国外数学名著系列

（影印版）31

Robert I. Soare

Recursively Enumerable Sets and Degrees

A Study of Computable Functions and Computably Generated Sets

递归可枚举集和图灵度

可计算函数与可计算生成集研究

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格（Springer）出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王　元

2005 年 12 月 3 日

To my parents Margaret and Irving,
and to my “second parents” Beryl and Lawrence

Only when one’s own cup has first
been filled with love abundantly
can one then give love to another

Preface to the Series

Perspectives in Mathematical Logic

(Edited by the Ω-group for "Mathematische Logik" of the Heidelberger Akademie der Wissenschaften)

On Perspectives. *Mathematical logic arose from a concern with the nature and the limits of rational or mathematical thought, and from a desire to systematise the modes of its expression. The pioneering investigations were diverse and largely autonomous. As time passed, and more particularly in the last two decades, interconnections between different lines of research and links with other branches of mathematics proliferated. The subject is now both rich and varied. It is the aim of the series to provide, as it were, maps of guides to this complex terrain. We shall not aim at encyclopaedic coverage; nor do we wish to prescribe, like Euclid, a definitive version of the elements of the subject. We are not committed to any particular philosophical programme. Nevertheless we have tried by critical discussion to ensure that each book represents a coherent line of thought; and that, by developing certain* themes, *it will be of greater interest than a mere assemblage of results and techniques.*

The books in the series differ in level: some are introductory, some highly specialised. They also differ in scope: some offer a wide view of an area, others present a single line of thought. Each book is, at its own level, reasonably self-contained. Although no book depends on another as prerequisite, we have encouraged authors to fit their book in with other planned volumes, sometimes deliberately seeking coverage of the same material from different points of view. We have tried to attain a reasonable degree of uniformity of notation and arrangement. However, the books in the series are written by individual authors, not by the group. Plans for books are discussed and argued about at length. Later, encouragement is given and revisions suggested. But it is the authors who do the work; if, as we hope, the series proves of value, the credit will be theirs.

History of the Ω-Group. *During 1968 the idea of an integrated series of monographs on mathematical logic was first mooted. Various discussions led to a meeting at Oberwolfach in the spring of 1969. Here the founding members of the group (R. O. Gandy, A. Levy, G. H. Müller, G. Sacks, D. S. Scott) discussed the project in earnest and decided to go ahead with it.*

Professor F. K. Schmidt and Professor Hans Hermes gave us encouragement and support. Later Hans Hermes joined the group. To begin with all was fluid. How ambitious should we be? Should we write the books ourselves? How long would it take? Plans for authorless books were promoted, savaged and scrapped. Gradually there emerged a form and a method. At the end of an infinite discussion we found our name, and that of the series. We established our centre in Heidelberg. We agreed to meet twice a year together with authors, consultants and assistants, generally in Oberwolfach. We soon found the value of collaboration: on the one hand the permanence of the founding group gave coherence to the overall plans; on the other hand the stimulus of new contributors kept the project alive and flexible. Above all, we found how intensive discussion could modify the authors' ideas and our own. Often the battle ended with a detailed plan for a better book which the author was keen to write and which would indeed contribute a perspective.

Oberwolfach, September 1975

Acknowledgements. *In starting our enterprise we essentially were relying on the personal confidence and understanding of Professor Martin Barner of the Mathematisches Forschungsinstitut Oberwolfach, Dr. Klaus Peters of Springer-Verlag and Dipl.-Ing. Penschuck of the Stiftung Volkswagenwerk. Through the Stiftung Volkswagenwerk we received a generous grant (1970-1973) as an initial help which made our existence as a working group possible.*

Since 1974 the Heidelberger Akademie der Wissenschaften (Mathematisch-Naturwissenschaftliche Klasse) has incorporated our enterprise into its general scientific program. The initiative for this step was taken by the late Professor F. K. Schmidt, and the former President of the Academy, Professor W. Doerr.

Through all the years, the Academy has supported our research project, especially our meetings and the continuous work on the Logic Bibliography, in an outstandingly generous way. We could always rely on their readiness to provide help wherever it was needed.

Assistance in many various respects was provided by Drs. U. Felgner and K. Gloede (till 1975) and Drs. D. Schmidt and H. Zeitler (till 1979). Last but not least, our indefatigable secretary Elfriede Ihrig was and is essential in running our enterprise.

We thank all those concerned.

Heidelberg, September 1982

R. O. Gandy *H. Hermes*
A. Levy *G. H. Müller*
G. Sacks *D. S. Scott*

Author's Preface

One of the fundamental contributions of mathematical logic has been the precise formulation and study of computable functions. This program received an enormous impetus in 1931 with Gödel's Incompleteness Theorem which used the notion of a primitive recursive function and led during the mid-1930's to a variety of definitions of a computable (i.e., *recursive*) function by Church, Gödel, Kleene, Post, and Turing. It was soon proved that these various definitions each gave rise to exactly the same class of mathematical functions, the class now generally accepted (according to Church's Thesis) as containing precisely those functions intuitively regarded as "effectively calculable." Informally, these are the functions which could be calculated by a modern computer if one ignores restrictions on the amount of computing time and storage capacity.

Closely associated is the notion of a computably listable (so-called *recursively enumerable* (*r.e.*)) set of numbers, namely one which can be generated by a computable procedure. Indeed the notions are, in a sense, interchangeable because one can begin the study of computable functions either: (1) with the notion of a recursive function, and can then define an r.e. set as the range of such a function on the integers; or (2) with the notion of an r.e. set, and can then define a recursive function as one whose graph is r.e. (The latter approach is sometimes preferable in generalized recursion theory.)

Thus although this book is ostensibly about r.e. sets and their degrees, it is intended more generally as an introduction to the theory of computable functions, and indeed it is intended as a replacement for the well-known book by Rogers [1967], which is now both out of date and out of print. This book will serve as an introduction for both mathematicians and computer scientists. The first four chapters cover the basic theory of computable functions and r.e. sets including the Kleene Recursion Theorem and the arithmetical hierarchy. Basic finite injury priority arguments appear in Chapter 7. Well grounded in the fundamentals, the computer scientist can then turn to computational complexity.

In his epochal address to the American Mathematical Society E. L. Post [1944] stripped away the formalism associated with the development of recursive functions in the 1930's and revealed in a clear informal style the essential properties of r.e. sets and their role in Gödel's incompleteness theorem. Recursively enumerable sets have later played an essential role in many other famous undecidability results (such as the Davis-Matijasevič-Putnam-Robinson resolution of Hilbert's tenth problem on the solution of certain Diophantine equations, or the Boone-Novikov theorem on the unsolvability of the word problem for finitely presented groups). This essential role of r.e. sets is because of: (1) the widespread occurrence of r.e. sets in many branches of mathematics; and (2) the fact that there exist r.e. sets which are not computable (i.e., not *recursive*). The first such set (constructed by Church, Rosser, and Kleene jointly) was called by Post *creative* because its existence together with the representability of all r.e. sets even in such a small fragment of mathematics as elementary number theory implied the impossibility of mechanically listing all statements true in such a fragment. Post remarked: "The conclusion is inescapable that even for such a fixed, well-defined body of mathematical propositions, *mathematical thinking is, and must remain, essentially creative*. To the writer's mind, this conclusion must inevitably result in at least a partial reversal of the entire axiomatic trend of the late nineteenth and early twentieth centuries, with a return to meaning and truth as being of the essence of mathematics."

This book represents a kind of progress report over the last forty years on the programs, ideas, and hopes expressed in Post's paper. It is intended to follow the same informal style as Post, but with full mathematical rigor. In doing so, this book is in the style of its principal predecessors on the subject: Kleene [1952a]; Rogers [1967]; and Shoenfield [1971]; to whom the author acknowledges a great debt. It differs from these predecessors in: (1) its emphasis on intuition and pictures of complicated constructions (often accompanied by suggestive terminology intended to create a diagram or image in the reader's mind); and (2) its modular approach of first describing the strategy for meeting each single requirement, and later describing the process by which these various and often conflicting strategies may be fitted together. In this way the book attempts to unveil some of the secrets of classical recursion theory whose seemingly formidable technical obstacles have tended to frighten away the novice from appreciating its considerable intrinsic beauty and elegance.

Classical recursion theory (CRT) is the study of computable functions on ω (the nonnegative integers) as opposed to generalized recursion theory (GRT) which deals with computation in certain ordinals or higher type objects. The beauty of CRT lies in the simplicity of its fundamental notions, just as a classical painting of the Renaissance is characterized by simplicity of line and composition. For example, the notion of an r.e. set as one which can be effectively listed is one of the few fundamental notions of higher mathematics which can easily be explained to the common man. The art and architecture of the Renaissance are characterized by balance, harmony, and a world on a human scale, where the human figure is not dwarfed by his surroundings. In CRT the universe is merely the natural numbers ω

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which the human mind can readily grasp and not some more abstract object. Further analogies between CRT and the classical art of the Renaissance may be found in the lectures in Bressanone, Italy (Soare [1981, §7]).

The ideas and methods of CRT (such as the priority method) have been useful not only in GRT but also in many other fields such as recursion theory on sets (so-called E-recursion), recursive model theory, the effective content of mathematics (particularly effective algebra and analysis), theoretical computer science and computational complexity, effective combinatorics (such as the extent to which classical combinatorial results like Ramsey's theorem can be effectivized), and models of formal systems such as Peano arithmetic. It is hoped that workers in these fields may find this book useful (particularly the first twelve chapters). The remainder of the book is written for the genuine devotee of recursion theory who wishes to be initiated into some of its inner mysteries.

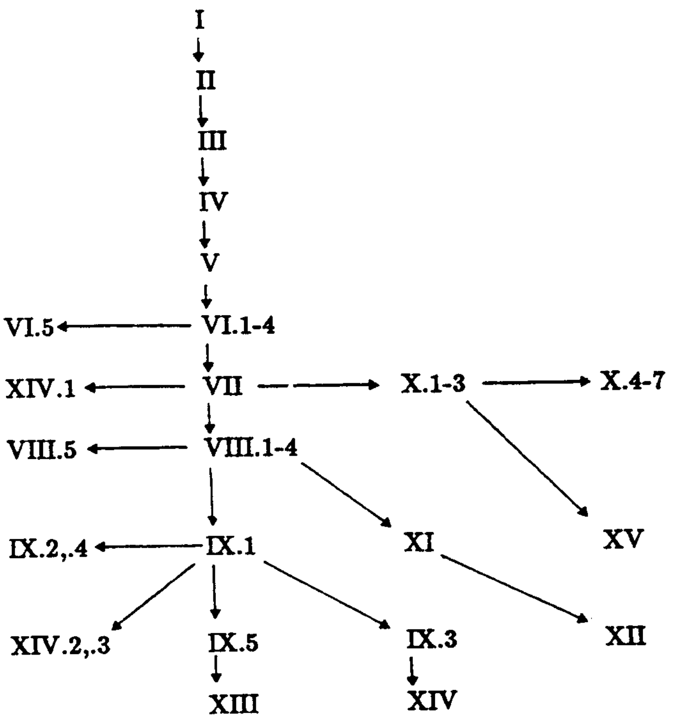
Much of the material of this book has been presented in courses and seminars at the University of Chicago, and in short courses at various international mathematical meetings, for instance: the C.I.M.E. conference in Bressanone, Italy during June, 1979 on *Recursive Theory and Computational Complexity;* the British Logic Colloquium in Leeds during August, 1979 on *Recursion Theory: Its Generalizations and Applications;* and the conference in Bielefeld, Germany during July, 1981 on the *Priority Method in Recursion Theory.* I am indebted to my Ph.D. students at the University of Chicago: Victor Bennison, Peter Fejer, Steffen Lempp, David Miller, Steven Schwarz, and Michael Stob; and to the other graduate students Kathy Edwards, Michael Mytilinaios, Nick Reingold, Francesco Ruggeri, Craig Smorynski, and Mitchell Stokes, all of whom have made the course stimulating and exciting to teach, and have contributed substantially to its present form. Special thanks are due for extensive contributions from L. Harrington, C. G. Jockusch, Jr., A. H. Lachlan, S. Lempp, M. Lerman, W. Maass, R. A. Shore, and T. Slaman. Many other mathematicians have supplied suggestions, corrections, stimulating conversations, or correspondence on the subject including among others F. Abramson, S. Ahmad, D. Alton, K. Ambos-Spies, K. Appel, M. Arslanov, M. Blum, T. Carlson, C. T. Chong, B. Cooper, J. Crossley, M. Davis, J. C. E. Dekker, A. Degtev, R. Downey, B. Dreben, R. Epstein, Y. Ershov, S. Friedman, R. O. Gandy, V. Harizanov, J. Hartmanis, L. Hay, P. Hinman, E. Herrmann, H. Hodes, S. Homer, Huang Wen Qi, I. Kalantari, E. B. Kinber, P. Kolaitis, G. Kreisel, S. Kurtz, R. Ladner, S. MacLane, A. Manaster, D. A. Martin, A. R. D. Mathias, T. McLaughlin, A. Meyer, T. Millar, A. Nerode, P. Odifreddi, J. Owings, D. Pokras, D. Posner, M. B. Pour-El, M. Ramachandran, J. Remmel, R. W. Robinson, J. Rosenstein, J. Royer, G. E. Sacks, L. Sasso, J. Shoenfield, S. Simpson, R. Smith, C. Smorynski, R. Solovay, S. Thomason, S. Wainer, Dong Ping Yang, C. E. M. Yates, and P. Young. Preliminary versions of this book were used in graduate courses by the following mathematicians and computer scientists who supplied very useful suggestions: T. Carlson and L. Harrington, University of California at Berkeley; M. Lerman, University of Connecticut; A. R. D. Mathias, Cambridge University; G. E. Sacks, Harvard University; B. Cooper, University of Leeds; L. Hay, University of Illinois at Chicago; C. G. Jockusch, Jr., University of Illinois at Champaign-Urbana; P. Hinman, University of Michigan; T. Millar, University of Wisconsin; D. Kozen, A. Nerode, and R. A. Shore, Cornell University; M. Stob, Massachusetts Institute of Technology; W. Schnyder, Purdue University; A. H. Lachlan, Simon Fraser University; and Dong Ping Yang, Institute of Software, Academica Sinica, Beijing.

The subject matter of this book includes the contributions of many fine mathematicians, but in particular the unusually innovative discoveries (in historical order) of Stephen C. Kleene, Emil Post, Clifford Spector, Richard Friedberg, A. A. Muchnik, Gerald E. Sacks, and Alistair Lachlan. The book itself reflects the enormous debt which the author owes to his mathematical forbears: Anil Nerode, his thesis advisor, who taught him not only recursion theory but also the enthusiasm and confidence so essential to mathematical success, and to his "mathematical grandfather," Saunders MacLane, whose mathematical vigor, commitment to excellence, and strength of character have deeply influenced the author since his arrival at the University of Chicago in 1975. The author is very grateful to the Heidelberger Akademie der Wissenschaften (Mathematisch-Naturwissenschaftliche Klasse) for its travel support to attend meetings of the Ω-group from 1974 to 1983 to discuss outlines and preliminary versions of the book. These meetings with the editors and other members of the Ω-group were very helpful as the author's view emerged and changed over that period. The Academy also provided support for Steffen Lempp to proofread the entire typescript. A debt of gratitude goes to the author's wife Pegeen for her patience and understanding during the preparation of the book, and for her proofreading parts of the manuscript. The author is indebted to Fred Flowers for typing the first draft of Chapters I–X, to Richard Carnes for typesetting the entire manuscript in TeX, and to Terry Brown for drawing the diagrams and for typing the bibliography.

Chicago
June 18, 1986

Robert I. Soare

Major Dependencies Diagram



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