse Crops Research Institute

Preface

One hundred and fifty years ago a chemist wrote:

> That it should be possible so to modify and intermingle a few simple substances, and thence produce all the variety of form, colour, odour, etc. which is observable in the different families of vegetables is a phenomenon too astonishing for our comprehension. Nothing short of omnipotence could have provided such a paradise for man.

It is to be hoped that today we would take a more rational and objective approach to the description of plants and plant function. Since Samuel Parkes wrote his book, 'The Chemical Catechism', in 1824 the plant sciences have developed. This development, through the processes of information gathering (that is observing plants and plant function), information collation and then analysis, has naturally led to the construction of hypotheses about plants and plant function. In general these hypotheses were initially qualitative, that is they were non-numeric descriptions of plants and plant function. With the rapid development of statistical theory during the past sixty years there has been the associated formulation of increasing numbers of quantitative, predictive hypotheses about plants and plant function. However, these hypotheses are firmly based on the *observed* behaviour of plants, and do not derive from a conscious attempt to understand the behaviour of the physical and chemical processes underlying plant growth and development. More recently, plant scientists have become more aware of, and taken more interest in, these underlying processes. This has led to the increasing use of mathematics to construct a framework within which hypotheses about the roles of these processes as determinants of plant function can be established. In short, we have begun to construct mathematical models of plants and plant function.

Since 1971, the Agricultural Research Council, UK, has sponsored an annual meeting for British scientists interested in the applications of mathematical models to problems in the plant sciences. This informal group of scientists, the ARC Crop Science Model Builders' Group, decided that for their tenth meeting in 1980 the normal one-day, informal meeting of the Group should be extended to a more formal, two-day symposium. This volume records the contributions to that symposium.

The symposium was structured so as to distinguish between models at the cell and organ levels, and at the crop level. Contributions were also invited on the techniques employed in mathematical modelling. Papers were, in the main, invited from those who had contributed consistently to the interests of the Group since its formation in 1971. Speakers were given complete freedom of the form of presentation of their material, within the broad framework of the areas they were invited to speak on. This volume reflects that freedom. Contributions range from essays to detailed mathematical treatments of particular problems, and from wide ranging surveys of work in particular areas to more general discussions of the philosophy of mathematical modelling. We hope that these varied contributions illustrate the thesis that mathematical modelling is not a discipline apart from the plant sciences but simply an integral and necessary part of these sciences.

Time and space dictated a less than comprehensive review of current activity in mathematical modelling. For example, topics such as translocation, assimilate partitioning and stomatal dynamics were omitted from our discussions. Modelling and prediction of agricultural production for commercial purposes necessarily involves some quantitative analysis of economic factors. However, discussions of 'economic models' have been outside the scope of the Group since its inception, and were not, therefore, included in this meeting. Undoubtedly, the interlacing of biological productivity models with economic models will become of increasing interest and importance during the next ten years as our ability to predict crop productivity improves; so also will models of the biological effects of pests and diseases. Perhaps the most exciting area is the use of models to examine the biological potential of novel agricultural systems and new crops. These possible developments may provide the impetus for a second formal meeting of the Group in due course.

September 1980

D. A. Rose
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